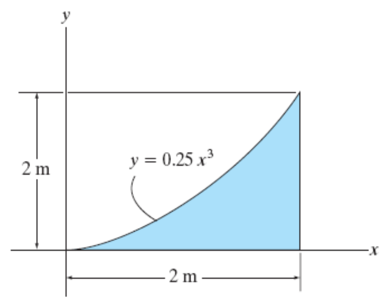
HW13 SOLUTIONS

•10–1. Determine the moment of inertia of the area about the x axis.



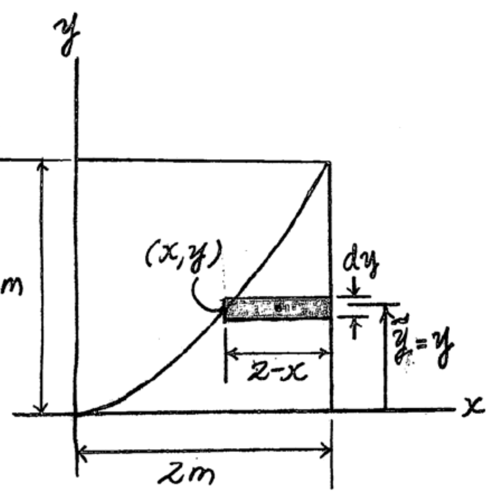
The area of the rectangular differential element in Fig. a is $dA = (2-x)\,dy$. Since $x = (4y)^{1/3}$ then $dA = \left[2-(4y)^{1/3}\right]dy$.

$$I_x = \int_A y^2 dA$$

$$= \int_0^{2\text{ m}} y^2\left[2-(4y)^{1/3}\right]dy$$

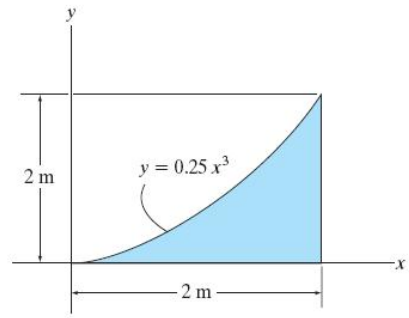
$$= \int_0^{2\text{ m}} \left(2y^2 - 4^{1/3}y^{7/3}\right)dy$$

$$= \left[\frac{2y^3}{3} - \frac{3}{10}\left(4^{1/3}\right)y^{10/3}\right]_0^{2\text{ m}} = 0.533\text{ m}^4$$ **Ans.**



(a)

10–2. Determine the moment of inertia of the area about the y axis.



The area of the rectangular differential element in Fig. *a* is $dA = y\,dx = \dfrac{x^3}{4}\,dx$.

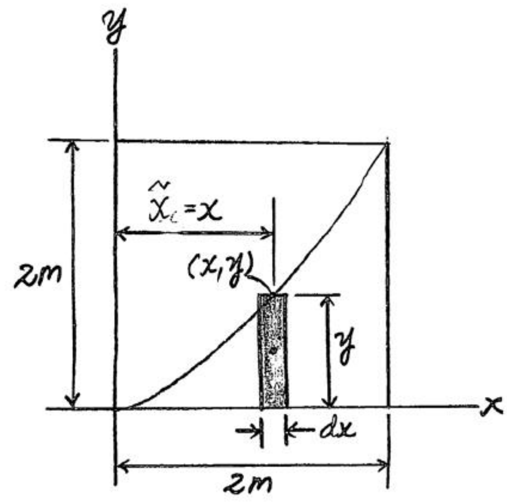
$$I_y = \int_A x^2 dA$$

$$= \int_0^{2\,\text{m}} x^2\left(\frac{x^3}{4}\right)dx$$

$$= \int_0^{2\,\text{m}} \frac{x^5}{4}\,dx$$

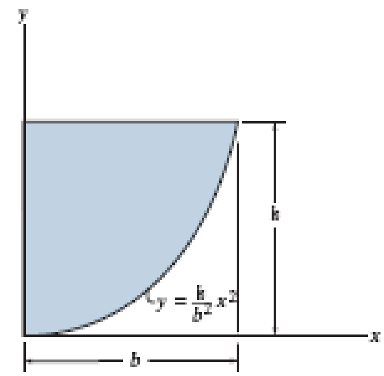
$$= \left(\frac{x^6}{24}\right)\Bigg|_0^{2\,\text{m}} = 2.67\,\text{m}^4$$

Ans.



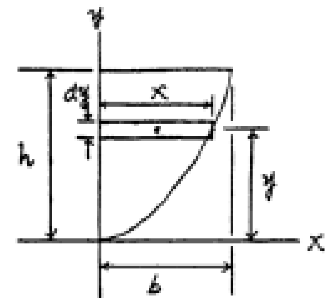
(a)

10–18. Determine the moment of inertia of the area about the x axis.



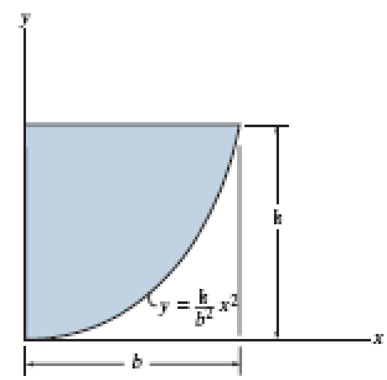
Differential Element : Here, $x = \frac{b}{\sqrt{h}}y^{\frac{1}{2}}$. The area of the differential element parallel to x axis is $dA = x\,dy = \left(\frac{b}{\sqrt{h}}y^{\frac{1}{2}}\right)dy$.

Moment of Inertia : Applying Eq. 10 – 1 and performing the integration, we have



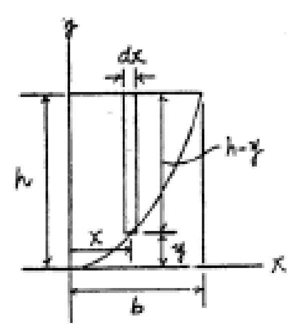
$$I_x = \int_A y^2 dA = \int_0^h y^2\left(\frac{b}{\sqrt{h}}y^{\frac{1}{2}}\right)dy$$
$$= \frac{b}{\sqrt{h}}\left(\frac{2}{7}y^{\frac{7}{2}}\right)\Big|_0^h$$
$$= \frac{2}{7}bh^3 \qquad \textbf{Ans}$$

10–19. Determine the moment of inertia of the area about the y axis.



Differential Element : The area of the differential element parallel to y axis is $dA = (h - y)dx = \left(h - \frac{h}{b^2}x^2\right)dx$.

Moment of Inertia : Applying Eq. 10 – 1 and performing the integration, we have



$$I_y = \int_A x^2 dA = \int_0^b x^2\left(h - \frac{h}{b^2}x^2\right)dx$$
$$= \left(\frac{h}{3}x^3 - \frac{h}{5b^2}x^5\right)\Big|_0^b$$
$$= \frac{2}{15}hb^3 \qquad \textbf{Ans}$$

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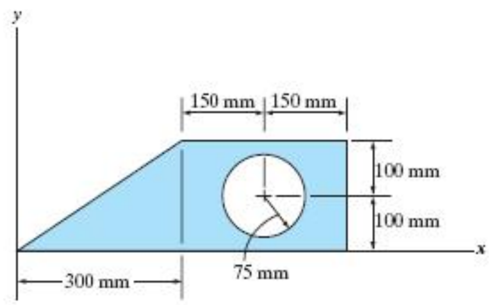
*10–32. Determine the moment of inertia of the composite area about the x axis.



Composite Parts: The composite area can be subdivided into three segments as shown in Fig. *a*. Since segment (3) is a hole, it contributes a negative moment of inertia. The perpendicular distance measured from the centroid of each segment to the x axis is also indicated.

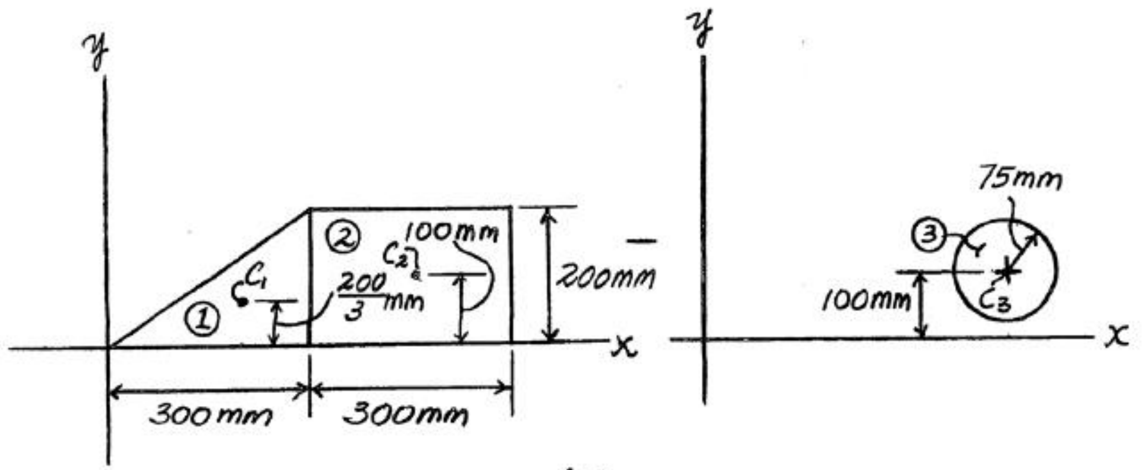
Moment of Inertia: The moment of inertia of each segment about the x axis can be determined using the parallel - axis theorem. Thus,

$$I_x = \bar{I}_{x'} + A(d_y)^2$$

$$= \left[\frac{1}{36}(300)(200^3) + \frac{1}{2}(300)(200)\left(\frac{200}{3}\right)^2\right] + \left[\frac{1}{12}(300)(200^3) + 300(200)(100)^2\right] + \left[-\frac{\pi}{4}(75^4) + \left(-\pi(75^2)\right)(100)^2\right]$$

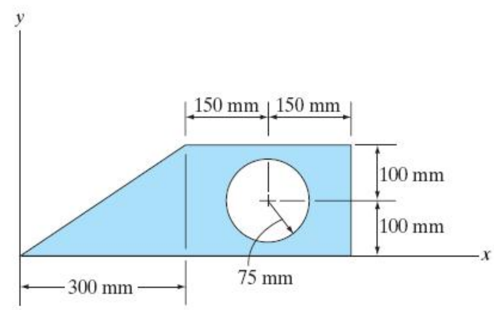
$$= 798(10^6)\ \text{mm}^4$$

Ans.



(a)

•**10–33.** Determine the moment of inertia of the composite area about the y axis.



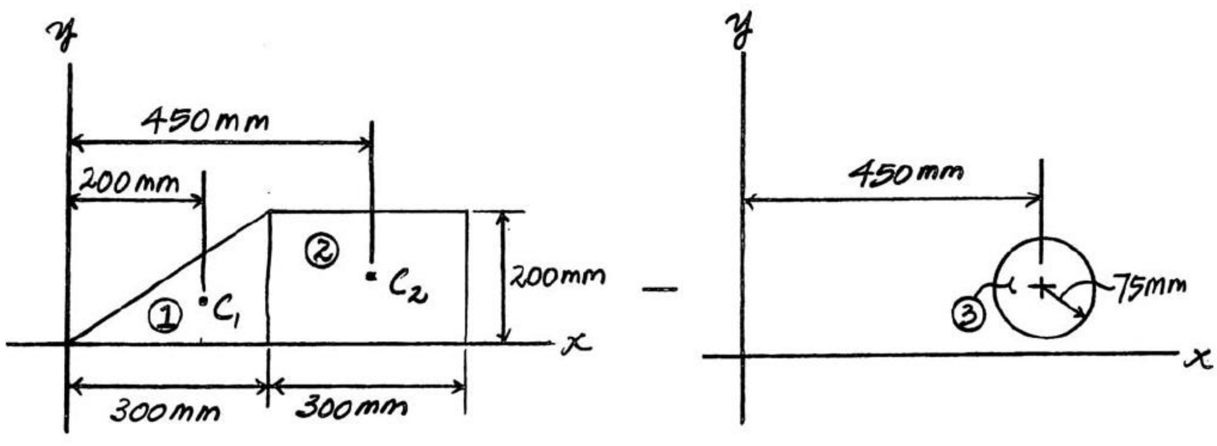
Composite Parts: The composite area can be subdivided into three segments as shown in Fig. *a*. Since segment (3) is a hole, it contributes a negative moment of inertia. The perpendicular distance measured from the centroid of each segment to the y axis is also indicated.

Moment of Inertia: The moment of inertia of each segment about the y axis can be determined using the parallel - axis theorem. Thus,

$$I_y = \bar{I}_{y'} + A(d_x)^2$$

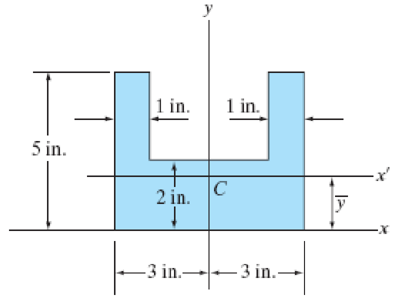
$$= \left[\frac{1}{36}(200)(300^3) + \frac{1}{2}(200)(300)(200)^2\right] + \left[\frac{1}{12}(200)(300^3) + 200(300)(450)^2\right] + \left[-\frac{\pi}{4}(75^4) + \left(-\pi(75^2)\right)(450)^2\right]$$

$$= 10.3(10^9)\ \text{mm}^4$$ **Ans.**



(a)

*10–36. Locate the centroid $\overline{y}$ of the composite area, then determine the moment of inertia of this area about the centroidal x' axis.



Composite Parts: The composite area can be subdivided into three segments The perpendicular distance measured from the centroid of each segment to the x axis is also indicated.

Centroid: The perpendicular distances measured from the centroid of each segment to the x axis are indicated in Fig. *a*.

$$\bar{y} = \frac{\Sigma y_C A}{\Sigma A} = \frac{(1)(6)(2) + 2[3.5(3)(1)]}{(6)(2) + 2[(3)(1)]} = 1.833 \text{ in.} = 1.83 \text{ in.} \qquad \textbf{Ans.}$$

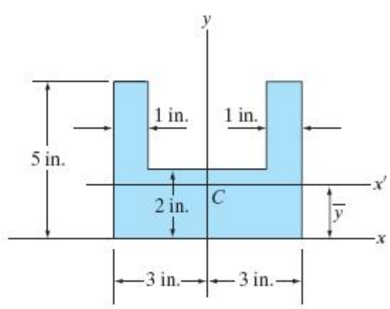
Moment of Inertia: The moment of inertia of each segment about the x' axis can be determined using the parallel - axis theorem. Thus,

$$I_{x'} = \bar{I}_x + A(d_y)^2$$

$$= \left[\frac{1}{12}(6)(2^3) + 6(2)(1.833 - 1)^2\right] + 2\left[\frac{1}{12}(1)(3^3) + 1(3)(3.5 - 1.833)^2\right]$$

$$= 33.5 \text{ in}^4 \qquad \textbf{Ans.}$$

•10–37. Determine the moment of inertia of the composite area about the centroidal y axis.



Moment of Inertia: The moment of inertia of each segment about the y axis can be determined using the parallel - axis theorem. Thus,

$$I_y = \bar{I}_y + A(d_x)^2$$

$$= \left[\frac{1}{12}(2)(6^3)\right] + 2\left[\frac{1}{12}(3)(1^3) + 3(1)(2.5)^2\right]$$

$$= 74 \text{ in}^4 \qquad \textbf{Ans.}$$
