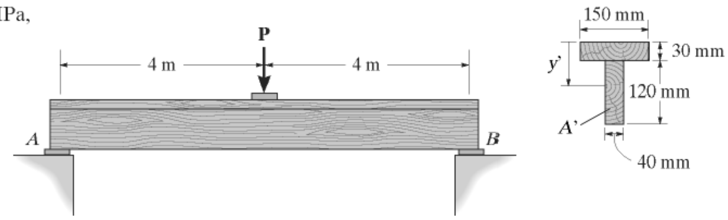
HW 24 Solutions

11–3. The timber beam is to be loaded as shown. If the ends support only vertical forces, determine the greatest magnitude of **P** that can be applied. $\sigma_{allow} = 25$ MPa, $\tau_{allow} = 700$ kPa.



$$\bar{y} = \frac{(0.015)(0.150)(0.03) + (0.09)(0.04)(0.120)}{(0.150)(0.03) + (0.04)(0.120)} = 0.05371 \text{ m}$$

$$I = \frac{1}{12}(0.150)(0.03)^3 + (0.15)(0.03)(0.05371 - 0.015)^2 + \frac{1}{12}(0.04)(0.120)^3 + (0.04)(0.120)(0.09 - 0.05371)^2 = 19.162(10^{-6}) \text{ m}^4$$

Maximum moment at center of beam:

$$M_{max} = \frac{P}{2}(4) = 2P$$

$$\sigma = \frac{Mc}{I}; \qquad 25(10^6) = \frac{(2P)(0.15 - 0.05371)}{19.162(10^{-6})}$$

$$P = 2.49 \text{ kN}$$

Maximum shear at end of beam:

$$V_{max} = \frac{P}{2}$$

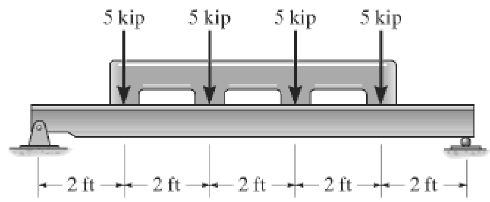
$$\tau = \frac{VQ}{It}; \qquad 700(10^3) = \frac{P\left[\frac{1}{2}(0.15 - 0.05371)(0.04)(0.15 - 0.05371)\right]}{19.162(10^{-6})(0.04)}$$

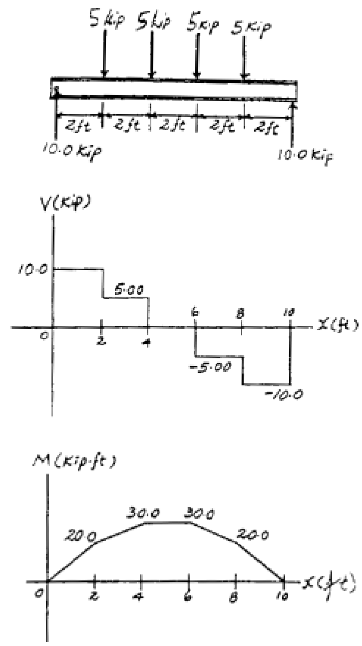
$$P = 5.79 \text{ kN}$$

Thus,

$P = 2.49$ kN **Ans**

*11–4. Select the lightest-weight steel wide-flange beam from Appendix B that will safely support the machine loading shown. The allowable bending stress is $\sigma_{allow} = 24$ ksi and the allowable shear stress is $\tau_{allow} = 14$ ksi.





Bending Stress: From the moment diagram, $M_{max} = 30.0$ kip · ft. Assume bending controls the design. Applying the flexure formula.

$$S_{req'd} = \frac{M_{max}}{\sigma_{allow}} = \frac{30.0(12)}{24} = 15.0 \text{ in}^3$$

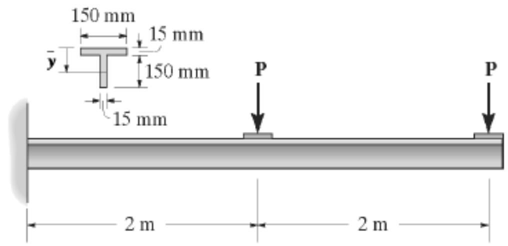
Select W12 × 16 $\left(S_x = 17.1 \text{ in}^3,\ d = 11.99 \text{ in.},\ t_w = 0.220 \text{ in.}\right)$

Shear Stress: Provide a shear stress check using $\tau = \frac{V}{t_w d}$ for the W12 × 16 wide-flange section. From the shear diagram, $V_{max} = 10.0$ kip

$$\tau_{max} = \frac{V_{max}}{t_w d} = \frac{10.0}{0.220(11.99)} = 3.79 \text{ ksi} < \tau_{allow} = 14 \text{ ksi } (O.K!)$$

Hence, **Use** W12 × 16 **Ans**

11–17. The steel cantilevered T-beam is made from two plates welded together as shown. Determine the maximum loads P that can be safely supported on the beam if the allowable bending stress is $\sigma_{allow} = 170$ MPa and the allowable shear stress is $\tau_{allow} = 95$ MPa.



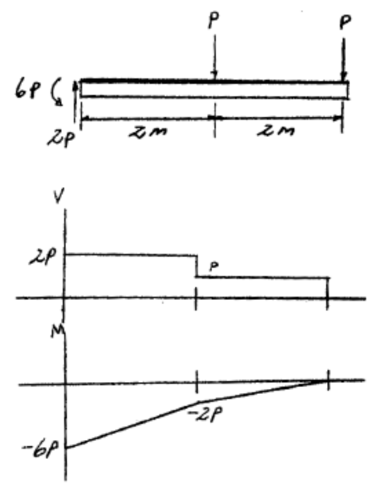
Section properties:

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{0.0075(0.15)(0.015) + 0.09(0.15)(0.015)}{0.15(0.015) + 0.15(0.015)} = 0.04875 \text{ m}$$

$$I = \frac{1}{12}(0.15)(0.015)^3 + 0.15(0.015)(0.04875 - 0.0075)^2$$
$$+ \frac{1}{12}(0.015)(0.15)^3 + 0.015(0.15)(0.09 - 0.04875)^2 = 11.9180(10^{-6}) \text{ m}^4$$

$$S = \frac{I}{c} = \frac{11.9180(10^{-6})}{(0.165 - 0.04875)} = 0.10252(10^{-3}) \text{ m}^3$$

$$Q_{max} = \bar{y}'A' = (\frac{0.165 - 0.04875}{2})(0.165 - 0.04875)(0.015) = 0.101355(10^{-3}) \text{ m}^3$$



Maximum load: **Assume failure due to bending moment.**

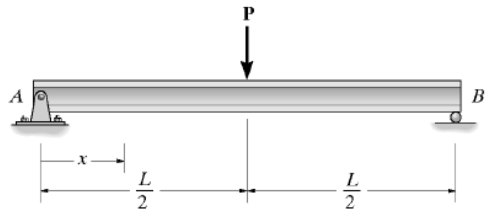
$$M_{max} = \sigma_{allow} S; \quad 6P = 170(10^6)(0.10252)(10^{-3})$$

$$P = 2904.7 \text{ N} = 2.90 \text{ kN} \qquad \textbf{Ans}$$

Check shear:

$$\tau_{max} = \frac{V_{max} Q_{max}}{I t} = \frac{2(2904.7)(0.101353)(10^{-3})}{11.9180(10^{-6})(0.015)} = 3.29 \text{ MPa} < \tau_{allow} = 95 \text{ MPa}$$

12–3. Determine the equation of the elastic curve for the beam using the x coordinate that is valid for $0 \le x < L/2$. Specify the slope at A and the beam's maximum deflection. EI is constant.



Support Reactions and Elastic Curve: As shown on FBD(a).

Moment Function: As shown on FBD(b).

Slope and Elastic Curve:

$$EI\frac{d^2v}{dx^2} = M(x)$$

$$EI\frac{d^2v}{dx^2} = \frac{P}{2}x$$

$$EI\frac{dv}{dx} = \frac{P}{4}x^2 + C_1 \qquad [1]$$

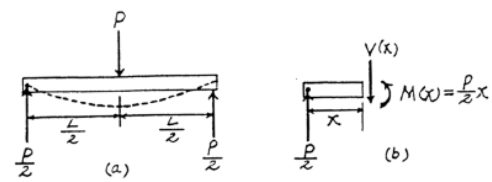
$$EI\,v = \frac{P}{12}x^3 + C_1x + C_2 \qquad [2]$$

Boundary Conditions: Due to symmetry, $\frac{dv}{dx} = 0$ at $x = \frac{L}{2}$.

Also, $v = 0$ at $x = 0$.

From Eq.[1] $\quad 0 = \frac{P}{4}\left(\frac{L}{2}\right)^2 + C_1 \qquad C_1 = -\frac{PL^2}{16}$

From Eq.[2] $\quad 0 = 0 + 0 + C_2 \qquad C_2 = 0$



The Slope: Substitute the value of C_1 into Eq.[1],

$$\frac{dv}{dx} = \frac{P}{16EI}\left(4x^2 - L^2\right)$$

$$\theta_A = \left.\frac{dv}{dx}\right|_{x=0} = -\frac{PL^2}{16EI} \qquad \textbf{Ans}$$

The negative sign indicates clockwise rotation.

The Elastic Curve: Substitute the values of C_1 and C_2 into Eq.[2],

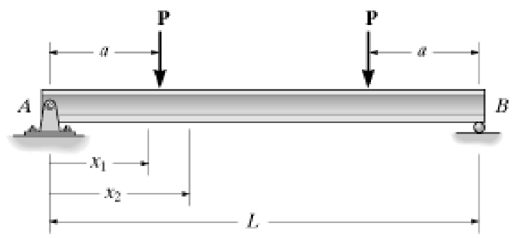
$$v = \frac{Px}{48EI}\left(4x^2 - 3L^2\right) \qquad \textbf{Ans}$$

v_{max} occurs at $x = \frac{L}{2}$,

$$v_{max} = -\frac{PL^3}{48EI} \qquad \textbf{Ans}$$

The negative sign indicates downward displacement.

12–7. Determine the equations of the elastic curve for the shaft using the x_1 and x_2 coordinates. Specify the slope at A and the displacement at the center of the shaft. EI is constant.



Elastic curve and slope:

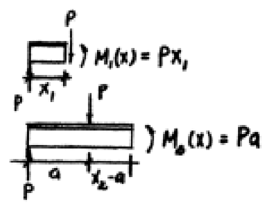
$$EI\frac{d^2v}{dx^2} = M(x)$$

For $M_1(x) = Px_1$

$$EI\frac{d^2v_1}{dx_1^2} = Px_1$$

$$EI\frac{dv_1}{dx_1} = \frac{Px_1^2}{2} + C_1 \qquad (1)$$

$$EIv_1 = \frac{Px_1^3}{6} + C_1x_1 + C_2 \qquad (2)$$



For $M_2(x) = Pa$

$$EI\frac{d^2v_2}{dx_2^2} = Pa$$

$$EI\frac{dv_2}{dx_2} = Pax_2 + C_3 \qquad (3)$$

$$EIv_2 = \frac{Pax_2^2}{2} + C_3x_2 + C_4 \qquad (4)$$

Boundary Conditions:

$v_1 = 0 \quad$ at $\quad x = 0$

From Eq. (2)

$C_2 = 0$

Due to symmetry:

$\frac{dv_2}{dx_2} = 0 \quad$ at $\quad x_2 = \frac{L}{2}$

From Eq. (3)

$$0 = Pa\frac{L}{2} + C_3$$

$$C_3 = -\frac{PaL}{2}$$

Continuity conditions:

$v_1 = v_2 \quad$ at $\quad x_1 = x_2 = a$

$$\frac{Pa^3}{6} + C_1a = \frac{Pa^3}{2} - \frac{Pa^2L}{2} + C_4$$

$$C_1a - C_4 = \frac{Pa^3}{3} - \frac{Pa^2L}{2} \qquad (5)$$

$\frac{dv_1}{dx_1} = \frac{dv_2}{dx_2} \quad$ at $\quad x_1 = x_2 = a$

$$\frac{Pa^2}{2} + C_1 = Pa^2 - \frac{PaL}{2}$$

$$C_1 = \frac{Pa^2}{2} - \frac{PaL}{2}$$

Substitute C_1 into Eq. (5)

$$C_4 = \frac{Pa^3}{6}$$

$$\frac{dv_1}{dx_1} = \frac{P}{2EI}(x_1^2 + a^2 - aL)$$

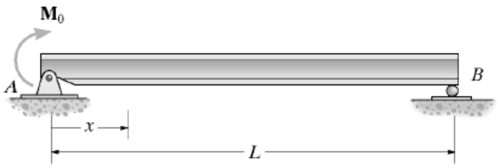
$$\theta_A = \frac{dv_1}{dx_1}\bigg|_{x_1=0} = \frac{Pa(a-L)}{2EI} \qquad \textbf{Ans}$$

$$v_1 = \frac{Px_1}{6EI}[x_1^2 + 3a(a-L)] \qquad \textbf{Ans}$$

$$v_2 = \frac{Pa}{6EI}[3x(x-L) + a^2] \qquad \textbf{Ans}$$

$$v_{max} = v_2\bigg|_{x=\frac{L}{2}} = \frac{Pa}{24EI}(4a^2 - 3L^2) \qquad \textbf{Ans}$$

12–15. Determine the deflection at the center of the beam and the slope at B. EI is constant.

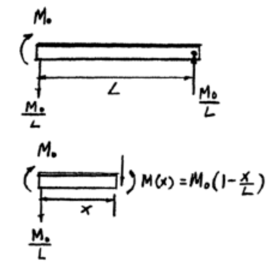


$$EI\frac{d^2v}{dx^2} = M(x)$$

$$EI\frac{d^2v}{dx^2} = M_0\left(1-\frac{x}{L}\right)$$

$$EI\frac{dv}{dx} = M_0\left(x-\frac{x^2}{2L}\right)+C_1 \qquad (1)$$

$$EI\,v = M_0\left(\frac{x^2}{2}-\frac{x^3}{6L}\right)+C_1x+C_2 \qquad (2)$$



Boundary conditions:

$v = 0$ at $x = 0$

From Eq. (2), $\quad C_2 = 0$

$v = 0$ at $x = L$

From Eq. (2),

$$0 = M_0\left(\frac{L^2}{2}-\frac{L^2}{6}\right)+C_1L; \qquad C_1 = -\frac{M_0L}{3}$$

$$\frac{dv}{dx} = \frac{M_0}{EI}\left(x-\frac{x^2}{2L}-\frac{L}{3}\right) \qquad (3)$$

$$\theta_A = \left.\frac{dv}{dx}\right|_{x=0} = -\frac{M_0L}{3EI}$$

$$\frac{dv}{dx} = 0 = \frac{M_0}{EI}\left(x-\frac{x^2}{2L}-\frac{L}{3}\right)$$

$$3x^2-6Lx+2L^2 = 0; \qquad x = 0.42265\,L$$

$$v = \frac{M_0}{6EIL}(3Lx^2-x^3-2L^2x) \qquad (4)$$

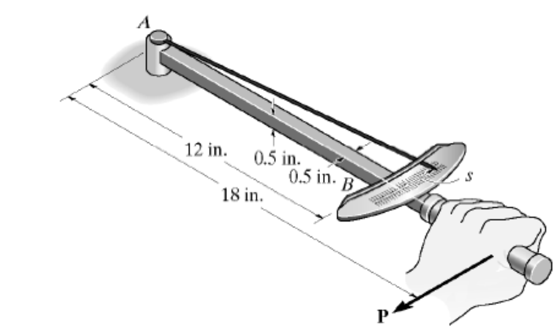
From Eq. (1) at $x = L$,

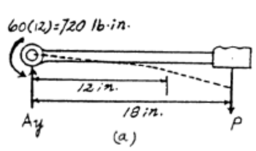
$$\theta_B = \left.\frac{dv}{dx}\right|_{x=L} = \frac{M_0L}{6EI} \qquad \textbf{Ans}$$

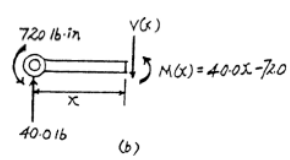
From Eq. (2),

$$v\Big|_{x=\frac{L}{2}} = \frac{-M_0L^2}{16EI} \qquad \textbf{Ans}$$

***12–16.** A torque wrench is used to tighten the nut on a bolt. If the dial indicates that a torque of 60 lb · ft is applied when the bolt is fully tightened, determine the force P acting at the handle and the distance s the needle moves along the scale. Assume only the portion AB of the beam distorts. The cross section is square having dimensions of 0.5 in. by 0.5 in. $E = 29(10^3)$ ksi.







Equations of Equilibrium: From FBD(a),

$$\circlearrowleft + \Sigma M_A = 0; \quad 720 - P(18) = 0 \quad P = 40.0 \text{ lb} \quad \textbf{Ans}$$

$$+\uparrow \Sigma F_y = 0; \quad A_y - 40.0 = 0 \quad A_y = 40.0 \text{ lb}$$

Moment Function: As shown on FBD(b).

Slope and Elastic Curve:

$$EI\frac{d^2v}{dx^2} = M(x)$$

$$EI\frac{d^2v}{dx^2} = 40.0x - 720$$

$$EI\frac{dv}{dx} = 20.0x^2 - 720x + C_1 \qquad [1]$$

$$EI\,v = 6.667x^3 - 360x^2 + C_1x + C_2 \qquad [2]$$

Boundary Conditions: $\frac{dv}{dx} = 0$ at $x = 0$ and $v = 0$ at $x = 0$.

From Eq. [1] $\quad 0 = 0 - 0 + C_1 \qquad C_1 = 0$

From Eq. [2] $\quad 0 = 0 - 0 + 0 + C_2 \qquad C_2 = 0$

The Elastic Curve: Substitute the values of C_1 and C_2 into Eq. [2],

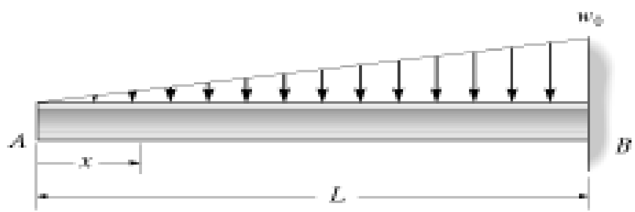
$$v = \frac{1}{EI}\left(6.667x^3 - 360x^2\right) \qquad [1]$$

At $x = 12$ in., $v = -s$. From Eq. [1],

$$-s = \frac{1}{(29)(10^6)\left(\frac{1}{12}\right)(0.5)(0.5^3)}\left[6.667\left(12^3\right) - 360\left(12^2\right)\right]$$

$$s = 0.267 \text{ in.} \qquad \textbf{Ans}$$

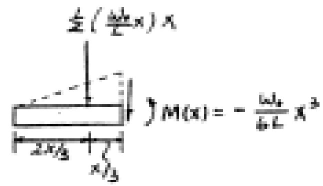
***12–28.** Determine the elastic curve for the cantilevered beam using the x coordinate. Also determine the maximum slope and maximum deflection. EI is constant.



$$EI\frac{d^2v}{dx^2} = M(x)\,; \qquad EI\frac{d^2v}{dx^2} = -\frac{w_0x^3}{6L}$$

$$EI\frac{dv}{dx} = -\frac{w_0x^4}{24L} + C_1 \qquad (1)$$

$$EI\,v = -\frac{w_0x^5}{120L} + C_1x + C_2 \qquad (2)$$



Boundary conditions:

$\frac{dv}{dx} = 0$ at $x = L$

From Eq. (1),

$$0 = -\frac{w_0}{24L}(L^4) + C_1\,; \qquad C_1 = \frac{w_0L^3}{24}$$

$v = 0$ at $x = L$

From Eq. (2),

$$0 = -\frac{w_0}{120\,L}(L^5) + \frac{w_0L^3}{24}(L) + C_2\,; \qquad C_2 = -\frac{w_0L^4}{30}$$

The slope:

From Eq.(1),

$$\frac{dv}{dx} = \frac{w_0}{24EIL}(-x^4 + L^4)$$

$$\theta_{max} = \left.\frac{dv}{dx}\right|_{x=0} = \frac{w_0L^3}{24EI} \qquad \textbf{Ans}$$

The elastic curve:

From Eq. (2),

$$v = \frac{w_0}{120EIL}(-x^5 + 5L^4x - 4L^5) \qquad \textbf{Ans}$$

$$v_{max} = v\Big|_{x=0} = \frac{w_0L^4}{30EI} \qquad \textbf{Ans}$$