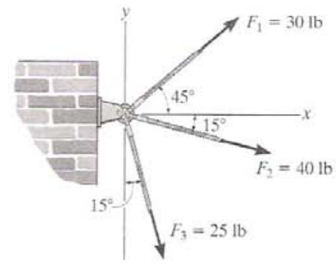
Homework Set #2

*2–32. Determine the magnitude of the resultant force acting on the pin and its direction measured clockwise from the positive x axis.



Rectangular Components: By referring to Fig. a, the x and y components of $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ can be written as

$$(F_1)_x = 30\cos 45° = 21.21 \text{ lb} \qquad (F_1)_y = 30\sin 45° = 21.21 \text{ lb}$$
$$(F_2)_x = 40\cos 15° = 38.64 \text{ lb} \qquad (F_2)_y = 40\sin 15° = 10.35 \text{ lb}$$
$$(F_3)_x = 25\sin 15° = 6.47 \text{ lb} \qquad (F_3)_y = 25\cos 15° = 24.15 \text{ lb}$$

Resultant Force: Summing the force components algebraically along the x and y axes,

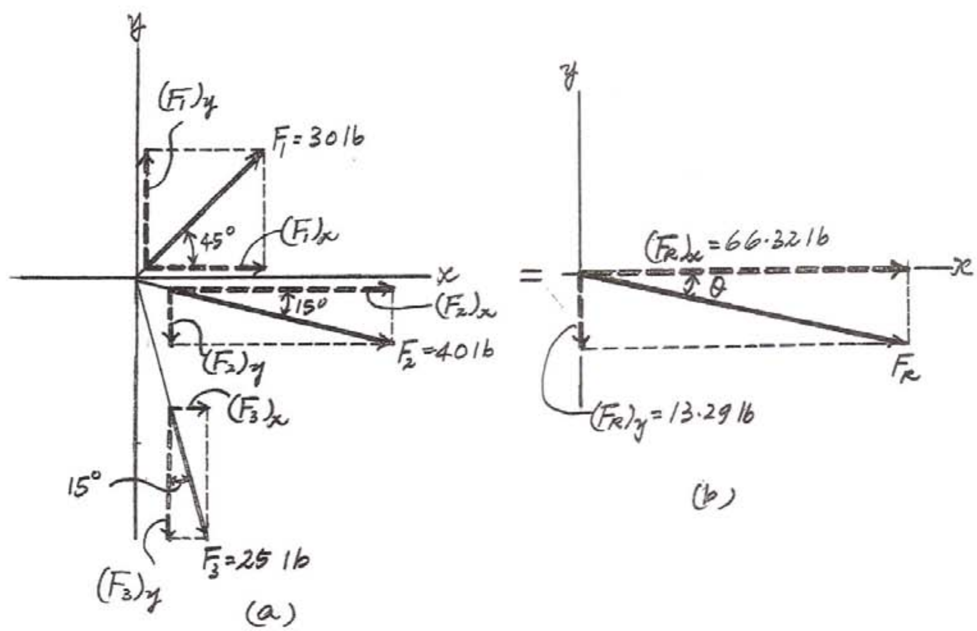
$$\overset{+}{\rightarrow}\Sigma(F_R)_x = \Sigma F_x; \quad (F_R)_x = 21.21 + 38.64 + 6.47 = 66.32 \text{ lb} \rightarrow$$
$$+\uparrow \Sigma(F_R)_y = \Sigma F_y; \quad (F_R)_y = 21.21 - 10.35 - 24.15 = -13.29 \text{ lb} = 13.29 \text{ lb} \downarrow$$

The magnitude of the resultant force $\mathbf{F}_R$ is

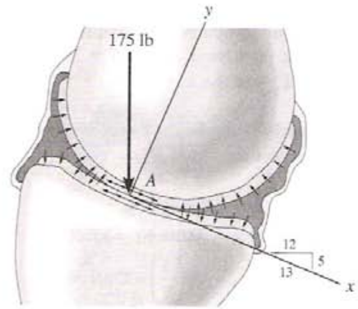
$$F_R = \sqrt{(F_R)_x{}^2 + (F_R)_y{}^2} = \sqrt{66.32^2 + 13.29^2} = 67.6 \text{ lb} \qquad \textbf{Ans.}$$

The direction angle θ of $\mathbf{F}_R$, measured clockwise from the positive x axis, is

$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left(\frac{13.29}{66.32}\right) = 11.3° \qquad \textbf{Ans.}$$

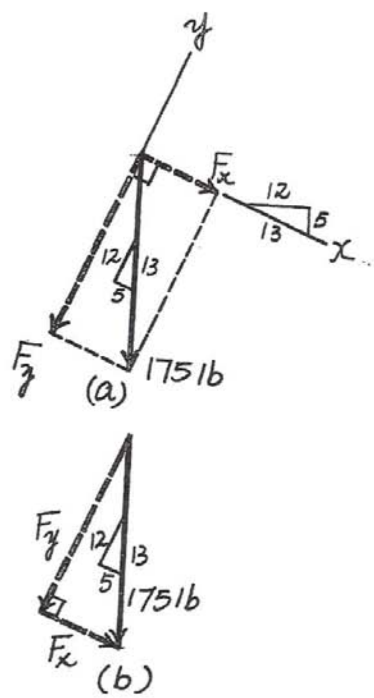


2–35. The contact point between the femur and tibia bones of the leg is at A. If a vertical force of 175 lb is applied at this point, determine the components along the x and y axes. Note that the y component represents the normal force on the load-bearing region of the bones. Both the x and y components of this force cause synovial fluid to be squeezed out of the bearing space.

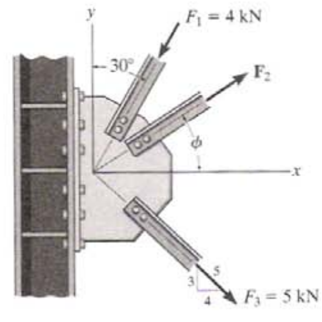


$$F_x = 175\left(\frac{5}{13}\right) = 67.3 \text{ lb} \qquad \textbf{Ans}$$

$$F_y = -175\left(\frac{12}{13}\right) = -162 \text{ lb} \qquad \textbf{Ans}$$



2–38. If $\phi = 30°$ and the resultant force acting on the gusset plate is directed along the positive x axis, determine the magnitudes of $\mathbf{F}_2$ and the resultant force.



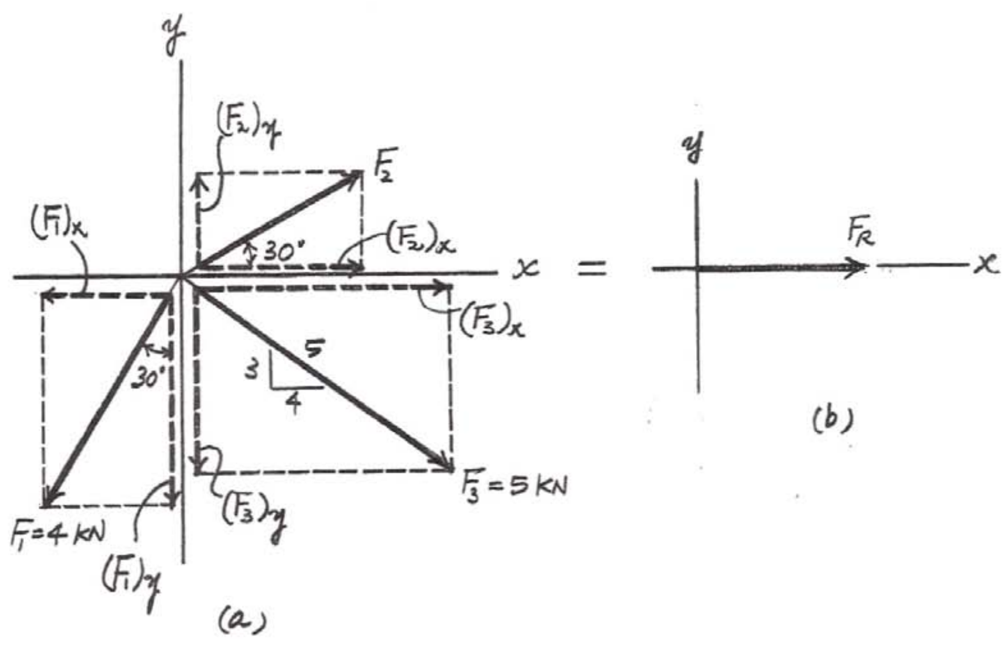
Rectangular Components: By referring to Fig. a, the x and y components of $\mathbf{F}_1$, $\mathbf{F}_2$, $\mathbf{F}_3$, and $\mathbf{F}_R$ can be written as

$$(F_1)_x = 4\sin 30° = 2\text{ kN} \qquad (F_1)_y = 4\cos 30° = 3.464\text{ kN}$$
$$(F_2)_x = F_2\cos 30° = 0.8660F_2 \qquad (F_2)_y = F_2\sin 30° = 0.5F_2$$
$$(F_3)_x = 5\left(\frac{4}{5}\right) = 4\text{ kN} \qquad (F_3)_y = 5\left(\frac{3}{5}\right) = 3\text{kN}$$
$$(F_R)_x = F_R \qquad (F_R)_y = 0$$

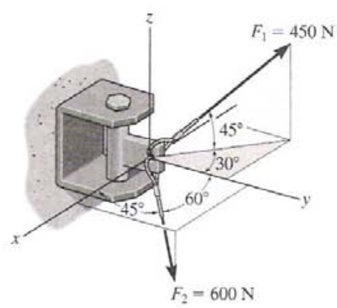
Resultant Force: Summing the force components algebraically along the x and y axes,

$$+\uparrow \Sigma(F_R)_y = \Sigma F_y; \quad 0 = -3.464 + 0.5F_2 - 3$$
$$F_2 = 12.93\text{ kN} = 12.9\text{ kN} \qquad \textbf{Ans.}$$

$$\overset{+}{\rightarrow}\Sigma(F_R)_x = \Sigma F_x; \quad F_R = -2 + 0.8660(12.93) + 4$$
$$= 13.2\text{ kN} \qquad \textbf{Ans.}$$

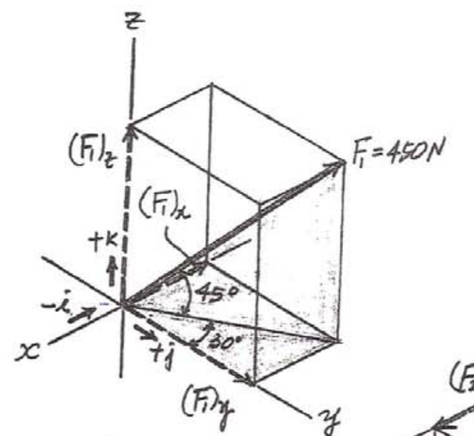


2–59. Determine the coordinate angle γ for $\mathbf{F}_2$ and then express each force acting on the bracket as a Cartesian vector.

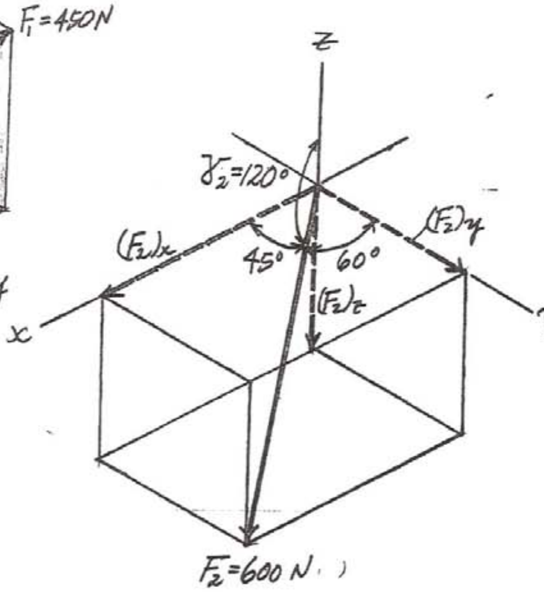


Rectangular Components: Since $\cos^2\alpha_2 + \cos^2\beta_2 + \cos^2\gamma_2 = 1$, then $\cos\gamma_{2z} = \pm\sqrt{1 - \cos^2 45° - \cos^2 60°} = \pm 0.5$. However, it is required that $\gamma_2 > 90°$, thus, $\gamma_2 = \cos^{-1}(-0.5) = 120°$. By resolving $\mathbf{F}_1$ and $\mathbf{F}_2$ into their x, y, and z components, as shown in Figs. *a* and *b*, respectively $\mathbf{F}_1$ and $\mathbf{F}_2$ can be expressed in Cartesian vector form as

$$\mathbf{F}_1 = 450\cos 45°\sin 30°(-\mathbf{i}) + 450\cos 45°\cos 30°(+\mathbf{j}) + 450\sin 45°(+\mathbf{k})$$
$$= \{-159\mathbf{i} + 276\mathbf{j} + 318\mathbf{k}\}\text{N} \qquad \textbf{Ans.}$$
$$\mathbf{F}_2 = 600\cos 45°\mathbf{i} + 600\cos 60°\mathbf{j} + 600\cos 120°\mathbf{k}$$
$$= \{424\mathbf{i} + 300\mathbf{j} - 300\mathbf{k}\}\text{ N} \qquad \textbf{Ans.}$$

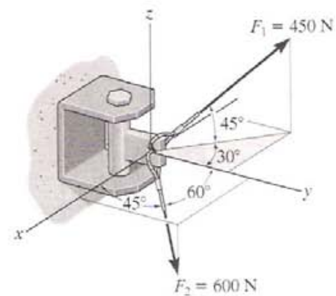


(a)



(b)

*2–60. Determine the magnitude and coordinate direction angles of the resultant force acting on the bracket.



Rectangular Components: Since $\cos^2\alpha_2 + \cos^2\beta_2 + \cos^2\gamma_2 = 1$, then $\cos\gamma_{2z} = \pm\sqrt{1-\cos^2 45° - \cos^2 60°} = \pm 0.5$. However, it is required that $\alpha_2 > 90°$, thus, $\gamma_2 = \cos^{-1}(-0.5) = 120°$. By resolving $\mathbf{F}_1$ and $\mathbf{F}_2$ into their x, y, and z components, as shown in Figs. a and b, respectively, $\mathbf{F}_1$ and $\mathbf{F}_2$, can be expressed in Cartesian vector form, as

$$\mathbf{F}_1 = 450\cos 45°\sin 30°(-\mathbf{i}) + 450\cos 45°\cos 30°(+\mathbf{j}) + 450\sin 45°(+\mathbf{k})$$
$$= \{-159.10\mathbf{i} + 275.57\mathbf{j} + 318.20\mathbf{k}\}\text{N} \qquad \textbf{Ans.}$$
$$\mathbf{F}_2 = 600\cos 45°\mathbf{i} + 600\cos 60°\mathbf{j} + 600\cos 120°\mathbf{k}$$
$$= \{424\mathbf{i} + 300\mathbf{j} - 300\mathbf{k}\}\text{ N} \qquad \textbf{Ans.}$$

Resultant Force: By adding $\mathbf{F}_1$ and $\mathbf{F}_2$ vectorally, we obtain $\mathbf{F}_R$.

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$$
$$= (-159.10\mathbf{i} + 275.57\mathbf{j} + 318.20\mathbf{k}) + (424.26\mathbf{i} + 300\mathbf{j} - 300\mathbf{k})$$
$$= \{265.16\mathbf{i} + 575.57\mathbf{j} + 18.20\mathbf{k}\}\text{ N}$$

The magnitude of $\mathbf{F}_R$ is

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2 + (F_R)_z^2}$$
$$= \sqrt{265.16^2 + 575.57^2 + 18.20^2} = 633.97\text{ N} = 634\text{ N} \qquad \textbf{Ans.}$$

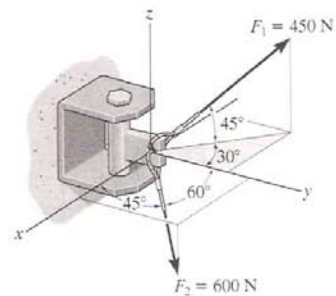
The coordinate direction angles of $\mathbf{F}_R$ are

$$\alpha = \cos^{-1}\left[\frac{(F_R)_x}{F_R}\right] = \cos^{-1}\left(\frac{265.16}{633.97}\right) = 65.3° \qquad \textbf{Ans.}$$

$$\beta = \cos^{-1}\left[\frac{(F_R)_y}{F_R}\right] = \cos^{-1}\left(\frac{575.57}{633.97}\right) = 24.8° \qquad \textbf{Ans.}$$

$$\gamma = \cos^{-1}\left[\frac{(F_R)_z}{F_R}\right] = \cos^{-1}\left(\frac{18.20}{633.97}\right) = 88.4° \qquad \textbf{Ans.}$$

*2–60. Determine the magnitude and coordinate direction angles of the resultant force acting on the bracket.



Rectangular Components: Since $\cos^2\alpha_2 + \cos^2\beta_2 + \cos^2\gamma_2 = 1$, then $\cos\gamma_{2z} = \pm\sqrt{1-\cos^2 45° - \cos^2 60°} = \pm 0.5$. However, it is required that $\alpha_2 > 90°$, thus, $\gamma_2 = \cos^{-1}(-0.5) = 120°$. By resolving $\mathbf{F}_1$ and $\mathbf{F}_2$ into their x, y, and z components, as shown in Figs. *a* and *b*, respectively, $\mathbf{F}_1$ and $\mathbf{F}_2$, can be expressed in Cartesian vector form, as

$$\mathbf{F}_1 = 450\cos 45°\sin 30°(-\mathbf{i}) + 450\cos 45°\cos 30°(+\mathbf{j}) + 450\sin 45°(+\mathbf{k})$$
$$= \{-159.10\mathbf{i} + 275.57\mathbf{j} + 318.20\mathbf{k}\}\text{N} \qquad \textbf{Ans.}$$
$$\mathbf{F}_2 = 600\cos 45°\mathbf{i} + 600\cos 60°\mathbf{j} + 600\cos 120°\mathbf{k}$$
$$= \{424\mathbf{i} + 300\mathbf{j} - 300\mathbf{k}\}\text{ N} \qquad \textbf{Ans.}$$

Resultant Force: By adding $\mathbf{F}_1$ and $\mathbf{F}_2$ vectorally, we obtain $\mathbf{F}_R$.

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$$
$$= (-159.10\mathbf{i} + 275.57\mathbf{j} + 318.20\mathbf{k}) + (424.26\mathbf{i} + 300\mathbf{j} - 300\mathbf{k})$$
$$= \{265.16\mathbf{i} + 575.57\mathbf{j} + 18.20\mathbf{k}\}\text{ N}$$

The magnitude of $\mathbf{F}_R$ is

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2 + (F_R)_z^2}$$
$$= \sqrt{265.16^2 + 575.57^2 + 18.20^2} = 633.97\text{ N} = 634\text{ N} \qquad \textbf{Ans.}$$

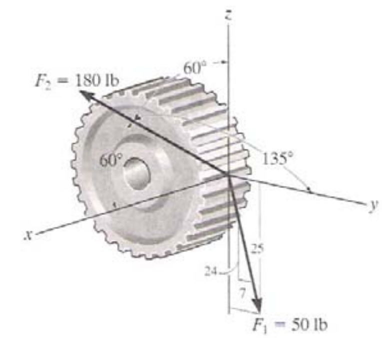
The coordinate direction angles of $\mathbf{F}_R$ are

$$\alpha = \cos^{-1}\left[\frac{(F_R)_x}{F_R}\right] = \cos^{-1}\left(\frac{265.16}{633.97}\right) = 65.3° \qquad \textbf{Ans.}$$

$$\beta = \cos^{-1}\left[\frac{(F_R)_y}{F_R}\right] = \cos^{-1}\left(\frac{575.57}{633.97}\right) = 24.8° \qquad \textbf{Ans.}$$

$$\gamma = \cos^{-1}\left[\frac{(F_R)_z}{F_R}\right] = \cos^{-1}\left(\frac{18.20}{633.97}\right) = 88.4° \qquad \textbf{Ans.}$$

2–67. The spur gear is subjected to the two forces caused by contact with other gears. Express each force as a Cartesian vector.

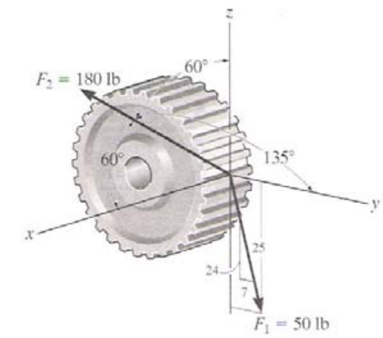


$$\mathbf{F}_1 = \frac{7}{25}(50)\mathbf{j} - \frac{24}{25}(50)\mathbf{k} = \{14.0\,\mathbf{j} - 48.0\,\mathbf{k}\}\ \text{lb} \qquad \text{Ans}$$

$$\mathbf{F}_2 = 180\cos 60°\mathbf{i} + 180\cos 135°\mathbf{j} + 180\cos 60°\mathbf{k}$$

$$= \{90\mathbf{i} - 127\mathbf{j} + 90\mathbf{k}\}\ \text{lb} \qquad \text{Ans}$$

***2–68.** The spur gear is subjected to the two forces caused by contact with other gears. Determine the resultant of the two forces and express the result as a Cartesian vector.



$$F_{Rx} = 180\cos 60° = 90$$

$$F_{Ry} = \frac{7}{25}(50) + 180\cos 135° = -113$$

$$F_{Rz} = -\frac{24}{25}(50) + 180\cos 60° = 42$$

$$\mathbf{F}_R = \{90\mathbf{i} - 113\mathbf{j} + 42\mathbf{k}\}\ \text{lb} \qquad \text{Ans}$$