# Advances in X-ray Computed Tomography Diagnostics of Ballistic Impact Damage

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With the relatively recent introduction of quantitative and volumetric X-ray computed tomography (XCT) applied to ballistic impact damage diagnostics, significant inroads have been made in expanding our knowledge base of the morphological variants of physical impact damage. Yet, the current state of the art in computational and simulation modeling of terminal ballistic performance remains predominantly focused on the penetration phenomenon, without detailed consideration of the physical characteristics of actual impact damage. Similarly, armor ceramic material improvements appear more focused on penetration resistance than on improved intrinsic damage tolerance and damage resistance. Basically, these approaches minimize our understanding of the potential influence that impact damage may play in the mitigation or prevention of ballistic penetration. Examples of current capabilities of XCT characterization, quantification, and visualization of complex impact damage variants are demonstrated and discussed for impacted ceramic and metallic terminal ballistic target materials. Potential benefits of incorporating such impact damage diagnostics in future ballistic computational modeling are also briefly discussed.

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# I. INTRODUCTION

THE mitigation and, where possible, the elimination of projectile penetration under various specific ballistic threat conditions would significantly improve the functionality and protective value of various armor materials and designs. However, a critical challenge is how to efficiently modify or create new materials and designs that will manifest such improved penetration resistance in either vehicular or personnel protection systems. The current extended development times and the increasing difficulty of achieving desired protection improvements with the empirical "shoot and look" approach used for many decades suggest that a more efficient and affordable approach to armor design is long overdue. The use of computational ballistic modeling to assist in the description, simulation, and, ultimately, the prediction of terminal ballistic impact performance remains a continuing technical challenge.<sup>[1-4]</sup> Anderson<sup>[5]</sup> has recently provided an excellent review of computational ceramic armor modeling. However, the predominant activities and limited successes with such modeling have been focused on the empirical penetration phenomena with minimal consideration of the details of physical impact damage in the target material, which occurs prior to and during the penetration process. Consequently, the linkage of the phenomenological penetration behavior to explicit material impact deformation and damage mechanisms has been minimal at best. This is, perhaps, not surprising since so little actual impact damage characterization data are available in detail, especially on a three-dimensional (3-D) volumetric basis.

It is suggested that renewed efforts are necessary to incorporate physical target damage into the development of numerical ballistic modeling for realistic armor performance predictions. A high-resolution volumetric damage knowledgebase consistent with both the ballistic conditions and the armor target material/architecture details is needed in order to incorporate physical damage into computational ballistic performance modeling. A significant factor required for the development of such a damage knowledgebase is the availability of a noninvasive, high-resolution damage diagnostic modality; such a modality exists in X-ray computed tomography (XCT). Exploratory XCT ballistic impact damage diagnostic capabilities have been developed over the past decade,<sup>[6–9]</sup> and the viability of this impact damage diagnostics approach has been convincingly demonstrated on several laboratory terminal ballistic targets.

The impact damage images presented here are not photographs, but rather are images reconstructed of the virtual X-ray data renderings. The multiple image processing capabilities of the voxel analysis and visualization software<sup>[10]</sup> provide dramatic visualization results not otherwise conveniently obtainable, and in some cases impossible, with conventional microscopy and photography.

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#### II. XCT DAMAGE DIAGNOSTIC MODALITY

X-ray computed tomography is a powerful 3-D noninvasive diagnostic modality that permits the interrogation of the external and internal design surfaces or damage features within the bulk of the original object. It uses the triangulation of volumetric X-ray absorption data to construct a completely digitized "density" map of a solid object, which accurately represents the dimensional and structural features of that object. The resolution level achievable with XCT depends upon both the object size and density, and the X-ray source and detector system. The nominal resolution level for the mesoscale tomography of a modest size target, e.g., a cube with a side dimension of  $\sim 25$  cm ( $\sim 1.5 \times$  $10^7 \text{ mm}^3$ ), is ~0.250 mm. Lower resolution levels are typically obtained on considerably larger objects. Greater resolution levels of  $< 20 \ \mu m$  are achievable with X-ray microtomography (XMCT) on relatively less dense or substantially smaller objects, such as a cube with side dimensions of  $\sim 25 \text{ mm}$  ( $\sim 1.5 \times$  $10^4 \text{ mm}^3$ ). If considerably higher resolution levels are still required, then destructive sectioning and highresolution electron techniques may augment the results of XCT.<sup>[11]</sup>

Once the physical target has been digitized by XCT scanning, all subsequent interrogations of that impacted target may be conducted in the virtual diagnostic domain. The original XCT scan data file is reconstructed into a virtual 3-D solid object using a commercially available advanced voxel analysis and visualization software package.<sup>[10]</sup> A variety of sophisticated image processing tools and routines are then used to create the XCT diagnostic actions, which contribute to the enhanced understanding of various physical impact damage details. This approach generally avoids the limitations of destructive sectioning, polishing, etching, and metallography. All XCT diagnostic results are repetitively reproducible, accessible, and amenable to digital archival file storage. These impact diagnostic capabilities typically include the following:

- (1) external features including radial and circular surface cracking and topological irregularities;
- (2) internal impact damage subsurface cracking morphology features;
- (3) orthogonal (axial, frontal, and sagittal) virtually sectioned views as well as arbitrary angle virtual sectioning;
- (4) impact-induced void characterizations including void sizes (typically larger than small intrinsic porosity occurring in as-manufactured bulk ceramics), spatial location, and volumetric distributions;
- (5) residual projectile fragment characterization including spatial position, size, morphology, surface area, and volume;
- (6) *in-situ* metrology including linear, area, volume, and angle dimensions and gray level differentiation; and
- (7) 3-D visualizations of all such damage features.

# III. EXAMPLES OF XCT IMPACT DAMAGE RESULTS

Several physical impact damage features have been identified with the XCT diagnostic approach in a limited number of laboratory armor material targets. It is reasonable to suspect that further impact damage manifestations may exist in cases of different combinations of armor materials, target design architectures, and ballistic impact conditions. It is important to identify and characterize such impact damage types and features in order to develop the necessary understanding of how such damage may affect the armor target material penetration resistance and overall ballistic performance. This section will present a limited but representative selection of impact damage features revealed via XCT in a monolithic Ti-6Al-4V alloy target and in various constrained armor ceramic materials. Several of the images are better viewed in both color and in 3-D stereo, details which, unfortunately, cannot be reproduced in the published article.

# A. 3-D Solid Object Reconstruction and Virtual Sectioning

The initial reconstruction of the XCT scan data file is invariably an opaque 3-D solid object, which reveals both internal and external features and defects. The external features are easily interrogated under variable magnification with the virtual rotation of the 3-D solid object. The presence and characteristics of internal damage features are not readily apparent until the target interior is probed with virtual sectioning, segmentation, and variable transparency image processing techniques. Examples of the virtual axial and sagittal planar sectioning of a  $TiB_2$  ceramic<sup>[6,12]</sup> 3-D solid object are shown in Figure 1. Several damage features are observed in this figure including a large penetration cavity, surface topological step rings with radial cracking, residual projectile fragments, and impact-induced voids (relatively large porosity). Segmentation results in Figure 2 reveal a large central interior residual heavy tungsten alloy projectile fragment and a complex intertwined cracking morphology in the virtually transparent surrounding bulk ceramic.

#### 1. Ballistic cavity development

Upon impacting a hard ceramic target, a high-speed kinetic energy projectile may experience various effects including blunting, complete destruction without significant penetration,\* or progressively penetrating and

\*This condition is frequently referred to as 100 pct dwell or interface defeat.

ultimately perforating the target ceramic. A wide shallow cavity is observed on the impact surface surrounded by wide circular rings in the above  $TiB_2$  ceramic target. The cavity is created by substantial amounts of pulverized ceramic ejected upon impact. The surface rings consist of remaining pulverized ceramic mixed with fine



Fig. 1—Reconstructed X-ray images of an opaque 3-D solid object visualizing an impacted  $TiB_2$  ceramic with virtually sectioned axial and sagittal planes, both revealing projectile fragments and impact-induced voids.



Fig. 2—Opaque 3-D solid object visualization of an impacted  $TiB_2$  ceramic with virtual transparency views of a segmented projectile fragment and complex internal cracking morphology. Image processing has resulted in the virtual transparency of the host  $TiB_2$  ceramic target.

projectile fragment debris, as evidenced by measured densities falling between that of the W-alloy projectile and the  $TiB_2$  ceramic. Cavity development further into the interior of the ceramic appears primarily filled with residual projectile material.

Another interesting example of cavity development is observed in the monolithic metallic target of Ti-6Al-4V alloy,<sup>[9]</sup> as shown in Figure 3. By filtering out the

opacity of the sample, the total through-thickness central penetration cavity is clearly visible. Of particular interest is the lack of residual projectile fragment material in the cavity, as well as the prominent spiral cracking morphology surrounding the last third of the cavity toward the exit face.

#### 2. Residual projectile fragmentation

Impacts of highly constrained laboratory ceramic targets by a high density tungsten alloy projectile at nominal velocities of ~1600 meters per second frequently result in projectile fragments embedded within the interior of the impacted ceramic. It is possible to use XCT diagnostic methods to noninvasively examine such fragments *in-situ* to determine the spatial position, dimensions, surface area, and volume of multiple residual fragments. Figures 4 and 5 present insightful visualizations for such segmented projectile fragments in separate Al<sub>2</sub>O<sub>3</sub>- and B<sub>4</sub>C- impacted ceramic targets,<sup>[13]</sup> respectively. Note the presence of distributed impactcreated voids, the continuity of the fragments in the Al<sub>2</sub>O<sub>3</sub> target, and the discontinuous separate projectile fragmentation in the B<sub>4</sub>C case.

#### B. Impact-Created Voids

The volumetric distribution of impact-induced voids of considerable size distribution has been observed with XCT in several laboratory ceramic ballistic materials following ballistic testing. A further example of such voids is shown in Figure 6. The outer dark circular ring is actually small (~0.20 mm<sup>3</sup>) voids at the interface of the  $TiB_2$  target sample with a surrounding containment ring. Many (though not all) of the voids internal to this TiB<sub>2</sub> ceramic target are observed along four broad radial bands. The lighter color areas are residual solid high-density residual projectile fragments. The total void level was measured at ~1.54 pct on a volumetric basis. Data for the spatial location, size distribution, volume, and surface area of all pores were obtainable with the defect analysis tool of the volume graphic voxel analysis software, but are not presented here.

#### IV. LINKAGE OF IMPACT DAMAGE TO COMPUTATIONAL MODELING

On February 14–15, 2006, the Army Research Office sponsored a technical workshop addressing the topic of "*The Role of Impact Damage on Armor Ceramic Performance*."<sup>[14]</sup> One issue identified at that workshop was a somewhat surprising lack of consensus of the definition of the word "damage" among the different technical specialties represented. The present authors prefer to consider impact damage as all irreversible physical changes to the target material that occur directly as a consequence of the ballistic insult. A more precise and generally accepted definition of damage is unlikely to emerge until the computational modeling, terminal ballistics testing, ceramic processing, and nondestructive evaluation communities become more familiar with the broad spectrum of physical changes in



Fig. 3—Reconstructed virtually transparent X-ray images demonstrate the ballistic cavity and spiral cracking in the lower section of the through thickness ballistic cavity in a Ti-6Al-4V target.



Fig. 4—An opaque 3-D solid object view and virtually transparent orthogonal segmented views revealing both residual projectile fragmentation and impact-induced voids *in-situ* within an  $Al_2O_3$  ballistic half cylinder target.

Projectile Fragment ID	3D Location – slice# < Sag- Frontal-Axia⊳	Bordering Dimensions, mm	Surface Area mm <sup>2</sup>	Volume mm <sup>3</sup>	9A, 10 <u>A.11A</u> 8A
P-F #1A	< \$212 F22 A84 >	14.82 x 4.51 x 19.12	416.30	264.32	74
P-F #2A	< S238 F29 A181 >	3.22 x 3.65 x 5.37	28.14	6.54	
P-F #3A	<\$252 F33 A172>	54.18 x 7.20 x 37.14	7.42	1.38	0A
P-F #4A	<\$247 F22 A326>	3.65 x 3.44 x 5.59	47.53	8.22	5 T
P-F #5A	< 5239 F22 A342>	1.50 x 0.86 x 1.29	4.17	0,60	A 4A
P-F #6A	<\$271 F22 A393>	1.07 x 0.64 x 1.72	2.73	0.32	
P-F #7A	< \$268 F22 A440 >	4.94 x 5.59 x 9.02	119.41	26.47	ZA
P-F #8A	< \$283 F22 A446>	2.15 x 1.72 x 1.93	7.52	1.13	ant 3
P-F #9A	< \$191 F22 A444>	2.36 x 1.93 x 1.50	9.13	1.27	
P-F #10A	<\$205 F33 A449>	3.01 x 1.50 x 3.01	14.38	2.37	
P-F #11A	< \$279 F26 A450>	2.15 x 1.72 x 1.93	7.35	1.08	- 100 - 1000 - 100

Fig. 5—Characterization data (left) and a transparent (right) visualization of segmented high-density residual projectile fragments in a  $B_4C$  ceramic target.

impacted target materials. The damage manifestation examples shown previously are descriptive of results for their particular laboratory impact conditions. Such damage features are anticipated to have multiple and complex variants, depending upon the specific target material/design architecture and the relevant ballistic impact conditions. Of even greater importance than the common definition(s) of damage are the following:

- (1) the identification, characterization, and analysis of significant damage morphological types under specific target/ballistic experimental conditions;
- (2) effective incorporation of such damage features into evolving computational ballistic damage modeling activities;
- (3) determination of the degradation of the structural integrity and the consequential effects of such damage details on the penetration phenomena and, ultimately, the overall ballistic performance; and
- (4) application of validated and verified ballistic damage models to guide the design and development of more damage-resistant/damage-tolerant armor materials.



Fig. 6—Porosity histogram of pore frequency with pore size (right) and a 3-D transparent visualization revealing the asymmetrical localization of porosity about a central W-alloy projectile fragment embedded within the TiB2 ceramic sample (left).



Fig. 7—Dynamic indentation of SiC-N ceramic by a tungsten carbide sphere. Left: experimentally observed impact crater and radial cracking (both highlighted for clarity). Middle: BFS model prediction of externally visible damage. Right: prediction of internal damage (suitable for validation against XCT data).

Penetration is only one form of ballistic impact damage. By using only penetration data, it is considered extremely unlikely, if not impossible, to systematically improve the damage tolerance/resistance of armor ceramics. Several different physical material impact damage manifestations have been observed both with and without projectile penetration<sup>[6-8,14-19]</sup> in a wide variety of ceramic ballistic targets. Certainly, the localized structural integrity of a ceramic is degraded with increasing impact damage. Penetration might increase as the target material's structural integrity becomes increasingly degraded. On the other hand, it is conceivable that penetration might be avoided if the projectile path is deflected by a pre-existing angled crack left behind from some earlier ballistic insult. Thus, it is quite probable that the ballistic penetration behavior is strongly influenced by the magnitude, morphology, and spatial distribution of the evolving impact damage on both the micro- and the mesoscale. By creating material compositional, microstructural, or macrostructural modifications (intrinsic or extrinsic) that would mitigate or redistribute the impact damage to benign or even protective localized concentrations, the structural integrity might remain adequate to resist not only a firststrike penetration, but also subsequent strikes as well.<sup>[7,14,15]</sup>

Axisymmetric *penetration* data invariably show dramatic deviations from axisymmetry in the form of

irregular localized *damage* (cracks, voids, and fragments). The material does not damage equally at all azimuthal angles. To the contrary, widely spaced radial cracks emanate from the impact crater, or, as illustrated in Figure 3, spiral cracking can occur internally. Conventional deterministic constitutive models (including nonlocal models) cannot reproduce such behavior. Whenever symmetry is broken in deterministic models, the predicted damage morphology is invariably tied to the underlying computational mesh texture, and it spuriously changes when the mesh is changed. To address this issue, recent modeling efforts<sup>[20,21]</sup> have incorporated realistic spatial uncertainty in strength to provide properly distributed weak points for failure initiation. This work has also included scale effects that are undeniably evident in strength testing data (i.e., small specimens are statistically stronger, but also more variable than large specimens). Imposing experimentally observed uncertainty and scale effects in the initial conditions of a simulation not only mitigates mesh dependencies, but also dramatically improves the predictive capabilities and numerical consistency of an otherwise conventional damage model. Results from such a simulation (Figure 7) are well suited for validation against XCT images.

Although this statistical modeling method is relatively immature, it seems to be promising for predicting damage morphologies with the richer spatial detail that is essential for second-strike predictions. The technique has also been shown to outperform conventional damage models in comparisons against time-resolved long rod penetration data.<sup>[20]</sup>

### V. SUMMARY AND CONCLUSIONS

Within the perspective of holistic armor ceramic materials research and development, notional approaches to improve damage tolerance may offer significant possibilities not otherwise considered explicitly with traditional penetration experiments or penetration modeling. However, before realistic notional approaches for the evolutionary or revolutionary enhancement of damage tolerance in armor ceramics can be formulated, an improved cognizance of the realistic damage processes and resulting complex physical damage modes actually occurring in various ceramics upon ballistic impact must be achieved. Nondestructive XCT diagnostic tools and techniques are evolving that permit extraordinary *in-situ* ballistic impact damage diagnostic and spatial characterization capabilities, which have previously been unavailable. These technological capabilities have been demonstrated to noninvasively interrogate and characterize actual volumetric impact damage in laboratory ballistic targets. Exploratory results include both qualitative and quantitative characterizations and visualizations of several physical impact damage manifestations in laboratory terminal ballistic targets of monolithic Ti-6Al-4V metallic and Al<sub>2</sub>O<sub>3</sub>, B<sub>4</sub>C, SiC, TiB<sub>2</sub>, and TiC ceramic materials.

Further results and improvements in these diagnostic techniques are anticipated. Such interrogative diagnostics of actual complex impact damage provide damage details beyond the current capabilities of computational ballistic modeling efforts. While much more needs to be done to construct a reasonable impact damage knowledgebase, sufficient information now exists to permit the development of the methodology for incorporating such 3-D physical damage considerations into evolving ballistic computational damage modeling. The XCT diagnostics appear essential for guiding improvements in theoretical models. Computational models must, for example, predict the breaks from symmetry in nominally axisymmetric penetration that are evident in XCT images. Doing so in a mesh-independent way requires supporting the same scale effects and (often massive) degree of uncertainty in strength that is observed in standard strength calibration testing. Many existing models can be "tuned" to match penetration depths. More discriminating validation metrics, such as matching time-resolved data and XCT damage morphology, are needed to identify which models truly support better physics.

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#### REFERENCES

- D.M. Stepp: in *Ceramic Armor Materials by Design*, J.W. McCauley, A.M. Rajendran, W.A. Gooch, Jr., S.J. Bless, S. Wax, A. Crowson, K.V. Logan, and M. Normandia, eds., *Ceram. Trans.*, ACERS, 735 Ceramic Place, Westerville, Ohio 43081, www.ceramics.org, 2002, vol. 134, pp. 421–28.
- A.M. Rajendran: in Ceramic Armor Materials by Design, J.W. McCauley, A.M. Rajendran, W.A. Gooch, Jr., S.J. Bless, S. Wax, A. Crowson, K.V. Logan, and M. Normandia, eds., Ceram. Trans., ACERS, 735 Ceramic Place, Westerville, Ohio 43081, www.ceramics.org, 2002, vol. 134, pp. 281–97.
- D.W. Templeton, T.J. Holmquist, H.W. Meyer, Jr., D.J. Grove, and B. Leavy: in *Ceramic Armor Materials by Design*, J.W. McCauley, A.M. Rajendran, W.A. Gooch, Jr., S.J. Bless, S. Wax, A. Crowson, K.V. Logan, and M. Normandia, eds., *Ceram. Trans.*, ACERS, 735 Ceramic Place, Westerville, Ohio 43081, www.ceramics.org, 2002, vol. 134, pp. 299–308.
- J.M. Winter, W.J. Bruchey, J.M. Wells, and N.L. Rupert: *Proc.* 21st Int. Symp. on Ballistics, Adelaide, Australia, 2004, ADPA, Defence Science and Technology organization, ISBN 0-9752028-0-4, 2004, vol. 1, pp. 111–17.
- 5. C.E. Anderson, Jr.: Proc. 30th Int. Conf. Advanced Ceramics & Composites—Advances in Ceramic Armor, ACERS, 735 Ceramic Place, Westerville, Ohio 43081, www.ceramics.org, 2006, vol. 27 (7), in press.
- J.M. Wells: *Proc. 22nd Int. Symp. Ballistics*, Vancouver, ADPA, DES Tech Publications, Inc., Lancaster, PA, ISBN 1-932078-52-5, 2005, vol. 1, pp. 793–800.
- H.T. Miller, W.H. Green, N.L. Rupert, and J.M. Wells: *Proc. 21st Int. Symp. Ballistics*, Adelaide, Australia, 2004, ADPA, Defence Science and Technology organization, ISBN 0-9752028-0-4, 2004, vol. 1, pp. 153–59.
- J.M. Wells: Proc. 30th Int. Conf. Advanced Ceramics & Composites—Advances in Ceramic Armor, 2006, vol. 27 (7), paper no. 6–17.
- J.M. Wells, W.H. Green, N.L. Rupert, J.R. Wheeler, S.J. Cimpoeru, and A.V. Zibarov: *Proc. 21st Int. Symp. Ballistics*, DSTO, Adelaide, Australia, ADPA, Defence Science and Technology organization, ISBN 0-9752028-0-4, 2004, vol. 2, pp. 125–31.
- 10. Volume Graphics StudioMax v1.2, www. volumegraphics.com. 11. N.K. Bourne, W.H. Green, and D.P. Dandekar: *Proc. R. Soc. A*,
- 11. N.K. Bourne, W.H. Green, and D.P. Dandekar: *Proc. R. Soc. A*, 2006, vol. 462, pp. 3197–3212.
- J.M. Wells, N.L. Rupert, W.J. Bruchey, and D.A. Shockey: *Proc.* 23rd Int. Symp. Ballistics, Tarragona, Spain, ADPA, Graficas Couche, S.I., c/Ceramica.\,64 – 28038 Madrid, Spain, ISBN 978-84-7493—379-6, 2007, vol. 2, in press.
- 13. J.M. Wells and D.M. Stepp: A Workshop on the Role of Impact Damage on the Performance of Armor Ceramics, ARO Report, 2006, in press.
- J.M. Wells: Proc. 30th Int. Conf. Advanced Ceramics & Composites—Advances in Ceramic Armor, 2006, vol. 27 (7), paper no. 6–19.
- J.M. Wells: Proc. 22nd Int. Symp. Ballistics, Vancouver, ADPA, DES Tech Publications, Inc., Lancaster, PA, ISBN 1-932078-52-5, 2005, vol. 2, pp. 1223–30.
- J.M. Wells: Proc. 31st Int. Conf. Advanced Ceramics & Composites—Advances in Ceramic Armor, ACERS, 735 Ceramic Place, Westerville, Ohio 43081, www.ceramics.org, 2007, in press.
- J.M. Wells: Proc. 3rd Annual Computational Ballistics Conf., New Forest, United Kingdom, June 6–9, 2007, in press.
- J.M. Wells, N.L. Rupert, and W.H. Green: Proc. 25th Army Science Conf., Orlando, FL, Nov. 27–30, 2006.
- J.M. Wells, N.L. Rupert, and W.H. Green: in *Ceramic Armor* Materials by Design, J.W. McCauley, A.M. Rajendran, W.A. Gooch, Jr., S.J. Bless, S. Wax, A. Crowson, K.V. Logan, and M. Normandia, eds., *Ceram. Trans.*, ACERS, 735 Ceramic

Place, Westerville, Ohio 43081, www.ceramics.org, 2002, vol. 134,

- pp. 441–48.
  20. R.M. Brannon: Proc. 9th ASCE Specialty Conf. Probabilistic Mechanics and Structural Reliability, PMC2004, July 26–28, 2004, Albuquerque, New Mexico, 2005.
- 21. R.M. Brannon and O.E. Strack: "The Influence of Micro-Heterogeneity and Failure Progression Variability on Mesh-Dependency of Conventional Damage Models," Draft Sandia Report, Sandia National Laboratory, 2005.