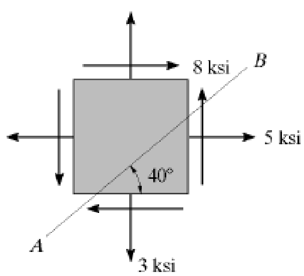
HW 22 Solutions

9–57. Solve Prob. 9–2 using Mohr's circle.



$$\frac{\sigma_x + \sigma_y}{2} = \frac{5+3}{2} = 4 \text{ ksi}$$

$$R = \sqrt{(5-4)^2 + 8^2} = 8.0623$$

$$\phi = \tan^{-1}\frac{8}{(5-4)} = 82.875°$$

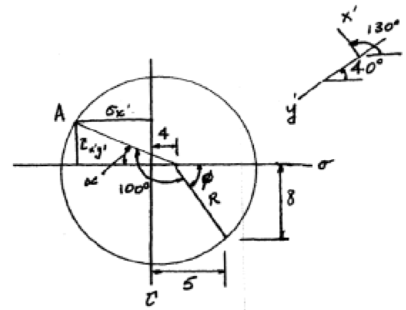
$2\theta = 2(130°) = 260°$

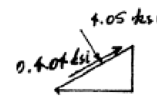
$360° - 260° = 100°$

$\alpha = 100° + 82.875° - 180° = 2.875°$

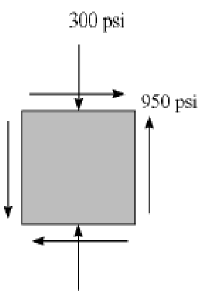
$\sigma_{x'} = 8.0623 \cos 2.875° - 4 = -4.05$ ksi **Ans**

$\tau_{x'y'} = -8.0623 \sin 2.875° = -0.404$ ksi **Ans**





9–59. Solve Prob. 9–10 using Mohr's circle.



$$\frac{\sigma_x + \sigma_y}{2} = \frac{0-300}{2} = -150 \text{ psi}$$

$$R = \sqrt{(150)^2 + (950)^2} = 961.769 \text{ psi}$$

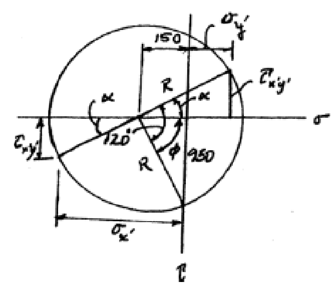
$$\phi = \tan^{-1}\frac{950}{150} = 81.0274°$$

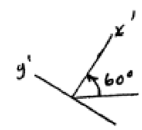
$\alpha = 180° - 60° - 81.0274° = 38.973°$

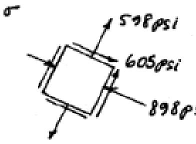
$\sigma_{x'} = -961.769 \cos 38.973° - 150 = -898$ psi **Ans**

$\tau_{x'y'} = 961.769 \sin 38.973° = 605$ psi **Ans**

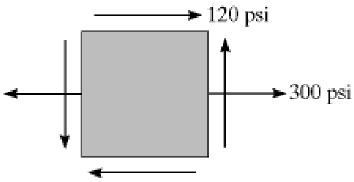
$\sigma_{y'} = 961.769 \cos 38.973 - 150 = 598$ psi **Ans**

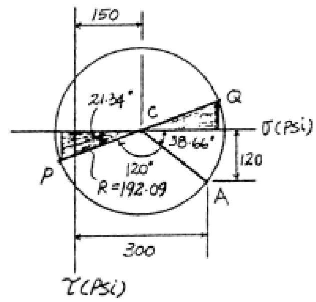






9–61. Solve Prob. 9–11 using Mohr's circle.





Construction of the Circle : In accordance with the sign convention, $\sigma_x = 300$ psi, $\sigma_y = 0$, and $\tau_{xy} = 120$ psi. Hence,

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{300 + 0}{2} = 150 \text{ psi}$$

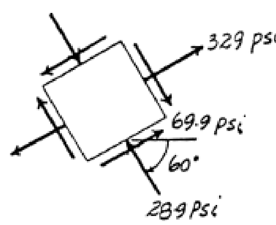
The coordinates for reference point A and C are

$$A(300,\ 120) \qquad C(150,\ 0)$$

The radius of the circle is

$$R = \sqrt{(300 - 150)^2 + 120^2} = 192.09 \text{ psi}$$

Stress on The Rotated Element : The normal and shear stress components $(\sigma_{x'}$ and $\tau_{x'y'})$ are represented by the coordinates of point P on the circle. $\sigma_{y'}$ can be determined by calculating the coordinates of point Q on the circle.

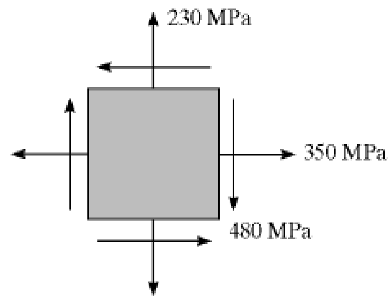


$$\sigma_{x'} = 150 - 192.09\cos 21.34° = -28.9 \text{ psi} \qquad \textbf{Ans}$$

$$\tau_{x'y'} = 192.09\sin 21.34° = 69.9 \text{ psi} \qquad \textbf{Ans}$$

$$\sigma_{y'} = 1.50 + 192.09\cos 21.34° = 329 \text{ psi} \qquad \textbf{Ans}$$

***9–68.** Determine the equivalent state of stress if an element is oriented 30° clockwise from the element shown.



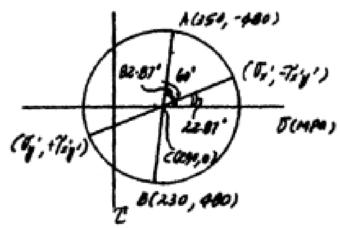
$A(350,-480)$ $\quad B(230,480)$ $\quad C(290,0)$

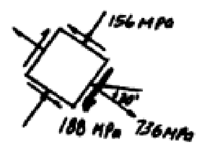
$R = \sqrt{60^2 + 480^2} = 483.73$

$\sigma_{x'} = 290 + 483.73 \cos 22.87° = 736$ MPa **Ans**

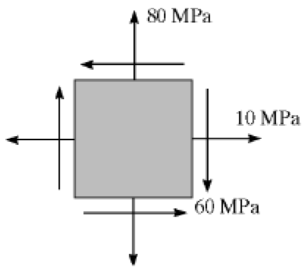
$\sigma_{y'} = 290 - 483.73 \cos 22.87° = -156$ MPa **Ans**

$\tau_{x'y'} = -483.73 \sin 22.87° = -188$ MPa **Ans**





9–71. Determine (a) the principal stresses and (b) the maximum in-plane shear stress and average normal stress. Specify the orientation of the element in each case.



Construction of the Circle: In accordance with the sign convention, $\sigma_x = 10$ MPa, $\sigma_y = 80$ MPa and $\tau_{xy} = -60$ MPa. Hence,

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{10 + 80}{2} = 45.0 \text{ MPa} \qquad \textbf{Ans}$$

The coordinates for reference points A and C are

$$A(10,\ -60) \qquad C(45.0,\ 0)$$

The radius of circle is

$$R = \sqrt{(45.0 - 10)^2 + 60^2} = 69.462 \text{ MPa}$$

a)

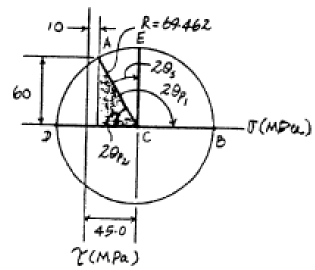
In - Plane Principal Stress: The coordinate of points B and D represent σ_1 and σ_2 respectively.

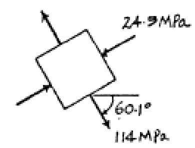
$$\sigma_1 = 45.0 + 69.462 = 114 \text{ MPa} \qquad \textbf{Ans}$$
$$\sigma_2 = 45.0 - 570.64 = -24.5 \text{ MPa} \qquad \textbf{Ans}$$

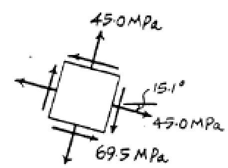
Orientation of Principal Plane: From the circle

$$\tan 2\theta_{p_2} = \frac{60}{45.0 - 10} = 1.7143 \qquad 2\theta_{p_2} = 59.74$$

$$2\theta_{p_1} = 180° - 2\theta_{p_2}$$
$$\theta_{p_1} = \frac{180° - 59.74°}{2} = 60.1° \quad (\textit{Clockwise}) \qquad \textbf{Ans}$$







b)

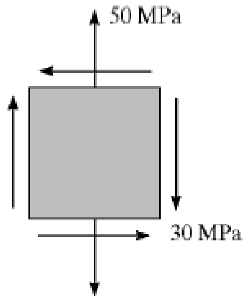
Maximum In - Plane Shear Stress: Represented by the coordinate of point E on the circle.

$$\tau_{\substack{max \\ in\text{-}plane}} = -R = -69.5 \text{ MPa} \qquad \textbf{Ans}$$

Orientation of the Plane for Maximum In - Plane Shear Stress: From the circle

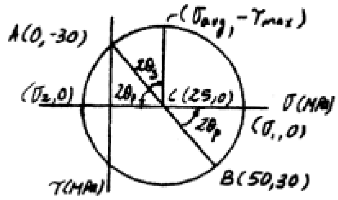
$$\tan 2\theta_s = \frac{45.0 - 10}{60} = 0.5833$$
$$\theta_s = 15.1° \quad (\textit{Clockwise}) \qquad \textbf{Ans}$$

***9–72.** Determine (a) the principal stresses and (b) the maximum in-plane shear stress and average normal stress. Specify the orientation of the element in each case.



$A(0,-30)$ $\quad B(50,30)$ $\quad C(25,0)$

$R = CA = CB = \sqrt{25^2 + 30^2} = 39.05$

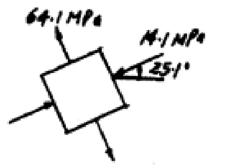


a)

$\sigma_1 = 25 + 39.05 = 64.1$ MPa **Ans**

$\sigma_2 = 25 - 39.05 = -14.1$ MPa **Ans**

$\tan 2\theta_p = \dfrac{30}{25}$ $\quad 2\theta_p = 50.19°$ $\quad \theta_p = 25.1°$



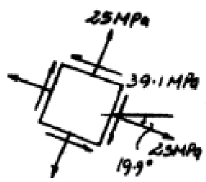
b)

$\tau_{\substack{max \\ in-plane}} = R = 39.1$ MPa **Ans**

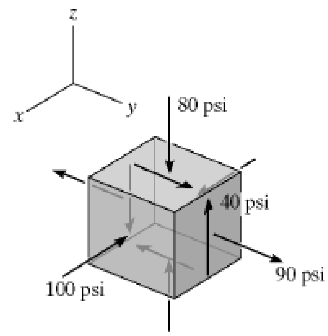
$\sigma_{avg} = 25$ MPa **Ans**

$2\theta_s = 90 - 2\theta_p$

$\theta_s = -19.9°$

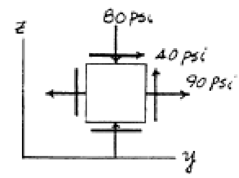


9–90. The stress at a point is shown on the element. Determine the principal stresses and the absolute maximum shear stress.



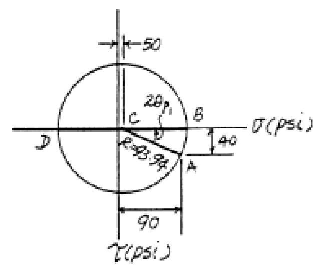
Construction of the Circle: Mohr's circle for the element in the $y-z$ plane is drawn first. In accordance with the sign convention, $\sigma_y = 90$ psi, $\sigma_z = -80$ psi, and $\tau_{yz} = 40$ psi. Hence,

$$\sigma_{avg} = \frac{\sigma_y + \sigma_z}{2} = \frac{90 + (-80)}{2} = 5.00 \text{ psi}$$



The coordinates for reference points A and C are $A(90, 40)$ $C(5.00, 0)$.

The radius of the circle is $R = \sqrt{(90 - 5.00)^2 + 40^2} = 93.94$ psi



In-Plane Principal Stress: The coordinates of points A and B represent σ_1 and σ_2, respectively.

$$\sigma_1 = 5.00 + 96.94 = 98.94 \text{ psi}$$
$$\sigma_2 = 5.00 - 96.94 = -88.94 \text{ psi}$$

Construction of Three Mohr's Circles: From the results obtained above,

$\sigma_{max} = 98.9$ psi $\quad \sigma_{int} = -88.9$ psi $\quad \sigma_{min} = -100$ psi **Ans**

Absolute Maximum Shear Stress: From the three Mohr's circles

$$\tau_{\substack{abs \\ max}} = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{98.94 - (-100)}{2} = 99.5 \text{ psi} \quad \textbf{Ans}$$