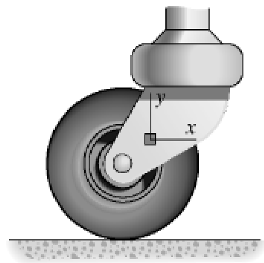
HW 23 SOLUTIONS

10–2. The state of strain at the point on the leaf of the caster assembly has components of $\epsilon_x = -400(10^{-6})$, $\epsilon_y = 860(10^{-6})$, and $\gamma_{xy} = 375(10^{-6})$. Use the strain-transformation equations to determine the equivalent in-plane strains on an element oriented at an angle of $\theta = 30°$ counterclockwise from the original position. Sketch the deformed element due to these strains within the x–y plane.



Normal Strain and Shear Strain: In accordance with the sign convention,

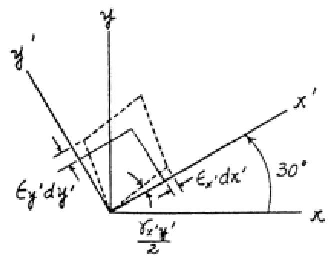
$$\varepsilon_x = -400(10^{-6}) \qquad \varepsilon_y = 860(10^{-6}) \qquad \gamma_{xy} = 375(10^{-6})$$
$$\theta = +30°$$

Strain Transformation Equations: Applying Eqs. 10 – 5, 10 – 6, and 10 – 7,

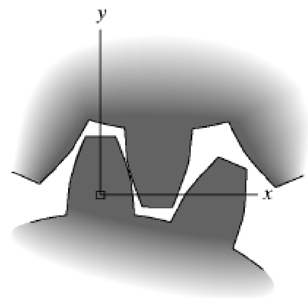
$$\varepsilon_{x'} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$
$$= \left(\frac{-400 + 860}{2} + \frac{-400 - 860}{2}\cos 60° + \frac{375}{2}\sin 60°\right)(10^{-6})$$
$$= 77.4(10^{-6}) \qquad \textbf{Ans}$$

$$\frac{\gamma_{x'y'}}{2} = -\frac{\varepsilon_x - \varepsilon_y}{2}\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta$$
$$\gamma_{x'y'} = [-(-400 - 860)\sin 60° + 375\cos 60°](10^{-6})$$
$$= 1279(10^{-6}) \qquad \textbf{Ans}$$

$$\varepsilon_{y'} = \frac{\varepsilon_x + \varepsilon_y}{2} - \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta - \frac{\gamma_{xy}}{2}\sin 2\theta$$
$$= \left(\frac{-400 + 860}{2} - \frac{-400 - 860}{2}\cos 60° - \frac{375}{2}\sin 60°\right)(10^{-6})$$
$$= 383(10^{-6}) \qquad \textbf{Ans}$$



*10–8. The state of strain at the point on the gear tooth has the components $\epsilon_x = 520(10^{-6})$, $\epsilon_y = -760(10^{-6})$, $\gamma_{xy} = -750(10^{-6})$. Use the strain-transformation equations to determine (a) the in-plane principal strains and (b) the maximum in-plane shear strain and average normal strain. In each case specify the orientation of the element and show how the strains deform the element within the x–y plane.

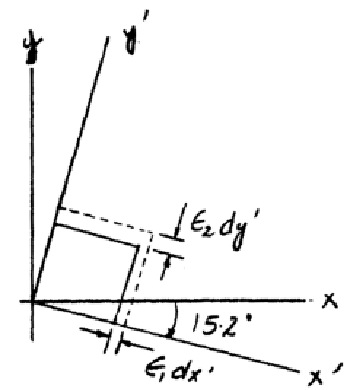


$$\varepsilon_x = 520(10^{-6}) \qquad \varepsilon_y = -760(10^{-6}) \qquad \gamma_{xy} = -750(10^{-6})$$

a) $$\varepsilon_{1,2} = \frac{\varepsilon_x + \varepsilon_y}{2} \pm \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$= \left[\frac{520 + (-760)}{2} \pm \sqrt{\left(\frac{520 - (-760)}{2}\right)^2 + \left(\frac{-750}{2}\right)^2}\,\right] 10^{-6}$$

$$\varepsilon_1 = 622\,(10^{-6})\,; \qquad \varepsilon_2 = -862(10^{-6})$$ **Ans**



Orientation of ε_1 and ε_2

$$\tan 2\theta_p = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \frac{-750}{[520 - (-760)]} = -0.5859\,; \qquad \theta_p = -15.18^\circ \text{ and } \theta_p = 74.82^\circ$$

Use Eq. 10 - 5 to determine the direction of ε_1 and ε_2.

$$\varepsilon_{x'} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$\theta = \theta_p = -15.18^\circ$$

$$\varepsilon_{x'} = \left[\frac{520 + (-760)}{2} + \frac{520 - (-760)}{2}\cos(-30.36^\circ) + \frac{-750}{2}\sin(-30.36^\circ)\right] 10^{-6}$$

$$= 622\,(10^{-6}) = \varepsilon_1$$

Therefore $\theta_{p_1} = -15.2^\circ$ and $\theta_{p_2} = 74.8^\circ$ **Ans**

b) $$\frac{\gamma_{\substack{max\\ in\text{-}plane}}}{2} = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

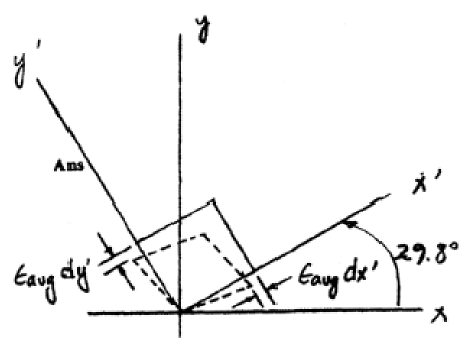
$$\gamma_{\substack{max\\ in\text{-}plane}} = 2\left[\sqrt{\left(\frac{520 - (-760)}{2}\right)^2 + \left(\frac{-750}{2}\right)^2}\,\right]10^{-6} = -1484\,(10^{-6})$$ **Ans**

$$\varepsilon_{avg} = \frac{\varepsilon_x + \varepsilon_y}{2} = \left[\frac{520 + (-760)}{2}\right]10^{-6} = -120\,(10^{-6})$$ **Ans**

Orientation of $\gamma_{\substack{max\\ in\text{-}plane}}$:

$$\tan 2\theta_s = \frac{-(\varepsilon_x - \varepsilon_y)}{\gamma_{xy}} = \frac{-[520 - (-760)]}{-750} = 1.7067$$

$$\theta_s = 29.8^\circ \text{ and } \theta_s = -60.2^\circ$$

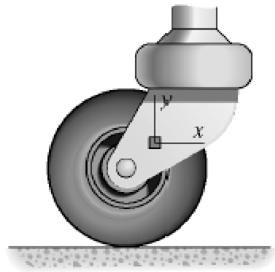


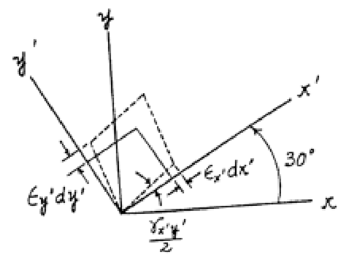
Use Eq. 10 - 6 to check the sign of $\gamma_{\substack{max\\ in\text{-}plane}}$.

$$\frac{\gamma_{x'y'}}{2} = -\frac{\varepsilon_x - \varepsilon_y}{2}\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta\,; \qquad \theta = \theta_s = 29.8^\circ$$

$$\gamma_{x'y'} = 2\left[-\frac{520 - (-760)}{2}\sin(59.6^\circ) + \frac{-750}{2}\cos(59.6^\circ)\right]10^{-6} = -1484\,(10^{-6})$$

10–15. Solve Prob. 10–2 using Mohr's circle.





Construction of the Circle: In accordance with the sign convent $\epsilon_x = -400(10^{-6})$, $\epsilon_y = 860(10^{-6})$ and $\frac{\gamma_{xy}}{2} = 187.5(10^{-6})$. Hence,

$$\epsilon_{avg} = \frac{\epsilon_x + \epsilon_y}{2} = \left(\frac{-400+860}{2}\right)(10^{-6}) = 230(10^{-6})$$

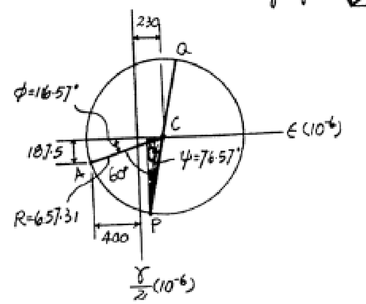
The coordinates for reference points A and C are

$$A(-400,\ 187.5)(10^{-6}) \qquad C(230,\ 0)(10^{-6})$$

The radius of the circle is

$$R = \left(\sqrt{(400+230)^2 + 187.5^2}\right)(10^{-6}) = 657.31(10^{-6})$$

Strain on the Inclined Element: The normal and shear strain $\left(\epsilon_{x'} \text{ and } \frac{\gamma_{x'y'}}{2}\right)$ are represented by the coordinates of point P on the circle. $\epsilon_{y'}$ can be determined by calculating the coordinates of point Q on the circle.



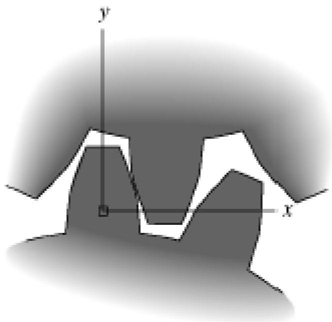
$$\epsilon_{x'} = (230 - 657.31\cos 76.57°)(10^{-6}) = 77.4(10^{-6}) \qquad \textbf{Ans}$$

$$\frac{\gamma_{x'y'}}{2} = (657.31\sin 76.57°)(10^{-6})$$

$$\gamma_{x'y'} = 1279(10^{-6}) \qquad \textbf{Ans}$$

$$\epsilon_{y'} = (230 + 657.31\cos 76.57°)(10^{-6}) = 383(10^{-6}) \qquad \textbf{Ans}$$

***10–20.** Solve Prob. 10–8 using Mohr's circle.



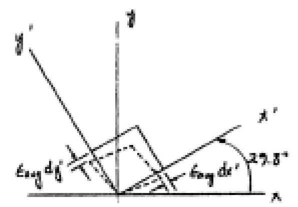
a) $\varepsilon_x = 520(10^{-6})$ $\quad \varepsilon_y = -760(10^{-6})$ $\quad \gamma_{xy} = -750(10^{-6})$ $\quad \frac{\gamma_{xy}}{2} = -375(10^{-6})$

$A\,(520,\ -375);\quad C\,(-120,\ 0)$

$R = \sqrt{(520 + 120)^2 + 375^2} = 741.77$

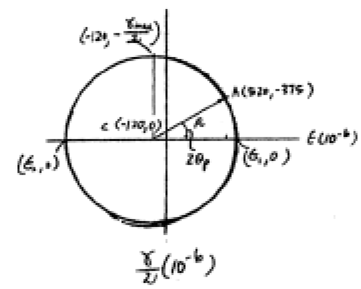
$\varepsilon_1 = 741.77 - 120 = 622(10^{-6})$ **Ans**

$\varepsilon_2 = -120 - 741.77 = -862(10^{-6})$ **Ans**



$$\tan 2\theta_{p_1} = \frac{375}{(120 + 520)} = 0.5859$$

$\theta_{p_1} = 15.2°$ **Ans**



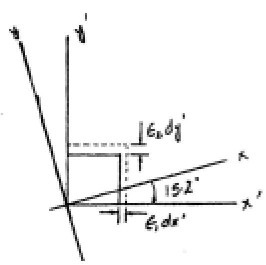
b) $\gamma_{\substack{max \\ in\text{-}plane}} = 2R = 2(741.77)$

$\gamma_{\substack{max \\ in\text{-}plane}} = -1484(10^{-6})$ **Ans**

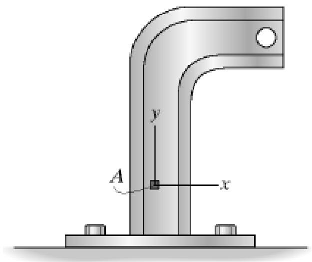
$\varepsilon_{avg} = -120\left(10^{-6}\right)$ **Ans**

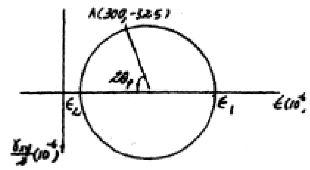
$$\tan 2\theta_s = \frac{(120 + 520)}{375} = 1.7067$$

$\theta_s = 29.8°$ **Ans**



10–23. The strain at point A on the bracket has components $\epsilon_x = 300(10^{-6})$, $\epsilon_y = 550(10^{-6})$, $\gamma_{xy} = -650(10^{-6})$, $\epsilon_z = 0$. Determine (a) the principal strains at A, (b) the maximum shear strain in the x–y plane, and (c) the absolute maximum shear strain.





$\varepsilon_x = 300(10^{-6})$ $\quad \varepsilon_y = 550(10^{-6})$ $\quad \gamma_{xy} = -650(10^{-6})$ $\quad \dfrac{\gamma_{xy}}{2} = -325(10^{-6})$

$A(300, -325)10^{-6}$ $\quad C(425, 0)10^{-6}$

$R = [\sqrt{(425-300)^2 + (-325)^2}]10^{-6} = 348.2(10^{-6})$

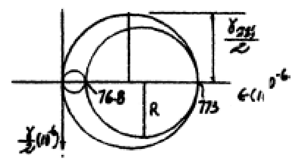
a)

$\varepsilon_1 = (425 + 348.2)(10^{-6}) = 773(10^{-6})$ **Ans**

$\varepsilon_2 = (425 - 348.2)(10^{-6}) = 76.8(10^{-6})$ **Ans**

b)

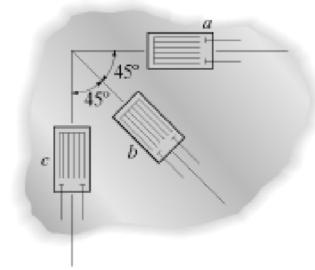
$\gamma_{\substack{max \\ in\text{-}plane}} = 2R = 2(348.2)(10^{-6}) = 696(10^{-6})$ **Ans**



c)

$\dfrac{\gamma_{\substack{abs \\ max}}}{2} = \dfrac{773(10^{-6})}{2}; \qquad \gamma_{\substack{abs \\ max}} = 773(10^{-6})$ **Ans**

***10–28.** The 45° strain rosette is mounted on the surface of an aluminum plate. The following readings are obtained for each gauge: $\epsilon_a = 475(10^{-6})$, $\epsilon_b = 250(10^{-6})$, and $\epsilon_c = -360(10^{-6})$. Determine the in-plane principal strains.



***Strain Rosettes* (45°):** Applying the equations in the text with $\varepsilon_a = 475(10^{-6})$, $\varepsilon_b = 250(10^{-6})$, $\varepsilon_c = -360(10^{-6})$, $\theta_a = 0°$, $\theta_b = -45°$, and $\theta_c = -90°$.

$$475(10^{-6}) = \varepsilon_x \cos^2 0° + \varepsilon_y \sin^2 0° + \gamma_{xy} \sin 0° \cos 0°$$

$$\varepsilon_x = 475(10^{-6})$$

$$250(10^{-6}) = 475(10^{-6})\cos^2(-45°) + \varepsilon_y \sin^2(-45°) + \gamma_{xy}\sin(-45°)\cos(-45°)$$

$$250(10^{-6}) = 237.5(10^{-6}) + 0.5\,\varepsilon_y - 0.5\,\gamma_{xy}$$

$$0.5\varepsilon_y - 0.5\,\gamma_{xy} = 12.5(10^{-6}) \qquad [1]$$

$$-360(10^{-6}) = 475(10^{-6})\cos^2(-90°) + \varepsilon_y \sin^2(-90°) + \gamma_{xy}\sin(-90°)\cos(-90°)$$

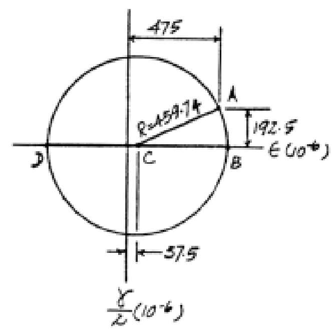
$$\varepsilon_y = -360(10^{-6})$$

From Eq. [1], $\gamma_{xy} = -385(10^{-6})$

Therefore, $\varepsilon_x = 475(10^{-6})$ $\quad \varepsilon_y = -360(10^{-6})$ $\quad \gamma_{xy} = -385(10^{-6})$

Construction of the Circle: With $\dfrac{\gamma_{xy}}{2} = -192.5(10^{-6})$ and

$$\varepsilon_{avg} = \frac{\varepsilon_x + \varepsilon_y}{2} = \left(\frac{475 + (-360)}{2}\right)(10^{-6}) = 57.5(10^{-6})$$



The coordinates for reference points A and C are

$$A(475,\ -192.5)(10^{-6}) \qquad C(57.5,\ 0)(10^{-6})$$

The radius of the circle is

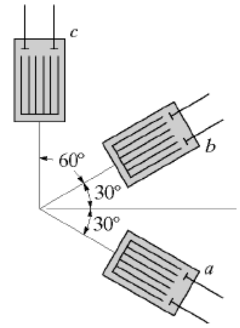
$$R = \left(\sqrt{(475 - 57.5)^2 + 192.5^2}\right)(10^{-6}) = 459.74(10^{-6})$$

In-Plane Principal Strain: The coordinates of points B and D represent ε_1 and ε_2, respectively.

$$\varepsilon_1 = (57.5 + 459.74)(10^{-6}) = 517(10^{-6}) \qquad \textbf{Ans}$$

$$\varepsilon_2 = (57.5 - 459.74)(10^{-6}) = -402(10^{-6}) \qquad \textbf{Ans}$$

10–31. The 60° strain rosette is mounted on a beam. The following readings are obtained from each gauge: $\epsilon_a = 150(10^{-6})$, $\epsilon_b = -330(10^{-6})$, and $\epsilon_c = 400(10^{-6})$. Determine (a) the in-plane principal strains and (b) the maximum in-plane shear strain and average normal strain. In each case show the deformed element due to these strains.



***Strain Rosettes* (60°):** Applying the equations in the text with $\varepsilon_a = 150(10^{-6})$, $\varepsilon_b = -330(10^{-6})$, $\varepsilon_c = 400(10^{-6})$, $\theta_a = -30°$, $\theta_b = 30°$ and $\theta_c = 90°$,

$$400(10^{-6}) = \varepsilon_x \cos^2 90° + \varepsilon_y \sin^2 90° + \gamma_{xy} \sin 90° \cos 90°$$

$$\varepsilon_y = 400(10^{-6})$$

$$400(10^{-6}) = \varepsilon_x \cos^2 90° + \varepsilon_y \sin^2 90° + \gamma_{xy} \sin 90° \cos 90°$$

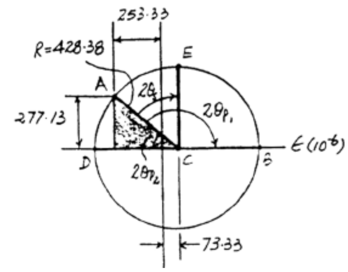
$$\varepsilon_y = 400(10^{-6})$$

$$150(10^{-6}) = \varepsilon_x \cos^2(-30°) + 400(10^{-6}) \sin^2(-30°) + \gamma_{xy} \sin(-30°)\cos(-30°)$$

$$50.0(10^{-6}) = 0.75\,\varepsilon_x - 0.4330\,\gamma_{xy} \qquad [1]$$

$$-330(10^{-6}) = \varepsilon_x \cos^2 30° + 400(10^{-6}) \sin^2 30° + \gamma_{xy} \sin 30° \cos 30°$$

$$-430(10^{-6}) = 0.75\,\varepsilon_x + 0.4330\,\gamma_{xy} \qquad [2]$$



Construction of the Circle: With $\varepsilon_x = -253.33(10^{-6})$, $\varepsilon_y = 400(10^{-6})$, and $\frac{\gamma_{xy}}{2} = -277.13(10^{-6})$

$$\varepsilon_{avg} = \frac{\varepsilon_x + \varepsilon_y}{2} = \left(\frac{-253.33+400}{2}\right)(10^{-6}) = 73.3(10^{-6}) \qquad \textbf{Ans}$$

Coordinates for reference points A and C are

$$A(-253.33,\ -277.13)(10^{-6}) \qquad C(73.33,\ 0)(10^{-6})$$

The radius of the circle is

$$R = \left(\sqrt{(253.33+73.33)^2 + 277.13^2}\right)(10^{-6}) = 428.38(10^{-6})$$

a)

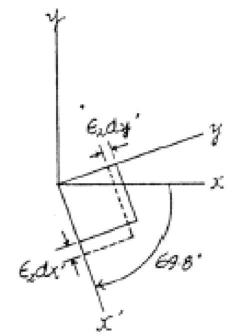
In-Plane Principal Strain: The coordinates of points B and D represent ε_1 and ε_2, respectively.

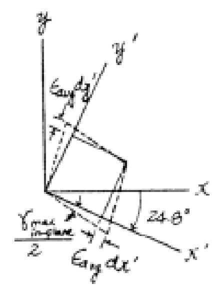
$$\varepsilon_1 = (73.33+428.38)(10^{-6}) = 502(10^{-6}) \qquad \textbf{Ans}$$
$$\varepsilon_2 = (73.33-428.38)(10^{-6}) = -355(10^{-6}) \qquad \textbf{Ans}$$

Orientation of Principal Strain: From the circle,

$$\tan 2\theta_{p_2} = \frac{277.13}{253.33+73.33} = 0.8484 \qquad 2\theta_{p_2} = 40.31°$$

$$2\theta_{p_1} = 180° - 2\theta_{p_2}$$
$$\theta_{p_1} = \frac{180° - 40.31°}{2} = 69.8° \quad \textit{(Clockwise)} \qquad \textbf{Ans}$$





b)

Maximum In-Plane Shear Strain: Represented by the coordinates of point E on the circle.

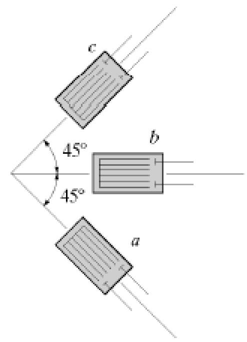
$$\frac{\gamma_{\substack{max\\in\text{-}plane}}}{2} = -R = -428.38(10^{-6})$$
$$\gamma_{\substack{max\\in\text{-}plane}} = -857(10^{-6}) \qquad \textbf{Ans}$$

Orientation of Maximum In-Plane Shear Strain: From the circle.

$$\tan 2\theta_s = \frac{253.33+73.33}{277.13} = 1.1788$$

$$\theta_s = 24.8° \quad \textit{(Clockwise)} \qquad \textbf{Ans}$$

***10–32.** The 45° strain rosette is mounted on a steel shaft. The following readings are obtained from each gauge: $\epsilon_a = 800(10^{-6})$, $\epsilon_b = 520(10^{-6})$, $\epsilon_c = -450(10^{-6})$. Determine the in-plane principal strains and their orientation.



$\varepsilon_a = 800(10^{-6})$ $\varepsilon_b = 520(10^{-6})$ $\varepsilon_c = -450(10^{-6})$

$\theta_a = -45°$ $\theta_b = 0°$ $\theta_c = 45°$

$$\varepsilon_b = \varepsilon_x \cos^2\theta_b + \varepsilon_y \sin^2\theta_b + \gamma_{xy}\sin\theta_b\cos\theta_b$$

$$520(10^{-6}) = \varepsilon_x \cos^2 0° + \varepsilon_y \sin^2 0° + \gamma_{xy}\sin 0° \cos 0°$$

$$\varepsilon_x = 520(10^{-6})$$

$$\varepsilon_a = \varepsilon_x \cos^2\theta_a + \varepsilon_y \sin^2\theta_a + \gamma_{xy}\sin\theta_a\cos\theta_a$$

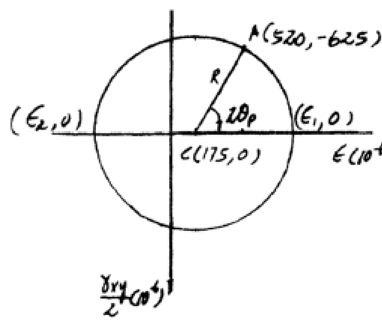
$$800(10^{-6}) = \varepsilon_x \cos^2(-45°) + \varepsilon_y \sin^2(-45°) + \gamma_{xy}\sin(-45°)\cos(-45°)$$

$$800(10^{-6}) = 0.5\varepsilon_x + 0.5\varepsilon_y - 0.5\gamma_{xy} \quad (1)$$

$$\varepsilon_c = \varepsilon_x \cos^2\theta_c + \varepsilon_y \sin^2\theta_c + \gamma_{xy}\sin\theta_c\cos\theta_c$$

$$-450(10^{-6}) = \varepsilon_x \cos^2 45° + \varepsilon_y \sin^2 45° + \gamma_{xy}\sin 45° \cos 45°$$

$$-450(10^{-6}) = 0.5\varepsilon_x + 0.5\varepsilon_y + 0.5\gamma_{xy} \quad (2)$$



Subtract Eq. (2) from Eq. (1)

$$1250(10^{-6}) = -\gamma_{xy}$$

$$\gamma_{xy} = -1250(10^{-6})$$

$$\varepsilon_y = -170(10^{-6})$$

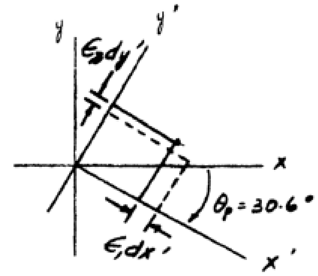
$$\frac{\gamma_{xy}}{2} = -625(10^{-6})$$

$A(520,-625)10^{-6}$ $C(175,0)10^{-6}$

$$R = [\sqrt{(520-175)^2 + 625^2}]10^{-6} = 713.90(10^{-6})$$

$$\varepsilon_1 = (175 + 713.9)10^{-6} = 889(10^{-6}) \quad \textbf{Ans}$$

$$\varepsilon_2 = (175 - 713.9)10^{-6} = -539(10^{-6}) \quad \textbf{Ans}$$



$$\tan 2\theta_p = \frac{625}{520-175}$$

$2\theta_p = 61.1°$ (Mohr's circle)

$\theta_p = -30.6°$ (element) **Ans**