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Validating Theories for Brittle Damage

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Validating simulated predictions of internal damage within armor ceramics is preferable to simply assessing a model's ability to predict penetration depth, especially if one hopes to perform subsequent "second strike" analyses. We present the results of a study in which crack networks are seeded by using a statistically perturbed strength, the median of which is inherited from a deterministic "smeared damage" model, with adjustments to reflect experimentally established size effects. This minor alteration of an otherwise conventional damage model noticeably mitigates mesh dependencies and, at virtually no computational cost, produces far more realistic cracking patterns that are well suited for validation against X-ray computed tomography (XCT) images of internal damage patterns. For Brazilian, spall, and indentation tests, simulations share qualitative features with externally visible damage. However, the need for more stringent quantitative validation, software quality testing, and subsurface XCT validation, is emphasized.

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I. INTRODUCTION

MODELING damage and the eventual failure of brittle materials continues to be a "grand challenge" in the theoretical and computational physics community. Often, the physical and mathematical underpinnings are more complicated for brittle materials than for other engineering materials, and damage diagnostic data have not, until recently, been detailed enough to discriminate between competing models. Fundamentally different models, for example, are capable of reproducing plate slap or penetration data.[1] Higher fidelity, more detailed, experimental data, such as time-resolved measurements of penetration[2] and images of crack network morphology,[3] seem essential to deciding whether one model is better than another. Regarding the usefulness of existing models for brittle failure, an Army experimentalist and modeler commented:[2]

> "In the hands of an experienced user with a good understanding of computational mechanics and ballistics, current computational tools can be effectively used to gain insight into the effects of specific design variables on various indicators of performance including overall performance. However, our understanding of the fundamental phenomena (such as contact, penetration, fragmentation, inelastic behavior, and failure) that are encountered in a ballistic event is still limited. This has been due in part to our failure or inability to accurately or directly study these complex phenomena under relevant conditions and at the length-scales required. Consequently, accurate prediction of the performance of ceramic armors is still a challenge."

Perhaps the greatest impediment to testing the merits of brittle failure models is that none of them (including ours) is convincingly verified, much less validated. Although scholarly definitions are available (*cf.* Reference 4), the distinction between verification and validation is frequently explained as follows: verification ensures that we are solving the equations right, whereas validation ensures that we are solving the right equations. Verification is a purely mathematical and comprehensive demonstration of the well-posedness of the equations and of the accuracy of their numerical implementation (preferably relative to simplified analytical solutions, because a converged result for this class of models cannot safely be presumed to actually solve the governing equations[5,6]). Validation, which should always come after verification, assesses the physical merits of the equations by confirming that they adequately reproduce all available data using a single material parameter set. Of course, what constitutes "adequate" validation is rather subjective, in that the answer depends on the class of problems to be solved as well as on what information is sought (*e.g.*, averages or distribution). A linear elastic model, for example, might be adequate for routine service conditions, but inadequate under abnormal conditions, in which failure might occur. Therefore, any assertion that a constitutive model is "well validated" must include a clear description of the model's domain of applicability. Finally, unless there are compelling arguments to the contrary, a verification and validation process must demonstrate that the governing equations are compatible with basic physical principles, such as thermodynamics and frame indifference, even in domains in which data are unavailable.

The verification of brittle damage theories is difficult because these theories tend to be notoriously mesh dependent. As discussed by Jirásek,[7] "the numerical

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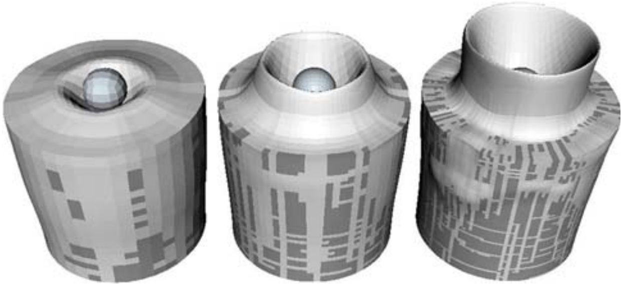

Fig. 1—Intolerable mesh sensitivity in dynamic indentation using a conventional strain-softening constitutive model (a version of Ref. 8 revised to emulate Ref. 9, which exhibited similar results). This image shows simulations at the same instant in time using three different mesh resolutions. SiC-N target cylinder: diameter 25.4 mm, height 25.4 mm; WC-6 pct Co spherical impactor: diameter 6.34 mm, velocity 500 m/s.

solution suffers by a pathological sensitivity to the finite element discretization." The simulations in Figure 1, for example, employed a conventional strain softening damage theory[8] (similar to References 9 and 10), to model a cylindrical ceramic target as it was impacted by a tungsten-carbide (WC) sphere. Not only does the simulated response fail to converge with mesh refinement, but radial cracks in this type of simulation are nonphysically tied to mesh texture. To illustrate this issue, Figure 2 sketches crack patterns typical of a deterministic model of the axisymmetric indentation of a homogeneous isotropic material. Quarter symmetry in the mesh causes quarter symmetry in the cracking patterns. This behavior contradicts the analytical prediction that the material response will be the same at all angular locations; it is nevertheless desirable from a verification perspective, since simulations of instabilities should always inherit symmetries from the perturbation sources (in this case, the perturbation source is the mesh).

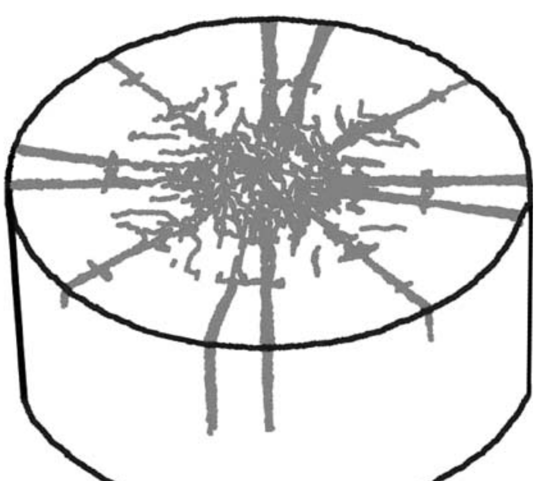

Fig. 2—Nonphysical "discretization texture bias" commonly seen in damage patterns for homogeneous deterministic models applied on a typical 1/4 symmetry cylindrical mesh. (Double cracks usually form in the coordinate directions, and, for finer resolutions, single cracks also form at 45 deg.)

Whereas linear constitutive models such as Hooke's Law are fairly insensitive to small errors in data,[11] such is not the case for highly nonlinear and inherently unstable brittle damage models. Failure to inherit response symmetries from dominant perturbation sources can indicate an order-of-operation bug. Order-of-operation differences in finite precision arithmetic (as when "$1 + x - 1$" differs from "$1 - 1 + x$") are usually benign, because these perturbations are orders of magnitude smaller than are perturbations from mesh irregularities. An intolerable and avoidable order-of-operation bug is a logical error in the solution algorithm that produces significantly different results for theoretically commutative or theoretically independent functions, depending on the order in which those operations are applied. If, for example, two elements have identical states at the beginning of a step, and if they are to be identically loaded through the step, then their updated states should be identical (at least to within numerical roundoff error, and, under certain circumstances, truly identical, since processor roundoff error is reproducible). If, on the other hand, the updated states are significantly different, beyond what can be attributed to finite-precision arithmetic errors, then an order-of-operation bug might exist since the update of the first element may have corrupted the data needed to update the second element. Symmetry testing for order-of-operation bugs is especially important for nonlocal models, which use information from neighboring elements; the material state in an element must not be updated within the data structures until the state at the beginning of the time step is no longer needed by neighboring elements.

Mesh sensitivity makes the results of deterministic damage models meaningless and, therefore, impossible to validate. Even though the model in Figure 1 was parameterized to reproduce plate impact data, it predicts far more material damage than was observed in the laboratory.[12,13] It would be unacceptable to extemporaneously alter the material strength properties to better match the indentation experiment. Doing so would make the new parameter set fail to match the plate impact data, which would therefore invalidate the model, by showing that a single parameter set cannot adequately reproduce all available data.

For verification or validation, the merits of a damage model cannot be assessed unless it is implemented in a host code that is compatible with the physics of its constitutive models. For example, default time-step control and boundary conditions are often incompatible with models that generate deformation-induced anisotropy. For large deformation problems, another serious verification issue is the tendency of remapping or advection schemes in finite-element or Eulerian codes to corrupt the integrity of internal variables. Consequently, a good model could easily yield poor results (or even *vice versa*). Figure 3, for example, illustrates that the host code's handling of state-variable advection can significantly affect results;[14] standard Eulerian state-variable tracking "smears" damage in the direction of motion, while a different tracking method better preserves the details of damage. A constitutive model (such

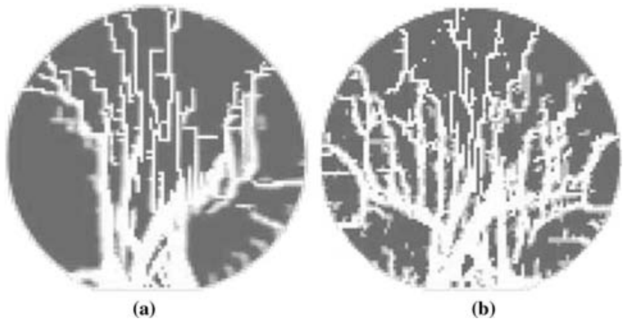

Fig. 3—Dramatic differences in predicted damage for impact of a brittle cylinder against a rigid wall. This statistical damage model[14] was run in two modes: (*a*) standard Eulerian state-variable mapping and (*b*) internal variables tied to Lagrangian tracer particles, to better preserve the integrity of the constitutive state. The conventional Eulerian scheme "streaks" damage in the direction of motion. Problem data: plane strain SiC-N cylinder: diameter 19 mm, velocity 125 m/s.

as in Reference 15) that explicitly evolves flaw size statistics tends to suffer so-called numerical healing under advection, because mixing the same material in weak and strong states produces an effective mixed state that is too strong and that is eventually almost as strong as the virgin (undamaged) material. Particle methods (*cf*. References 16 through 20) show tremendous potential for resolving these issues, because they support both arbitrarily large deformations and Lagrangian state-variable tracking. Conventional host codes (*e.g.*, finite elements with rezoning or Eulerian finite difference methods) can induce zeroth-order errors in constitutive response. When compared with conventional host codes, particle methods might solve the momentum equation somewhat less accurately. However, the overall error is likely to be significantly reduced, because the constitutive model response for particle methods cannot be corrupted by advection errors. The quantification of overall simulation error to account for constitutive errors remains an essentially unexplored avenue of research in the verification community, possibly because analytical solutions for nontrivial constitutive models are intractable except in single-element tests (in which the momentum equation plays no role).

Interestingly, high-fidelity—even low-fidelity—imaging of induced damage networks is an underutilized source of validation data that has guided our verification efforts to reduce mesh sensitivity. Figure 4, for example, illustrates the well-known (but not well-modeled) tendency of brittle materials to break symmetry in nominally axisymmetric loading by forming radial cracks,[21] and Figure 5 shows nominally-symmetry-preserving—but still not uniform—fragmentation under homogeneous loading.[22] Because radial crack spacing is fairly reproducible in the laboratory, we attribute radial cracking to statistical distributions of weak points within the material. A deterministic damage model can predict radial cracks, but the locations of the cracks usually track the mesh texture, as in Figure 2. This discretization texture bias is desirable from a verification perspective, because breaking from mesh texture in an

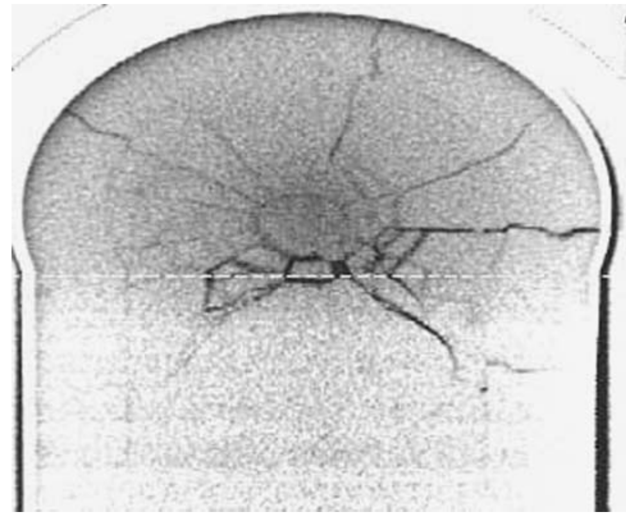

Fig. 4—Strongly-symmetry-breaking radial cracking and weakly-symmetry-preserving cone cracking revealed in XCT for axisymmetric dynamic indentation.[21] SiC-N target cylinder: diameter 25.4 mm, height 25.4 mm; sectioned ~4 mm from impact center. WC-6 pct Co spherical impactor: diameter 6.34 mm, velocity 385 m/s.

otherwise axisymmetric and deterministic simulation usually indicates an order-of-operation bug in the algorithm. However, the inability of deterministic algorithms to realistically break symmetries is a serious validation issue.

Statistical mesoscale heterogeneity in strength is responsible for well-documented size effects that must not be ignored in brittle failure: large samples have, on average, lower strength than small samples; this is not

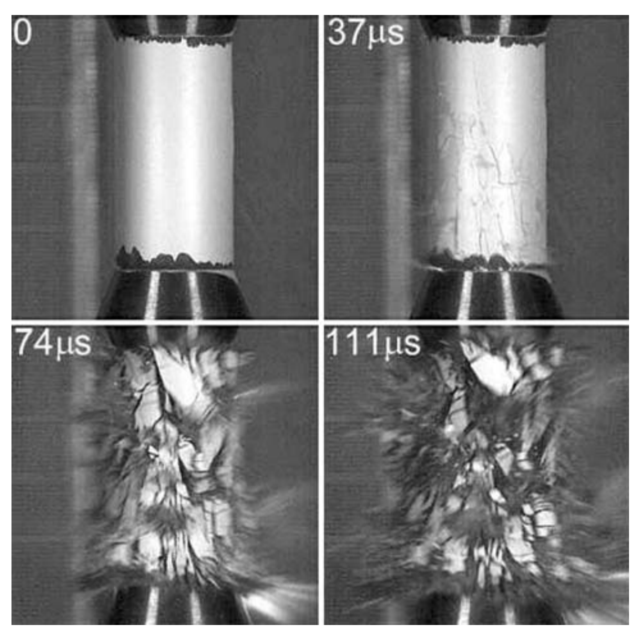

Fig. 5—Weakly-symmetry-preserving fragmentation under initially quasistatic compression of SiC-N ceramic;[22] nearly homogeneous loading, obtained by a new fixturing design and confirmed *via* redundant gage records, may allow correlating the fragment sizes with the weak tail of the strength distribution. This, in turn, may be an indicator of penetration resistance of brittle material.[23]

surprising, since large samples are more likely to contain a critically large microscopic flaw that will nucleate macroscale catastrophic failure. Uncertainty in micromorphology is also responsible for nonreproducibility in strength measurements that cannot be attributed to instrumentation uncertainty. For ceramics, the standard deviation in brittle strength (often ± 20 pct) far exceeds that of metals. These simple observations have provided the motivation for revising the code used in Figure 1 so that, as discussed in the next section, each finite element is assigned a statistical variation in strength and a size effect consistent with that observed in standard calibration experiments. It will be shown that this minor time-zero alteration of the initial state dramatically mitigates mesh sensitivity in dynamic indentation and produces radial cracking that compares favorably with observed external damage. Because the strength perturbations are computed at initialization, this solution method entails essentially no computational overhead compared to deterministic simulations. After contrasting the merits of this approach with its known inadequacies, this article concludes by pointing out the tremendous potential for X-ray computed tomography (XCT) to further validate (or invalidate) this model or any other model capable of predicting detailed spatial distributions of internal damage, which seems to be essential for subsequent second-strike analyses.

II. INCORPORATION OF ALEATORY UNCERTAINTY IN DAMAGE SIMULATIONS

In a recent experimental quest for correlations between penetration resistance and standard material properties (such as hardness, strength, grain size, *etc.*), the only property that seemed promising as a predictor for armor performance was the "weakest link" tail of the strength distribution.[23] As illustrated in Figures 6 and 7, strength data for brittle armor ceramics are statistically distributed and are potentially consistent with Weibull theory. More data points for a variety of sample geometries are needed to better quantify the size effect and to resolve the important weak tail of the distribution. Scale effects and uncertainty in the available strength data seem to be non-negligible, because including them in simulations dramatically influences the results. Similar observations of strength statistics and size effects have been made for other brittle or quasibrittle materials, such as concrete.[24] In this section, we aim to illustrate that incorporating strength statistics and scale effects in simulations favorably improves the qualitative aspects of the predictions. For these types of problems, quantitative verification and validation metrics remain a subject of debate. When statistical data are involved, however, it seems clear that one must compare not just expected values, but also other properties of the distribution, such as the standard deviation and skewness.

Under Weibull theory, which accounts for both strength variability and size effects, the probability P_s that a sample of size V is safe from failure under an applied stress σ is given by

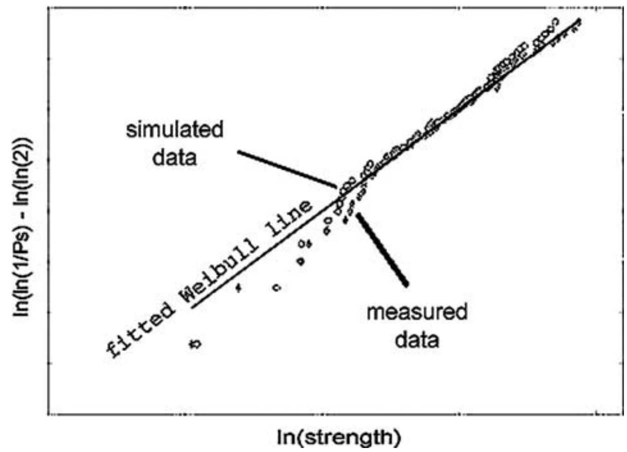

Fig. 6—Weibull diagram of 73 spall strengths[28] measured simultaneously (*i.e.*, in a single experiment on one small sample) using line-VISAR techniques[29] (numbering of axes intentionally omitted, pending separate publication of the data). The data are compared with 73 simulated exactly Weibull distributed points, to illustrate that deviations from the Weibull line are typical of finite-sampling errors.

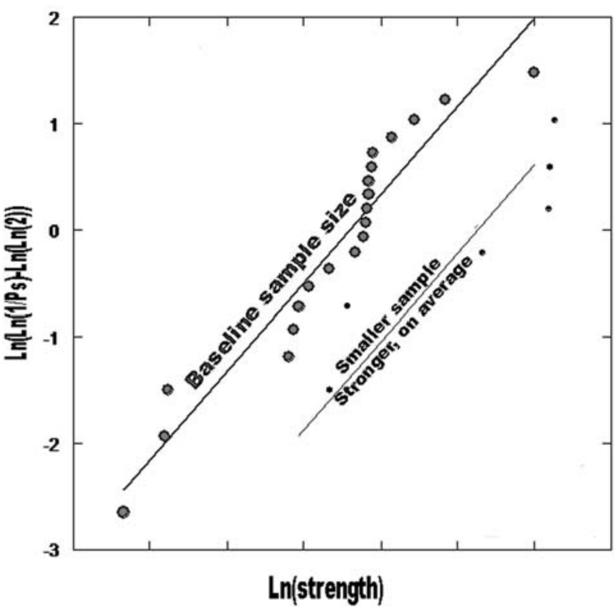

Fig. 7—Weibull plot of Brazilian strengths for many samples of two different sizes, illustrating that a small sample is, on average, stronger than a larger sample. The "stair-stepping" is a typical finite-sampling effect that diminishes as the number of experiments is increased.

$$P_s = 2^{-\frac{V}{\bar{V}}\left(\frac{\sigma}{\bar{\sigma}}\right)^m}$$

where $\bar{\sigma}$ is the median strength measured for a sample of size $\bar{V}$, and m is a constant called the Weibull modulus (*cf.* Reference 25). Small values of the Weibull modulus correspond to a large variability in strength. Strength becomes deterministic in the limit, as $m \to \infty$. Simply recognizing that flaw sizes and orientations are random will provide a microphysical basis for a distribution of this general character.[26] Because samples that are smaller than the reference size ($V < \bar{V}$) have a lower probability of failure at an applied stress σ, they have a larger median strength, but also greater variability. Note that P_s is the complementary cumulative distribution (CCD) of measured strengths. Setting values to the parameters ($\bar{\sigma}$, $\bar{V}$, and m) requires repeatability testing, using a single sample size ($V = \bar{V}$) to obtain the CCD. Then ln (ln $(1/P_s)) - \ln$ (ln (2)) *vs* $\ln(\sigma)$ is fitted to a straight line, the slope of which is the Weibull modulus m. Somewhat more accurate estimators for the Weibull modulus can be applied to well-populated data sets (30 or more data points).[27] If, as in Figure 7, the Weibull line translates toward higher strengths for smaller samples, then the need for a size effect is validated and should, therefore, be included in simulations.

Recognizing that codes treat finite elements as if homogeneously deformed, we treat each element as if it were a very small sample, and we assign it a statistically variable strength the median size of which is dependent, according to Weibull theory, so that onset-of-failure probabilities for a finite domain are preserved regardless of whether that domain is subdivided into few or many elements.[14] This spatial conservation of probability is appealing for eliminating mesh-size dependence for the onset of localized failure, while simultaneously providing a physical basis for the heterogeneity of damage in nominally homogeneous loading. For certain classes of problems, spatial conservation of failure probability also mitigates the mesh-dependence for progression of failure when used in conjunction with size effects for time to failure. As will be discussed here, however, other problems seem to require more sophisticated (correlated) seeding, nonlocal terms, and/or a non-Weibull size effect, in order to account for flaw interactions in a mesh-independent way. Size-dependent time to failure [which distinguishes our approach from other models (*e.g.*, Reference 9) that employ a non-size-dependent plastic strain to failure] regularizes the energy release rate by recognizing that a damage front traverses a small element in less time than it traverses a large element. The theory is upgraded to tensor stress states by allowing greater variability in strength at low pressures than under high confinement, again consistent with available (and limited) data for armor ceramics. In other words, rather than statistically perturbing strength itself, the parameters defining the tensor strength envelope in a deterministic damage model are perturbed statistically in a way that reproduces the strength variability observed in different laboratory tests having different tensor directionalities. As illustrated in Figures 8 and 9, incorporating these enhancements into an otherwise conventional damage model dramatically reduces mesh dependence and improves comparisons with externally visible radial cracking for that problem. Moreover, repeating the simulations with different random seeds produces similar radial cracking patterns but at random angular positions, which is an important verification of minimal discretization texture bias.

The results in Figure 8 are clearly less mesh dependent (and more similar to laboratory observations of surface damage) than those in Figure 1, but we wonder about the meaning of such phrases as "less mesh dependent" and "more similar to." We say that the simulations in Figure 8 are less mesh dependent because the time-resolved displacement histories of the indentor at the three resolutions were closer to each other (by any norm) than they were for the simulations in Figure 1. Unlike the simulations in Figure 1, the simulations in Figure 8 predicted the rebound of the indentor, as was observed in the experiment. We say that our simulated cracking pattern compares favorably with laboratory observations, because both the simulation and the experiment have five major radial cracks (*i.e.*, extending to the boundary), and these cracks are separated unevenly in the simulations and in the experiment. (Since ours is a smeared damage model for which the locations of cracks within an element are not resolved, we define a "crack" to be a contiguous line or plane of failed elements.) As was the case in the experiments, the angular positions of the simulated radial cracks is random when the statistical model is run using different random seeds. The simulation predicts zones of isolated minor cracking (not reaching the boundary), which were also seen in higher-resolution inspections of the laboratory samples. Beyond essentially qualitative observations like these, more quantitative and objective validation metrics for crack pattern validation are needed. A sufficient number of simulations and experiments have not yet been

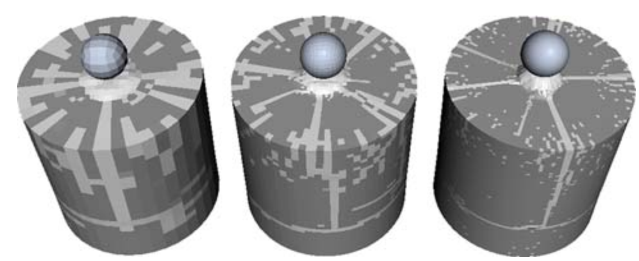

Fig. 8—Dramatic reduction in mesh sensitivity using the same model and same median properties as were used in Fig. 1, but with the strength of each element statistically perturbed about the median appropriate for the element size.

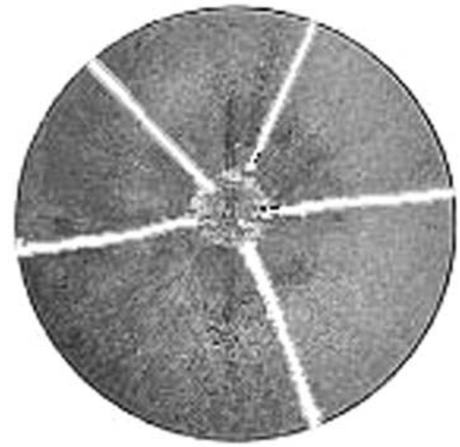

Fig. 9—The laboratory-observed crack pattern[12] (enhanced in white for clarity) corresponding to the simulation in Fig. 8.

performed to compare the mean, variance, and skewness of simulated and measured distributions. The expense of such testing cannot be justified without at least two models that are qualitatively competitive under a variety of loading conditions. To judge the "competitiveness" of existing ceramics models, greater attention to model verification is needed.

For verification, when two models are compared, we regard one model to be less mesh dependent than the other if it exhibits a "smaller change" in the displacement and damage fields (or other response functions of interest) upon a change in the mesh size and/or mesh texture. For models that cannot be compared using identical meshes (*e.g.*, when one of them uses particle methods), then the baseline for comparisons could be the central processing unit (CPU) cost. Quantitative metrics for changes in displacement or damage will be needed once damage models progress to the point at which differences between two simulations are no longer obvious to the human eye. Moreover, since damage and displacement vary considerably in each experiment and since a model such as ours is statistical, verification and validation metrics need to be applied to response *distributions*. This goal falls beyond the financial resources of most research projects. The simulations in Figures 10 and 11 might seem "qualitatively reasonable" or "encouraging" in the sense that the irregular and highly localized damage are typical of a brittle material. Plots of tracer velocities or load *vs* displacement for those simulations are also encouraging in that they are qualitatively similar in shape to what is observed in the laboratory for brittle materials. By using unacceptable "parameter tuning," those simulations could have been forced to quantitatively agree with measured velocity data or force displacement curves. However, as discussed in Section III, more aggressive assessment of those simulations reveals that verification (and, therefore, validation) issues are not yet fully resolved.[14] Specifically, distributions of simulated response functions for the simulations in Figures 10 and 11 continue to be mesh sensitive, even though the simulation of Figure 8 was relatively mesh insensitive.

III. AVENUES FOR DISCRIMINATING VALIDATION

Although the simulated damage zones in Figures 10 and 11 appear to compare well with externally visible damage in experiments, one or two qualitatively favorable results against such limited data hardly constitute a full validation of any model. Not only are simulations of other experiments called for, but more aggressive verification and more discriminating quantitative validation are needed. We have, for example, determined that our own quasistatic Brazilian simulations in Figure 11 reproduce several qualitative features, such as the strength distribution shape and the size effect trends, but these simulations currently suffer far more mesh sensitivity than was observed in the highly compressive dynamic indentation simulations in Figure 8. Until resolved, this mesh sensitivity disallows quantitative validation or calibration.

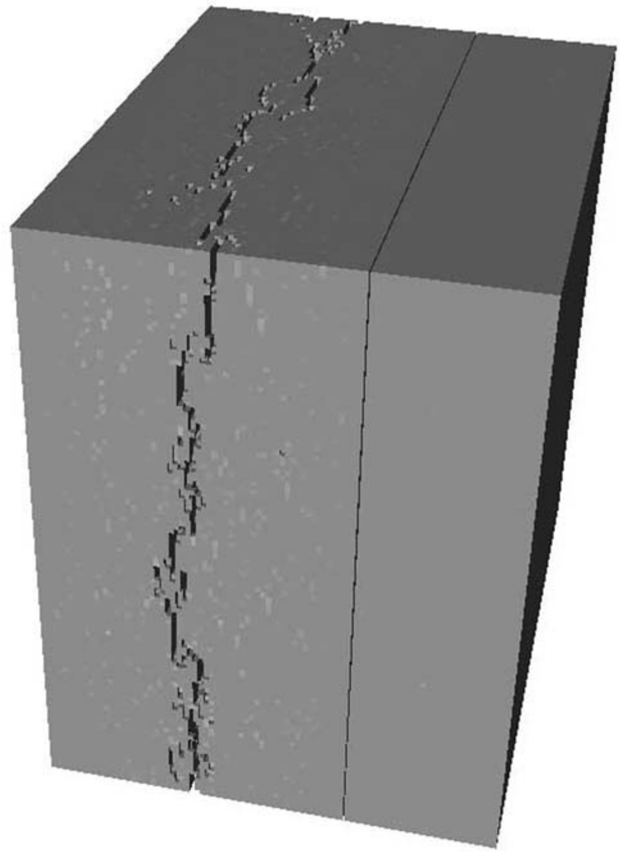

Fig. 10—Realistic-looking, uneven spall plane prediction,[14] but quantitative predictions of time-resolved statistics are not yet reproduced adequately by our model or by any other macroscale model (to our knowledge) without the tuning of material parameters away from their optimal values for other problems. Simulation data: SiC-N ceramic, target thickness 10 mm, impactor thickness 5 mm, impact speed 250 m/s.

In addition to reproducing failure distributions rather than isolated realizations, another underutilized and more demanding validation metric requires comparing predictions against time-resolved observations of

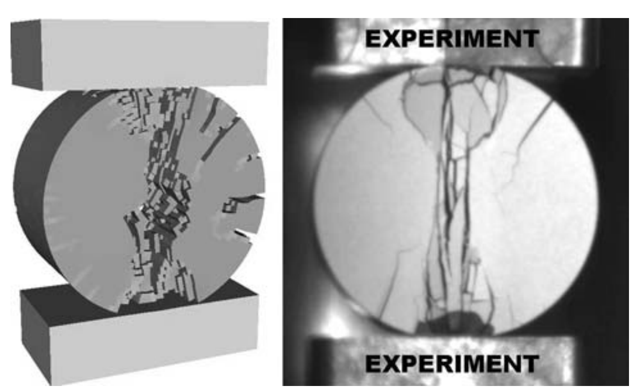

Fig. 11—Realistic-looking, uneven damage zones in Brazilian simulations[14] compare favorably with laboratory data for observable damage, but more discriminating verification and validation metrics reveal continued discretization sensitivity due to neglect of correlated flaw interactions. Simulation data: SiC-N ceramic, cylinder diameter 19 mm, cylinder thickness 9.53 mm, loading rate 100 m/s.

dynamically forming three-dimensional (3-D) damage.[2] Aside from assessing predictions of surface damage, more extensive validation of predicted 3-D internal damage also seems essential if one is interested in, for example, second-strike simulations, in which damage from a first impact must be well modeled in order to determine the ability of a sample to resist subsequent mechanical insult. Ongoing advances in XCT recently described by Wells[30] seem ideally suited for this type of validation. To be well validated, a brittle failure model designed for impact and penetration applications must be accurate for all of these tests using a single parameter set (which is a goal that has not yet, to our knowledge, been achieved and independently confirmed).

Several examples of XCT images of localized internal damage created by high-speed kinetic-energy long-rod penetrators impacting encapsulated ceramic targets can be found in a companion article (Wells and Brannon) within this issue. In addition to the various cracking morphologies frequently reported, additional damage types observed using XCT include ceramic fragmentation and embedded projectile fragmentation, intermixed regions of debris from both the projectile and the host target ceramic, and impact-induced porosity.[31] Such ballistic impact damage features are generally asymmetrically and inhomogeneously distributed within the brittle ceramic target. Clearly, these complexities of ballistic impact damage present unique challenges for the computational mechanics community that could easily take decades to address.

Another recent example of XCT images of damage is shown in Figures 12 and 13; this example illustrates that internal damage can be significantly different from surface damage.[32] Note, in particular, that damage and intact regions are layered, probably as a result of the interaction of release waves from boundaries, which makes this experiment an ideal candidate for the validation testing of damage models.

Crack separations of less than 200 to 250 microns would not be detectable at the resolution levels typically associated with meso- or macroscale XCT, which was used in the companion article (Wells and Brannon, within this issue). Smaller crack separations are, however, detectable with microfocus XCT techniques, which have an order-of-magnitude greater resolution. The scans in Figures 12 and 13, for example, were obtained using a 420-keV X-ray tube with two focal spot sizes and a 225-keV microfocus X-ray tube with a variable focal spot size down to 5 μm. Even with microfocus XCT, Bourne *et al.*[32] found it necessary to loosen the sample from its cup and remove the cover plate, in order to reveal more detail in the damage morphology (where, by quantitative measurement, radial cracking was then found to be between 60 to 90 μm in width, while lateral cracking was in the same range or larger). Loosening a sample raises the concern of introducing additional damage in the sample. Therefore, the loosening process itself should be simulated to most effectively assess the predictive ability of a damage model.

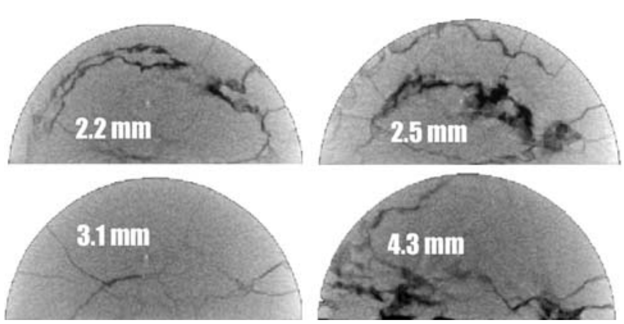

Fig. 12—XCT images of impact-induced damage on planar cross sections at indicated distances from the impact face of the target cylinder.[32] Composite alumina target ~17-mm diameter; copper impactor 500 m/s.

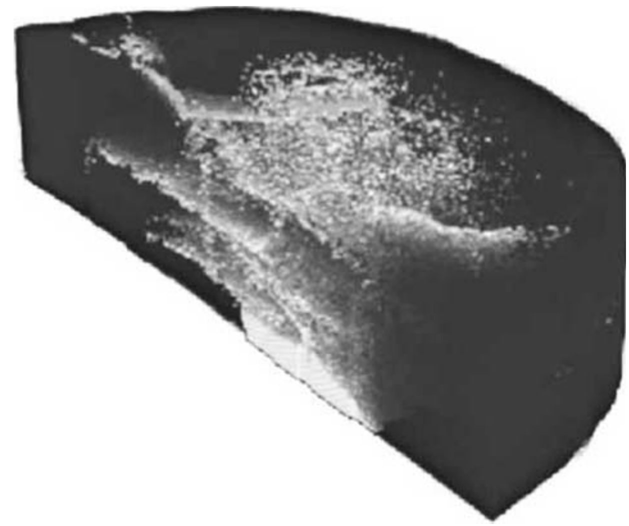

Fig. 13—Composite reconstruction[32] of 3-D damage of the sample in Fig. 12.

IV. SUMMARY AND CONCLUSIONS

Few (if any) brittle damage theories are verified well enough to be truly ready for validation. Clearer definitions of verification and validation success metrics are needed in order to determine the merits of one model as compared to another. The primary verification challenge is to eliminate mesh dependence using a single input parameter set in a suite of problems covering the application domain for the model. For armor applications, the application domain includes nonmonotonic loading in tension and compression over a broad range of loading rates and at a variety of size scales. A validation challenge is to reproduce the non-negligible uncertainty and size effects that are evident in strength data. Endowing finite elements with the same uncertainty and size effects as seen in laboratory data seems to mitigate mesh dependence in some problems but exacerbate it in others.

Even though no brittle damage model (including our own) has been sufficiently validated to be trustworthy in design contexts, the effort to develop predictive damage models remains worthwhile in light of the many situations for which brittle damage can occur in grand challenge scenarios that are too difficult, expensive, or dangerous to test directly in the laboratory. While model verification efforts proceed, experimental technology can also be improved, to offer higher-fidelity validation data that are detailed enough to discriminate between the many brittle damage models that can reproduce low-fidelity validation data, such as penetration depth. These more advanced validation experiments include time-resolved measurements of dynamically forming damage and post-mortem assessments of internal damage, using XCT damage diagnostics and similar methods. In all cases, the irreproducibility and size effects of brittle damage must be quantified *via* enough testing to obtain response distributions for a variety of sample sizes. Reproducibility testing is not only costly, but also introduces additional uncertainties (such as sample-to-sample variability, changing ambient conditions, *etc*.). Consequently, calibration of scale-dependent statistical damage models might be more tractable by using advanced diagnostics such as the line-VISAR[29] used in Figure 6, where 73 strength values were inferred from a single experiment.

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