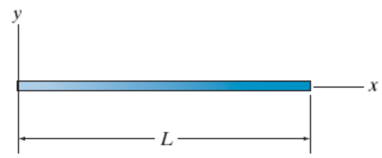
HW 12 SOLUTIONS

•9–5. Determine the mass and the location of the center of mass $\bar{x}$ of the rod if its mass per unit length is $m = m_0(1 + x/L)$.

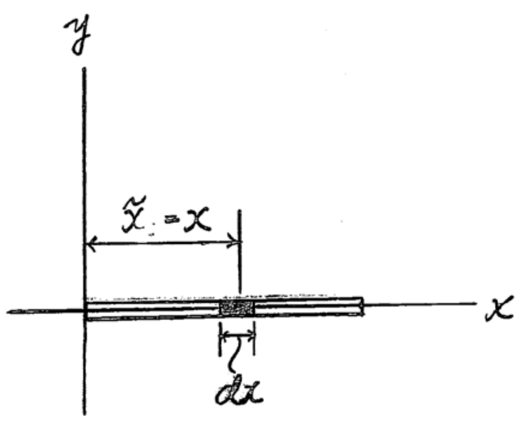


Differential Element. The element shown shaded in Fig. *a* has a mass of

$$\int_m dm = \int_0^L m_0\left(1+\frac{x}{L}\right)dx = \frac{3}{2}m_0L \qquad \textbf{Ans.}$$

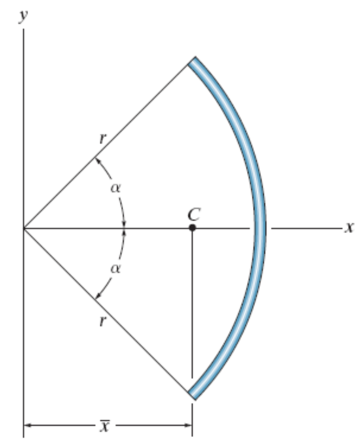
The centroid of the differential element is located at $x_c = x$.

$$\bar{x} = \frac{\int_m \tilde{x}\,dm}{\int_m dm} = \frac{\int_0^L x\left[m_0\left(1+\frac{x}{L}\right)dx\right]}{\int_0^L m_0\left(1+\frac{x}{L}\right)dx} = \frac{\int_0^L \left(x+\frac{x^2}{L}\right)dx}{\int_0^L \left(1+\frac{x}{L}\right)dx} = \frac{5}{9}L \qquad \textbf{Ans.}$$



(a)

9–7. Locate the centroid $\bar{x}$ of the circular rod. Express the answer in terms of the radius r and semiarc angle α.



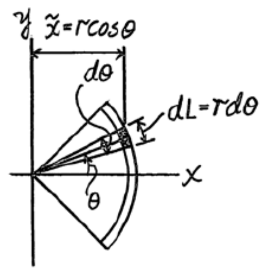
$L = 2r\alpha$

$\tilde{x} = r\cos\theta$

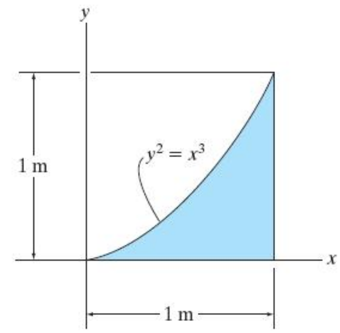
$\int \tilde{x}\, dL = \int_{-\alpha}^{\alpha} r\cos\theta\, r\,d\theta$

$= 2r^2 \sin\alpha$

$\bar{x} = \dfrac{2r^2 \sin\alpha}{2r\alpha} = \dfrac{r\sin\alpha}{\alpha}$ **Ans**



•9–9. Determine the area and the centroid $(\overline{x}, \overline{y})$ of the area.



Differential Element: The area element parallel to the y axis shown shaded in Fig. a will be considered. The area of the element is

$$dA = y\,dx = x^{3/2}\,dx$$

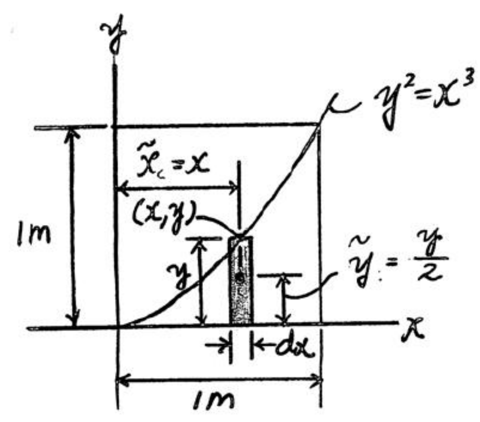
Centroid: The centroid of the element is located at $\tilde{x} = x$ and $\tilde{y} = y/2 = \dfrac{x^{3/2}}{2}$.

Area: Integrating,

$$A = \int_A dA = \int_0^{1\text{ m}} x^{3/2}\,dx = \frac{2}{5}x^{5/2}\Big|_0^{1\text{ m}} = \frac{2}{5}\text{ m}^2 = 0.4\text{ m}^2 \qquad \textbf{Ans.}$$

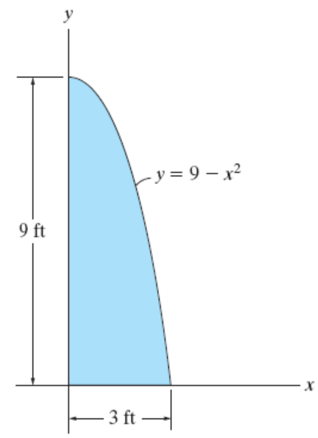
$$\bar{x} = \frac{\int_A \tilde{x}\,dA}{\int_A dA} = \frac{\int_0^{1\text{ m}} x\left(x^{3/2}\,dx\right)}{2/5} = \frac{\int_0^{1\text{ m}} x^{5/2}\,dx}{2/5} = \frac{\left(\frac{2}{7}x^{7/2}\right)\Big|_0^{1\text{ m}}}{2/5} = \frac{5}{7}\text{ m} = 0.714\text{ m} \qquad \textbf{Ans.}$$

$$\bar{y} = \frac{\int_A \tilde{y}\,dA}{\int_A dA} = \frac{\int_0^{1\text{ m}} \left(\frac{x^{3/2}}{2}\right)\left(x^{3/2}\,dx\right)}{2/5} = \frac{\int_0^{1\text{ m}} \frac{x^3}{2}\,dx}{2/5} = \frac{\frac{x^4}{8}\Big|_0^{1\text{ m}}}{2/5} = \frac{5}{16}\text{ m} = 0.3125\text{ m} \qquad \textbf{Ans.}$$



(a)

*9–24. Locate the centroid $(\bar{x}, \bar{y})$ of the area.



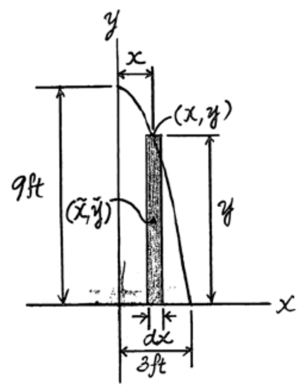
$dA = y\,dx = (9 - x^2)\,dx$

$\tilde{x} = x$

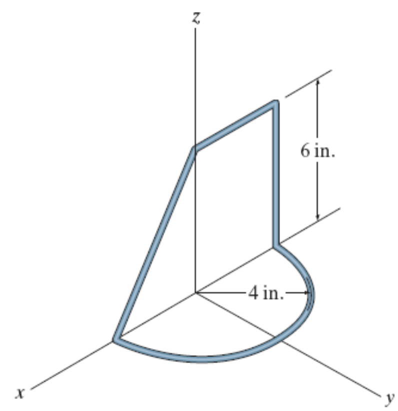
$\tilde{y} = \frac{y}{2} = \frac{1}{2}(9 - x^2)\,dx$

$\bar{x} = \frac{\int_A \tilde{x}\,dA}{\int_A dA} = \frac{\int_0^3 x(9 - x^2)\,dx}{\int_0^3 (9 - x^2)\,dx} = 1.125$ Ans

$\bar{y} = \frac{\int_A \tilde{y}\,dA}{\int_A dA} = \frac{\frac{1}{2}\int_0^3 (9 - x^2)^2\,dx}{\int_0^3 (9 - x^2)\,dx} = 3.60 \text{ ft}$ Ans



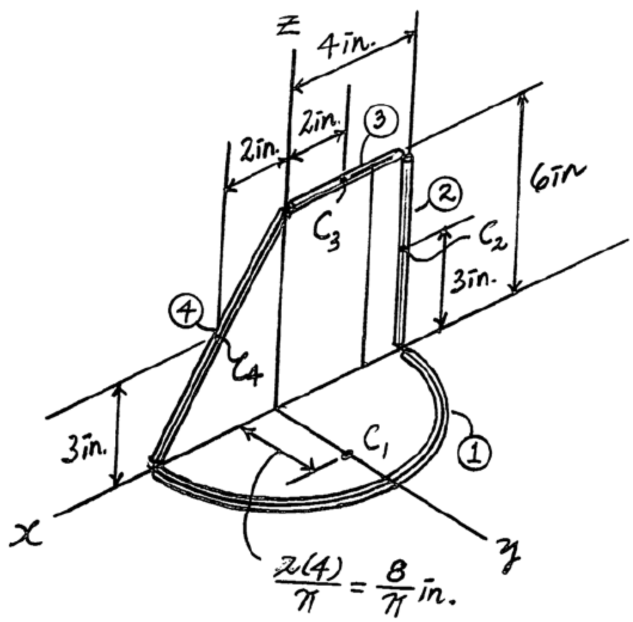
9–46. Locate the centroid $(\bar{x}, \bar{y}, \bar{z})$ of the wire.



$$\bar{x} = \frac{\Sigma \tilde{x} L}{\Sigma L} = \frac{(0)\pi(4) + (-4)(6) + (-2)(4) + (2)\left(\sqrt{4^2 + 6^2}\right)}{\pi(4) + 6 + 4 + \sqrt{4^2 + 6^2}} = \frac{-17.58}{29.78} = -0.590 \text{ in.}$$ **Ans.**

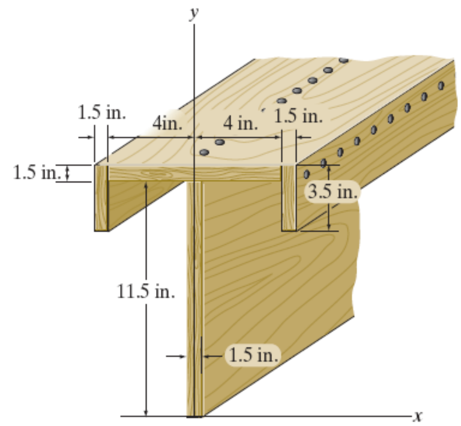
$$\bar{y} = \frac{\Sigma \tilde{y} L}{\Sigma L} = \frac{(8/\pi)\pi(4) + 0(6) + 0(4) + 0\left(\sqrt{4^2 + 6^2}\right)}{\pi(4) + 6 + 4 + \sqrt{4^2 + 6^2}} = \frac{32}{29.78} = 1.07 \text{ in.}$$ **Ans.**

$$\bar{z} = \frac{\Sigma \tilde{z} L}{\Sigma L} = \frac{(0)\pi(4) + 3(6) + 6(4) + 3\left(\sqrt{4^2 + 6^2}\right)}{\pi(4) + 6 + 4 + \sqrt{4^2 + 6^2}} = \frac{63.63}{29.78} = 2.14 \text{ in.}$$ **Ans.**



(a)

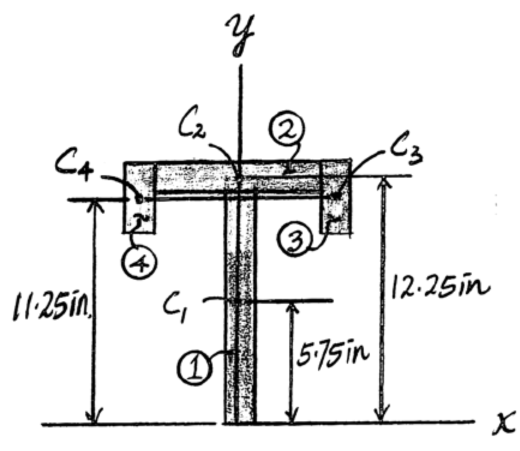
*9–56. Locate the centroid $\bar{y}$ of the cross-sectional area of the built-up beam.



Centroid: The centroid of each composite segment is shown in Fig. *a*.

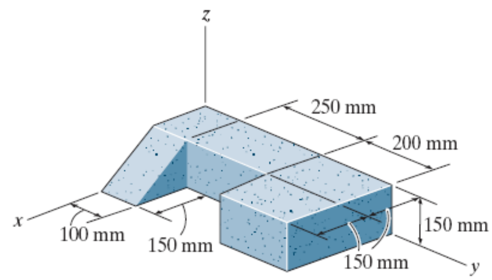
$$\bar{y} = \frac{\Sigma\tilde{y}A}{\Sigma A} = \frac{5.75(11.5)(1.5) + 12.25(8)(1.5) + 11.25(3.5)(1.5) + 11.25(3.5)(1.5)}{11.5(1.5) + 8(1.5) + 3.5(1.5) + 3.5(1.5)}$$

$= 9.17$ in. **Ans.**



(a)

*9–72. Locate the center of mass $(\bar{x}, \bar{y}, \bar{z})$ of the homogeneous block assembly.

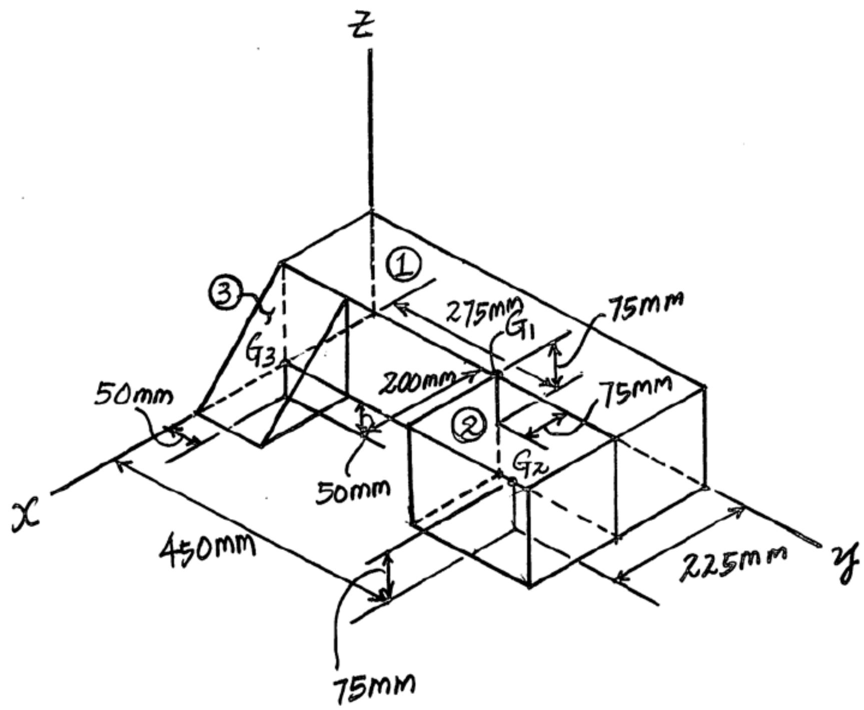


Centroid: Since the block is made of a homogeneous material, the center of mass of the block coincides with the centroid of its volume. The centroid of each composite segment is shown in Fig. *a*.

$$\bar{x} = \frac{\Sigma \tilde{x} V}{\Sigma V} = \frac{(75)(150)(150)(550) + (225)(150)(150)(200) + (200)\left(\frac{1}{2}\right)(150)(150)(100)}{(150)(150)(550) + (150)(150)(200) + \frac{1}{2}(150)(150)(100)} = \frac{2.165625(10^9)}{18(10^6)} = 120 \text{ mm} \qquad \textbf{Ans.}$$

$$\bar{y} = \frac{\Sigma \tilde{y} V}{\Sigma V} = \frac{(275)(150)(150)(550) + (450)(150)(150)(200) + (50)\left(\frac{1}{2}\right)(150)(150)(100)}{(150)(150)(550) + (150)(150)(200) + \frac{1}{2}(150)(150)(100)} = \frac{5.484375(10^9)}{18(10^6)} = 305 \text{ mm} \qquad \textbf{Ans.}$$

$$\bar{z} = \frac{\Sigma \tilde{z} V}{\Sigma V} = \frac{(75)(150)(150)(550) + (75)(150)(150)(200) + (50)\left(\frac{1}{2}\right)(150)(150)(100)}{(150)(150)(550) + (150)(150)(200) + \frac{1}{2}(150)(150)(100)} = \frac{1.321875(10^9)}{18(10^6)} = 73.4 \text{ mm} \qquad \textbf{Ans.}$$



(a)