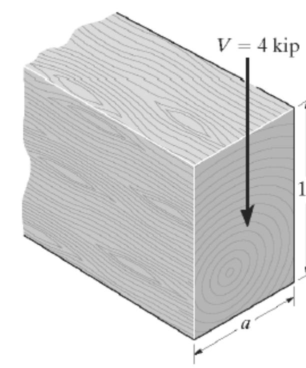
# HW 20 SOLUTIONS

**7–6.** The beam has a rectangular cross section and is made of wood having an allowable shear stress of $\tau_{allow} = 1.6$ ksi. If it is subjected to a shear of $V = 4$ kip, determine the smallest dimension $a$ of its bottom and $1.5a$ of its sides.




***Section Properties:***

$$I = \frac{1}{12}(a)(1.5a)^3 = 0.28125\,a^4$$

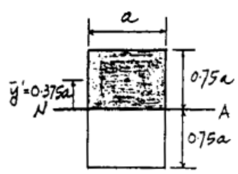
$$Q_{max} = \bar{y}'A' = (0.375a)(0.75a)(a) = 0.28125a^3$$

***Allowable Shear Stress:*** Applying the shear formula

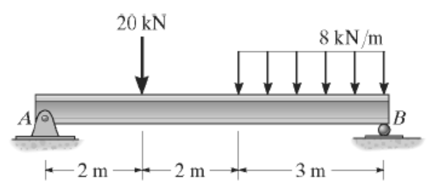
$$\tau_{max} = \tau_{allow} = \frac{VQ_{max}}{It}$$

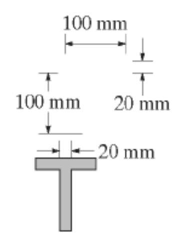
$$1.6 = \frac{4(0.28125a^3)}{0.28125a^4(a)}$$

$$a = 1.58 \text{ in.} \qquad \textbf{Ans}$$



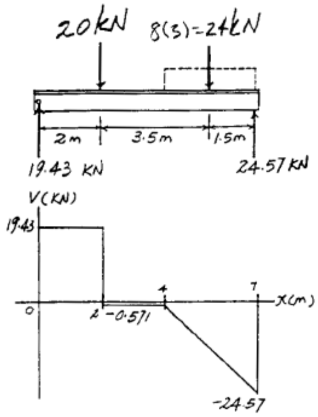

***7–16.** The T-beam is subjected to the loading shown. Determine the maximum transverse shear stress in the beam at the critical section.




***Support Reactions:*** As shown on FBD.

***Internal Shear Force:*** As shown on Shear diagram, $V_{max} = 24.57$ kN.




***Section Properties:***

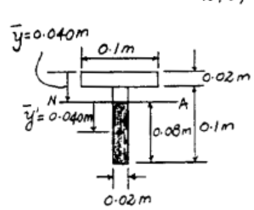
$$\bar{y} = \frac{\Sigma \tilde{y}A}{\Sigma A} = \frac{0.01(0.1)(0.02) + 0.07\,(0.1)(0.02)}{0.1(0.02) + 0.1(0.02)} = 0.0400 \text{ m}$$

$$I_{NA} = \frac{1}{12}(0.1)\left(0.02^3\right) + 0.1(0.02)(0.0400 - 0.01)^2 + \frac{1}{12}(0.02)\left(0.1^3\right) + (0.02)(0.1)(0.07 - 0.0400)^2 = 5.3333\left(10^{-6}\right) \text{ m}^4$$

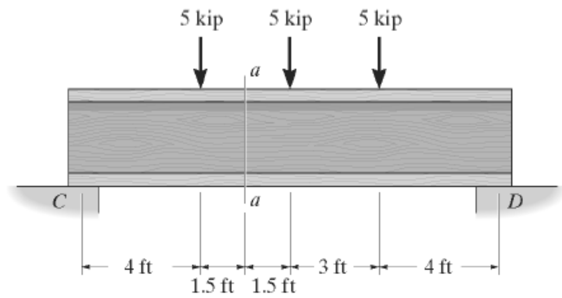
$$Q_{max} = \bar{y}'A' = 0.04(0.02)(0.08) = 64.0\left(10^{-6}\right) \text{ m}^3$$

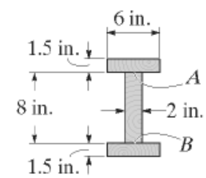
***Maximum Shear Stress:*** Maximum shear stress occurs at the point where the neutral axis passes through the section. Applying the shear formula

$$\tau_{max} = \frac{VQ_{max}}{It} = \frac{24.57(10^3)\,64.0(10^{-6})}{5.3333(10^{-6})(0.02)} = 14.7 \text{ MPa} \qquad \textbf{Ans}$$

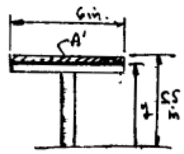


**7–26.** The beam is made from three boards glued together at the seams $A$ and $B$. If it is subjected to the loading shown, determine the maximum vertical shear force resisted by the top flange of the beam. The supports at $C$ and $D$ exert only vertical reactions on the beam.




$V_{max} = 7.5$ kip (at $C$ or $D$)




$$I = \frac{1}{12}(6)(11)^3 - \frac{1}{12}(4)(8)^3 = 494.83 \text{ in}^4$$

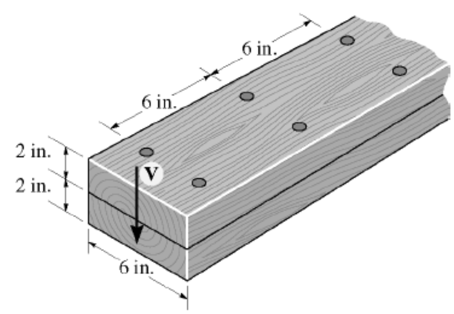
$$F_f = \int_{A_f} \tau dA$$

$$\tau = \frac{VQ}{It} = \frac{7.5(10^3)(5.5-y)(6)[(5.5+y)/2]}{494.83(6)} = 7.57836(30.25 - y^2)$$

$$F_f = \int_4^{5.5} 7.57836(30.25 - y^2)(6dy)$$

$$= 45.4702(30.25y - \frac{1}{3}y^3)\Big|_4^{5.5} = 512 \text{ lb} \quad \textbf{Ans}$$

***7–36.** The beam is constructed from two boards fastened together at the top and bottom with two rows of nails spaced every 6 in. If each nail can support a 500-lb shear force, determine the maximum shear force $V$ that can be applied to the beam.

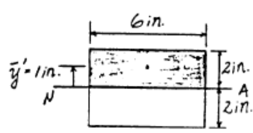



***Section Properties:***

$$I = \frac{1}{12}(6)(4^3) = 32.0 \text{ in}^4$$

$$Q = \bar{y}'A' = 1(6)(2) = 12.0 \text{ in}^4$$

***Shear Flow:*** There are two rows of nails. Hence, the allowable shear flow $q = \frac{2(500)}{6} = 166.67$ lb/in.

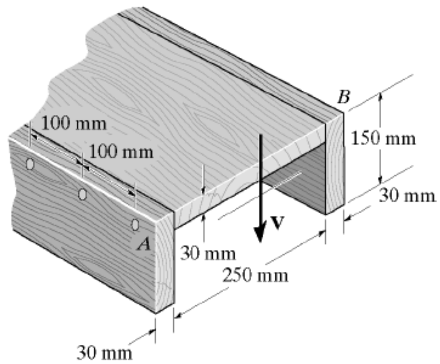



$$q = \frac{VQ}{I}$$

$$166.67 = \frac{V(12.0)}{32.0}$$

$$V = 444 \text{ lb} \quad \textbf{Ans}$$

***7–40.** The beam is subjected to a shear of $V = 800$ N. Determine the average shear stress developed in the nails along the sides $A$ and $B$ if the nails are spaced $s = 100$ mm apart. Each nail has a diameter of 2 mm.




$$\bar{y} = \frac{0.015\,(0.03)(0.25) + 2\,(0.075)(0.15)(0.03)}{0.03(0.25) + 2(0.15)(0.03)} = 0.04773 \text{ m}$$

$$I = \frac{1}{12}(0.25)(0.03^3) + (0.25)(0.03)(0.04773 - 0.015)^2$$

$$+ (2)(\frac{1}{12})(0.03)(0.15^3) + 2(0.03)(0.15)(0.075 - 0.04773)^2$$

$$= 32.164773(10^{-6}) \text{ m}^4$$

$$Q = \bar{y}'A' = 0.03273(0.25)(0.03) = 0.245475(10^{-3}) \text{ m}^3$$

$$q = \frac{VQ}{I} = \frac{800\,(0.245475)(10^{-3})}{32.164773(10^{-6})} = 6105.44 \text{ N/m}$$


$$F = q\,s = 6105.44\,(0.1) = 610.544 \text{ N}$$

Since each side of the beam resists this shear force then

$$\tau_{avg} = \frac{F}{2A} = \frac{610.544}{2(\frac{\pi}{4})(0.002^2)} = 97.2 \text{ MPa} \quad \textbf{Ans}$$