HW 21 SOLUTIONS

8–1. A spherical gas tank has an inner radius of $r = 1.5$ m. If it is subjected to an internal pressure of $p = 300$ kPa, determine its required thickness if the maximum normal stress is not to exceed 12 MPa.

$$\sigma_{allow} = \frac{p\,r}{2\,t}; \qquad 12(10^6) = \frac{300(10^3)(1.5)}{2\,t}$$

$$t = 0.0188 \text{ m} = 18.8 \text{ mm} \qquad \textbf{Ans}$$

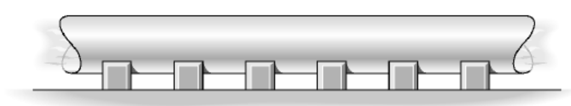
8–2. A pressurized spherical tank is to be made of 0.5-in.-thick steel. If it is subjected to an internal pressure of $p = 200$ psi, determine its outer radius if the maximum normal stress is not to exceed 15 ksi.

$$\sigma_{allow} = \frac{p\,r}{2\,t}; \qquad 15(10^3) = \frac{200\,r_i}{2(0.5)}$$

$$r_i = 75 \text{ in.}$$

$$r_o = 75 \text{ in.} + 0.5 \text{ in.} = 75.5 \text{ in.} \qquad \textbf{Ans}$$

8–6. The open-ended polyvinyl chloride pipe has an inner diameter of 4 in. and thickness of 0.2 in. If it carries flowing water at 60 psi pressure, determine the state of stress in the walls of the pipe.

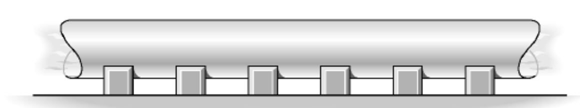


$$\sigma_1 = \frac{p\,r}{t} = \frac{60(2)}{0.2} = 600 \text{ psi} \qquad \textbf{Ans}$$

$$\sigma_2 = 0 \qquad \textbf{Ans}$$

There is no stress component in the longitudinal direction since the pipe has open ends.

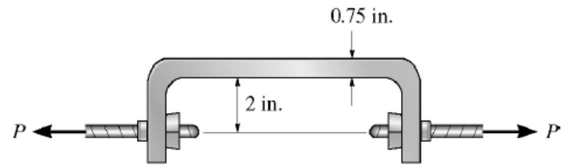
8–7. If the flow of water within the pipe in Prob. 8–6 is stopped due to the closing of a valve, determine the state of stress in the walls of the pipe. Neglect the weight of the water. Assume the supports only exert vertical forces on the pipe.



$$\sigma_1 = \frac{p\,r}{t} = \frac{60(2)}{0.2} = 600 \text{ psi} \qquad \textbf{Ans}$$

$$\sigma_2 = \frac{p\,r}{2\,t} = \frac{60(2)}{2(0.2)} = 300 \text{ psi} \qquad \textbf{Ans}$$

8–15. The steel bracket is used to connect the ends of two cables. If the allowable normal stress for the steel is $\sigma_{allow} = 24$ ksi, determine the largest tensile force P that can be applied to the cables. The bracket has a thickness of 0.5 in. and a width of 0.75 in.

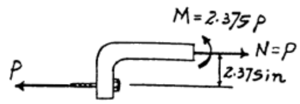


Internal Force and Moment: As shown on FBD.

Section Properties:

$$A = 0.5(0.75) = 0.375 \text{ in}^2$$

$$I = \frac{1}{12}(0.5)\left(0.75^3\right) = 0.01758 \text{ in}^4$$



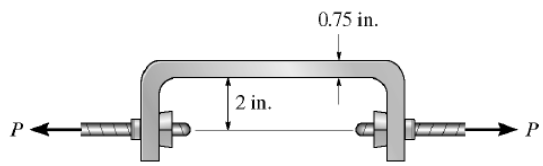
Allowable Normal Stress: The maximum normal stress occurs at the bottom of the steel bracket.

$$\sigma_{max} = \sigma_{allow} = \frac{N}{A} + \frac{Mc}{I}$$

$$24\left(10^3\right) = \frac{P}{0.375} + \frac{2.375P(0.375)}{0.01758}$$

$$P = 450 \text{ lb}$$ **Ans**

***8–16.** The steel bracket is used to connect the ends of two cables. If the applied force $P = 500$ lb, determine the maximum normal stress in the bracket. The bracket has a thickness of 0.5 in. and a width of 0.75 in.



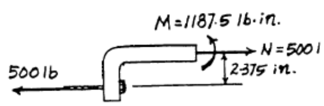
Internal Force and Moment: As shown on FBD.

Section Properties:

$$A = 0.5(0.75) = 0.375 \text{ in}^2$$

$$I = \frac{1}{12}(0.5)\left(0.75^3\right) = 0.01758 \text{ in}^4$$

Maximum Normal Stress: The maximum normal stress occurs at the bottom of the steel bracket.

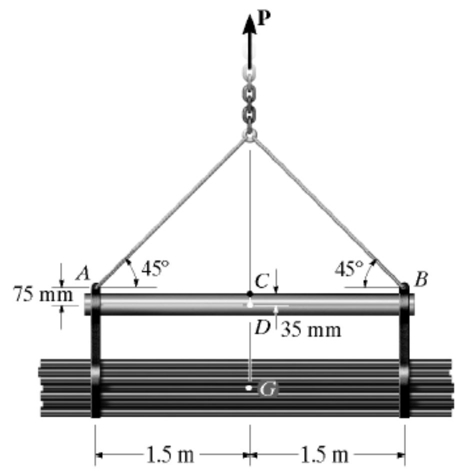


$$\sigma_{max} = \frac{N}{A} + \frac{Mc}{I}$$

$$= \frac{500}{0.375} + \frac{1187.5(0.375)}{0.01758}$$

$$= 26.7 \text{ ksi}$$ **Ans**

***8–48.** The strongback AB consists of a pipe that is used to lift the bundle of rods having a total mass of 3 Mg and center of mass at G. If the pipe has an outer diameter of 70 mm and a wall thickness of 10 mm, determine the state of stress acting at point D. Show the results on a differential volume element located at this point. Neglect the weight of the pipe.



Support Reactions:

$$+\uparrow \Sigma F_y = 0; \qquad 2F\sin 45° - 2(14715) = 0$$
$$F = 20\,810 \text{ N}$$

Internal Forces and Moment:

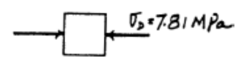
$$\xrightarrow{+} \Sigma F_x = 0; \qquad 20\,810\cos 45° + N = 0 \qquad N = -14715 \text{ N}$$

$$+\uparrow \Sigma F_y = 0; \qquad V + 20\,810\sin 45° - 14\,715 = 0 \qquad V = 0$$

$$\circlearrowleft + \Sigma M_O = 0; \qquad M + 14\,715(1.5) - 20\,810\cos 45°(0.075) + 20\,810\sin 45°(1.5) = 0$$
$$M = 1103.625 \text{ N}\cdot\text{m}$$

Section Properties:

$$A = \pi\left(0.035^2 - 0.025^2\right) = 0.600\pi\left(10^{-3}\right)\ \text{m}^2$$
$$I = \frac{\pi}{4}\left(0.035^4 - 0.025^4\right) = 0.2775\pi\left(10^{-6}\right)\ \text{m}^4$$



Normal Stress:

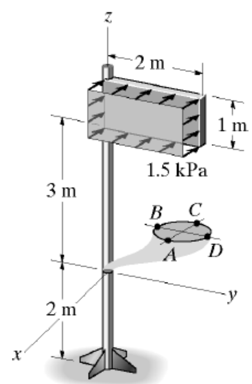
$$\sigma = \frac{N}{A} \pm \frac{My}{I}$$

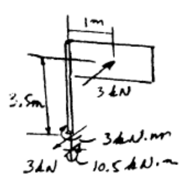
$$\sigma_D = \frac{-14\,715}{0.600\pi(10^{-3})} + \frac{1103.625(0)}{0.2775\pi(10^{-6})}$$
$$= -7.81 \text{ MPa} = 7.81 \text{ MPa (C)} \qquad \textbf{Ans}$$

Shear Stress: Since $V = 0$, then

$$\tau_D = 0 \qquad \textbf{Ans}$$

8–49. The sign is subjected to the uniform wind loading. Determine the stress components at points A and B on the 100-mm-diameter supporting post. Show the results on a volume element located at each of these points.

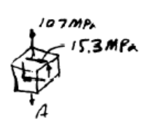




Point A:

$$\sigma_A = \frac{Mc}{I} = \frac{10.5(10^3)(0.05)}{\frac{\pi}{4}(0.05)^4} = 107 \text{ MPa} \qquad \textbf{Ans}$$

$$\tau_A = \frac{Tc}{J} = \frac{3(10^3)(0.05)}{\frac{\pi}{2}(0.05)^4} = 15.279(10^6) = 15.3 \text{ MPa} \qquad \textbf{Ans}$$

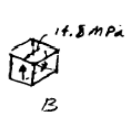


Point B:

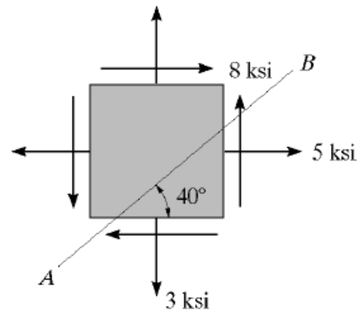
$\sigma_B = 0$ **Ans**

$$\tau_B = \frac{Tc}{J} - \frac{VQ}{It} = 15.279(10^6) - \frac{3000(4(0.05)/3\pi)(\frac{1}{2})(\pi)(0.05)^2}{\frac{\pi}{4}(0.05)^4(0.1)}$$

$\tau_B = 14.8$ MPa **Ans**



9–7. Solve Prob. 9–2 using the stress-transformation equations developed in Sec. 9.2.



$\sigma_x = 5$ ksi $\quad \sigma_y = 3$ ksi $\quad \tau_{xy} = 8$ ksi $\quad \theta = 130°$

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= \frac{5+3}{2} + \frac{5-3}{2}\cos 260° + 8\sin 260° = -4.05 \text{ ksi} \qquad \textbf{Ans}$$

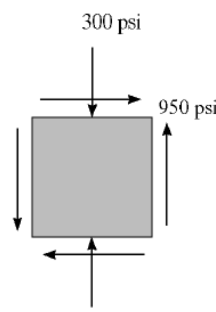
The negative sign indicates $\sigma_{x'}$ is a compressive stress.

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= -\left(\frac{5-3}{2}\right)\sin 260° + 8\cos 260° = -0.404 \text{ ksi} \qquad \textbf{Ans}$$

The negative sign indicates $\tau_{x'y'}$ is in the $-y'$ direction.

9–10. Determine the equivalent state of stress on an element if the element is oriented 30° counterclockwise from the element shown. Use the stress-transformation equations.



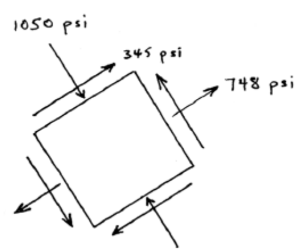
$\sigma_x = 0 \quad \sigma_y = -300$ psi $\quad \tau_{xy} = 950$ psi $\quad \theta = 30°$

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= \frac{0-300}{2} + \frac{0-(-300)}{2}\cos(60°) + 950\sin(60°) = 748 \text{ psi} \qquad \textbf{Ans}$$

$$\tau_{x'y'} = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta$$

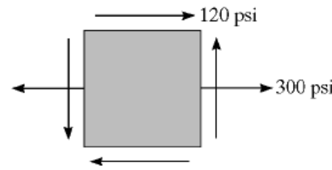
$$= -\left(\frac{0-(-300)}{2}\right)\sin(60°) + 950\cos(60°) = 345 \text{ psi} \qquad \textbf{Ans}$$



$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$= \frac{0-300}{2} - \left(\frac{0-(-300)}{2}\right)\cos(60°) - 950\sin(60°) = -1050 \text{ psi} \qquad \textbf{Ans}$$

9–11. Determine the equivalent state of stress on an element if the element is oriented 60° clockwise from the element shown.



Normal and Shear Stress: **In accordance with the established sign convention,**

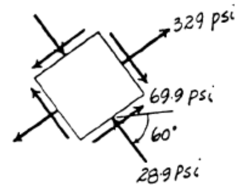
$\theta = -60°$ $\quad \sigma_x = 300$ psi $\quad \sigma_y = 0$ $\quad \tau_{xy} = 120$ psi

Stress Transformation Equations: Applying Eqs. 9 – 1, 9 – 2 and 9 – 3.

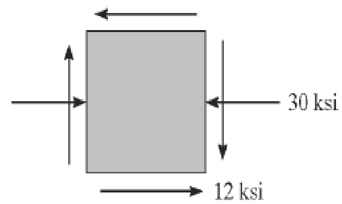
$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$
$$= \frac{300+0}{2} + \frac{300-0}{2}\cos(-120°) + [120\sin(-120°)]$$
$$= -28.9 \text{ psi} \qquad \textbf{Ans}$$

$$\sigma_{y'} = \frac{\sigma_x - \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$
$$= \frac{300-0}{2} - \frac{300-0}{2}\cos(-120°) - [120\sin(-120°)]$$
$$= 329 \text{ psi} \qquad \textbf{Ans}$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$
$$= -\frac{300-0}{2}\sin(-120°) + [120\cos(-120°)]$$
$$= 69.9 \text{ psi} \qquad \textbf{Ans}$$



9–15. The state of stress at a point is shown on the element. Determine (a) the principal stresses and (b) the maximum in-plane shear stress and average normal stress at the point. Specify the orientation of the element in each case.



$\sigma_x = -30$ ksi $\qquad \sigma_y = 0 \qquad \tau_{xy} = -12$ ksi

a)

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \frac{-30+0}{2} \pm \sqrt{\left(\frac{-30-0}{2}\right)^2 + (-12)^2}$$

$\sigma_1 = 4.21$ ksi **Ans** $\qquad \sigma_2 = -34.2$ ksi **Ans**

Orientation of principal stress:

$$\tan 2\theta_p = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2} = \frac{-12}{(-30-0)/2} = 0.8$$

$\theta_p = 19.33°$ and $-70.67°$

Use Eq. 9 - 1 to determine the principal plane of σ_1 and σ_2.

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy} \sin 2\theta$$

$\theta = 19.33°$

$$\sigma_{x'} = \frac{-30+0}{2} + \frac{-30-0}{2}\cos 2(19.33°) + (-12)\sin 2(19.33°) = -34.2 \text{ ksi}$$

Therefore $\theta_{p_2} = 19.3°$ **Ans** $\qquad$ and $\theta_{p_1} = -70.7°$ **Ans**

b)

$$\tau_{\max_{\text{in-plane}}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{-30-0}{2}\right)^2 + (-12)^2} = 19.2 \text{ ksi} \quad \textbf{Ans}$$

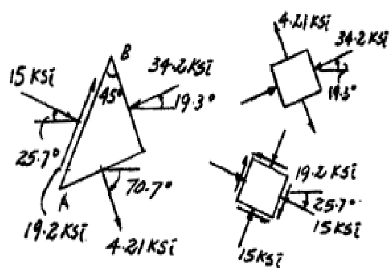
$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{-30+0}{2} = -15 \text{ ksi} \quad \textbf{Ans}$$

Orientation of max. in - plane shear stress :

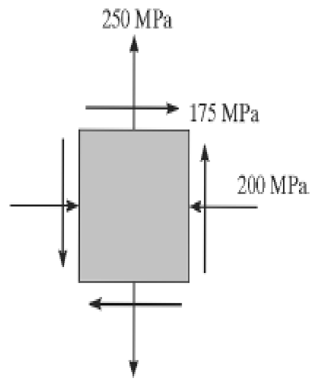
$$\tan 2\theta_s = \frac{-(\sigma_x - \sigma_y)/2}{\tau_{xy}} = \frac{-(-30-0)/2}{-12} = -1.25$$

$\theta_s = -25.7°$ and $64.3°$ **Ans**

By observation, in order to preserve equilibrium along *AB*, $\tau_{\max}$ has to act in the direction shown in the figure.



***9–16.** The state of stress at a point is shown on the element. Determine (a) the principal stresses and (b) the maximum in-plane shear stress and average normal stress at the point. Specify the orientation of the element in each case.



a) $\sigma_x = -200 \text{ MPa} \qquad \sigma_y = 250 \text{ MPa} \qquad \tau_{xy} = 175 \text{ MPa}$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

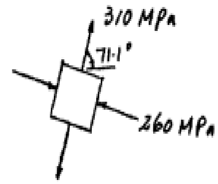
$$= \frac{-200 + 250}{2} \pm \sqrt{\left(\frac{-200 - 250}{2}\right)^2 + 175^2}$$

$\sigma_1 = 310 \text{ MPa} \qquad \sigma_2 = -260 \text{ MPa}$ **Ans**

Orientation of principal stress:

$$\tan 2\theta_p = \frac{\tau_{xy}}{\frac{\sigma_x - \sigma_y}{2}} = \frac{175}{\frac{-200-250}{2}} = -0.7777$$

$\theta_p = -18.94^\circ$ and 71.06°



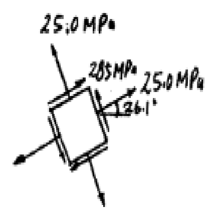
Use Eq. 9-1 to determine the principal plane of σ_1 and σ_2

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$\theta = \theta_p = -18.94^\circ$

$$\sigma_{x'} = \frac{-200 + 250}{2} + \frac{-200 - 250}{2}\cos(-37.88^\circ) + 175\sin(-37.88^\circ) = -260 \text{ MPa} = \sigma_2$$

Therefore $\theta_{p_1} = 71.1^\circ \qquad \theta_{p_2} = -18.9^\circ$ **Ans**



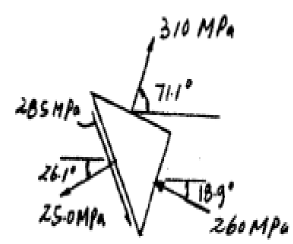
b) $$\tau_{\substack{max \\ in\text{-}plane}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{-200 - 250}{2}\right)^2 + 175^2} = 285 \text{ MPa}$$ **Ans**

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{-200 + 250}{2} = 25.0 \text{ MPa}$$ **Ans**

Orientation of maximum in-plane shear stress:

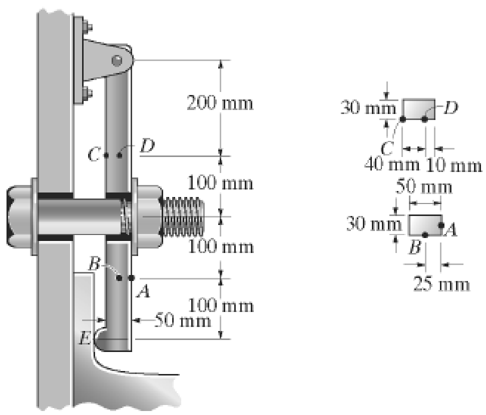
$$\tan 2\theta_s = -\frac{\frac{(\sigma_x - \sigma_y)}{2}}{\tau_{xy}} = -\frac{\frac{-200-250}{2}}{175} = 1.2857$$

$\theta_s = 26.1^\circ$ **Ans** and -63.9° **Ans**

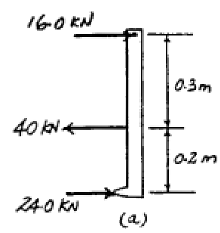


By observation, in order to preserve equilibrium, $\tau_{max} = 285$ MPa has to act in the direction shown in the figure.

9–22. The clamp bears down on the smooth surface at E by tightening the bolt. If the tensile force in the bolt is 40 kN, determine the principal stresses at points A and B and show the results on elements located at each of these points. The cross-sectional area at A and B is shown in the adjacent figure.



Support Reactions: As shown on FBD(a).
Internal Forces and Moment: As shown on FBD(b).



Section Properties:

$$I = \frac{1}{12}(0.03)\left(0.05^3\right) = 0.3125\left(10^{-6}\right)\ \text{m}^4$$

$$Q_A = 0$$

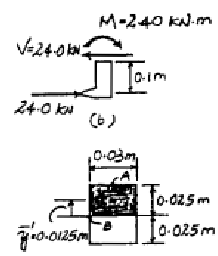
$$Q_B = \bar{y}'A' = 0.0125(0.025)(0.03) = 9.375\left(10^{-6}\right)\ \text{m}^3$$

Normal Stress: Applying the flexure formula $\sigma = -\dfrac{My}{I}$,

$$\sigma_A = -\frac{2.40(10^3)(0.025)}{0.3125(10^{-6})} = -192\ \text{MPa}$$

$$\sigma_B = -\frac{2.40(10^3)(0)}{0.3125(10^{-6})} = 0$$

Shear Stress: Applying the shear formula $\tau = \dfrac{VQ}{It}$.



$$\tau_A = \frac{24.0(10^3)(0)}{0.3125(10^{-6})(0.03)} = 0$$

$$\tau_B = \frac{24.0(10^3)\left[9.375(10^{-6})\right]}{0.3125(10^{-6})(0.03)} = 24.0\ \text{MPa}$$

In-Plane Principal Stresses: $\sigma_x = 0$, $\sigma_y = -192$ MPa, and $\tau_{xy} = 0$ for point A. Since no shear stress acts on the element.

$$\sigma_1 = \sigma_x = 0 \qquad \textbf{Ans}$$

$$\sigma_2 = \sigma_y = -192\ \text{MPa} \qquad \textbf{Ans}$$

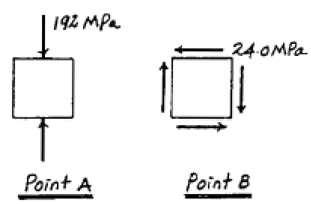
$\sigma_x = \sigma_y = 0$ and $\tau_{xy} = -24.0$ MPa for point B. Applying Eq. 9 – 5

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= 0 \pm \sqrt{0 + (-24.0)^2}$$

$$= 0 \pm 24.0$$

$$\sigma_1 = 24.0 \qquad \sigma_2 = -24.0\ \text{MPa} \qquad \textbf{Ans}$$



Orientation of Principal Plane: Applying Eq. 9 – 4 for point B,

$$\tan 2\theta_p = \frac{\tau_{xy}}{\left(\sigma_x - \sigma_y\right)/2} = \frac{-24.0}{0} = -\infty$$

$$\theta_p = -45.0^\circ \quad \text{and} \quad 45.0^\circ$$

Subsututing the results into Eq. 9 - 1 with $\theta = -45.0^\circ$ yields

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= 0 + 0 + [-24.0\sin(-90.0^\circ)]$$

$$= 24.0\ \text{MPa} = \sigma_1$$

Hence,

$$\theta_{p_1} = -45.0^\circ \qquad \theta_{p_2} = 45.0^\circ \qquad \textbf{Ans}$$