Decomposition and Visualization of Soil Stiffness Tensors with VEES

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NEES Project Number NSF--CMS--0324661

We present a decomposition to reduce the number of components of fourth-order elastic-plastic stiffness tensors to second-order 3x3 tensors for easier visualization.

Approach

Polar decomposition, followed by eigen-decomposition on the polar "stretch".

If any resulting eigenvalue is significantly *lower* than the others, the material is less stiff in that eigen-direction.

The associated second-order eigentensor represents the mode of stress to which the material becomes most vulnerable.

Limitations

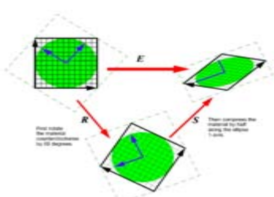
- Small Deformation Theory
- Minor symmetry in tensors
- No elastic-plastic coupling

Stage One: Polar decomposition Uniquely separates a matrix into two components:

E = RS

S : stretch (symmetric positive-semidefinite matrix)

R: rotation (orthonormal)

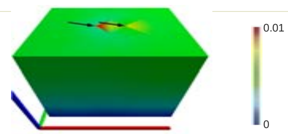


For *non-associated* materials, we conjecture the polar rotation **R** quantifies misalignment of the yield surface normal and the plastic potential surface normal.

Stage Two: Eigen-decomposition

- Reduced eigenvalue: solid is ***less stiff.***
- Visualize associated second-order eigentensor

Experiment

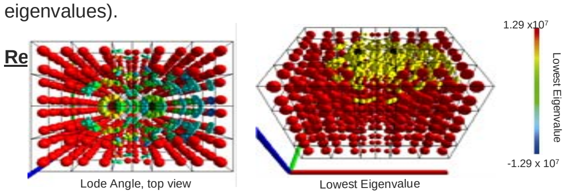


Deformation of volume at end of Stage 1 and Stage 2. Arrows indicate location of point loads and direction

- Stage 1: Self-weight compression (–Z)
- Stage 2: Two point loads; –Z component (0.9659 kN) and +X component (1294 kN)

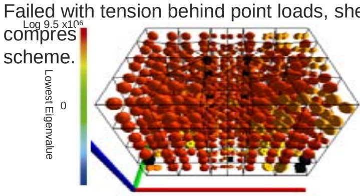
Eigentensor glyphs colored by lowest eigenvalue, scaled across all time steps. Black cubes indicate singularity (negative stiffness eigenvalues).

Re



Drucker-Prager: associated, elastic perfectly-plastic.

Failed with tension behind point loads, shear around, and triaxial compres with the Lode angle color scheme.



Dafalias and Manzari's sand model: pressure-dependent and non-associated.

Stage 1 induced hardening at the bottom, with some singularity. Stage

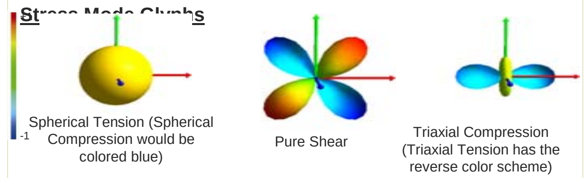
Visualization

Tensor glyphs were drawn by stretching a unit sphere according to the formula

$$\sigma_N = \sigma_{ij}\, n_i\, n_j$$

n is a unit length direction vector from the center of the sphere to a point on the surface.

Stress Mode Glyphs



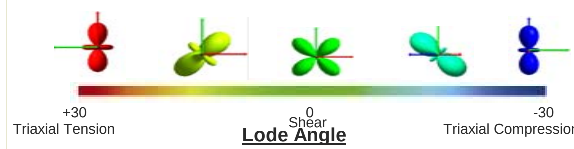
Associated Scalar Fields For Color

1.Lowest Eigenvalue: highlights loss of stiffness

2.Lode Angle: categorizes deviatoric mode for stress tensors

$$\theta = \tfrac{1}{3}\sin^{-1}\left[\left(\frac{J_3}{2}\right)\left(\frac{3}{J_2}\right)^{3/2}\right]$$

Where $J_2 = \frac{1}{2}\|S\|^2$ and $J_3 = \det S$ and *S* is the deviatoric part of the stress tensor



Lode Angle

Additional support from a GAANN fellowship, UCSC Dissertation Year fellowship, UCSC/Los Alamos Institute for Scalable Scientific Data Management (ISSDM)

For source code: aneeman@cse.ucsc.edu

Beta pprogram (without stiffness visualization)
http://neesforge.nees.org/projects/vees/