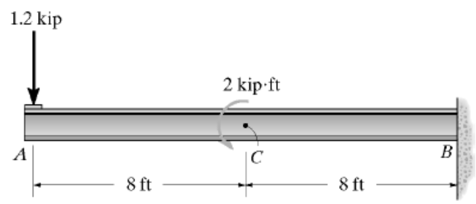
HW 25 Solutions

12–89. The W8 × 48 cantilevered beam is made of A-36 steel and is subjected to the loading shown. Determine the displacement at its end A.



Elastic Curve: The elastic curves for the concentrated load and couple moment are drawn separately as shown.

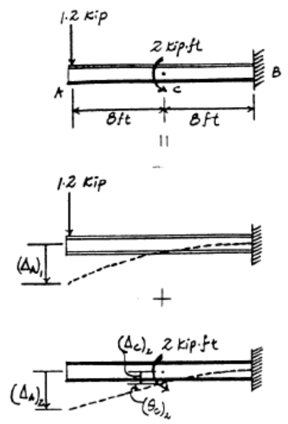
Method of Superposition: Using the table in Appendix C, the required slope and displacement are

$$(\Delta_A)_1 = \frac{PL_{AB}^3}{3EI} = \frac{1.2(16^3)}{3EI} = \frac{1638.4 \text{ kip}\cdot\text{ft}^3}{EI} \downarrow$$

$$(\Delta_C)_2 = \frac{M_0 L_{BC}^2}{2EI} = \frac{2(8^2)}{2EI} = \frac{64.0 \text{ kip}\cdot\text{ft}^3}{EI}$$

$$(\theta_C)_2 = \frac{M_0 L_{BC}}{EI} = \frac{2(8)}{EI} = \frac{16.0 \text{ kip}\cdot\text{ft}^2}{EI}$$

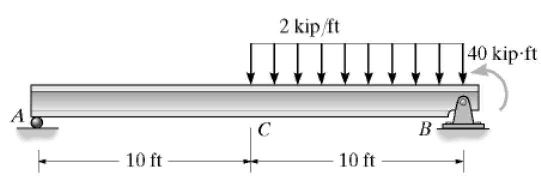
$$(\Delta_A)_2 = (\Delta_C)_2 + (\theta_C)_2 L_{AC} = \frac{64.0}{EI} + \frac{16.0}{EI}(8) = \frac{192 \text{ kip}\cdot\text{ft}^3}{EI} \downarrow$$



The displacement at A is

$$\Delta_A = (\Delta_A)_1 + (\Delta_A)_2 = \frac{1638.4}{EI} + \frac{192}{EI} = \frac{1830.4 \text{ kip}\cdot\text{ft}^3}{EI} = \frac{1830.4(1728)}{29.0(10^3)(184)} = 0.593 \text{ in.} \downarrow \qquad \textbf{Ans}$$

***12–92.** The W14 × 43 simply supported beam is made of A-36 steel and is subjected to the loading shown. Determine the slope at A and B.

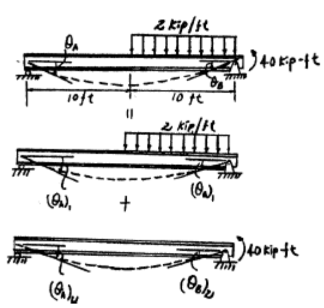


$$\theta_A = \theta_{A_1} + \theta_{A_2} = \frac{7wL^3}{384\,EI} + \frac{ML}{6\,EI} = \frac{\frac{7(2)}{12}(240^3)}{384\,EI} + \frac{40(12)(240)}{6EI} = \frac{61{,}200}{29(10^3)(428)}$$

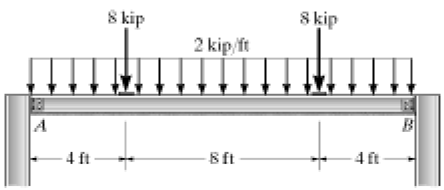
$$= 0.00493 \text{ rad} = 0.283^\circ \qquad \textbf{Ans}$$

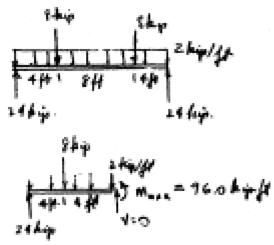
$$\theta_B = \theta_{B_1} + \theta_{B_2} = \frac{3wL^3}{128\,EI} + \frac{ML}{3\,EI} = \frac{\frac{3(2)}{12}(240^3)}{128\,EI} + \frac{40(12)(240)}{3EI} = \frac{92{,}400}{29(10^3)(428)}$$

$$= 0.007444 \text{ rad} = 0.427^\circ \qquad \textbf{Ans}$$



12–95. The simply supported beam carries a uniform load of 2 kip/ft. Code restrictions, due to a plaster ceiling, require the maximum deflection not to exceed 1/360 of the span length. Select the lightest-weight A-36 steel wide-flange beam from Appendix B that will satisfy this requirement and safely support the load. The allowable bending stress is $\sigma_{allow} = 24$ ksi and the allowable shear stress is $\tau_{allow} = 14$ ksi. Assume A is a pin and B a roller support.





$M_{max} = 96$ kip · ft

Strength criterion:

$$\sigma_{allow} = \frac{M}{S_{req'd}}$$

$$24 = \frac{96(12)}{S_{req'd}}$$

$$S_{req'd} = 48 \text{ in}^3$$

Choose W14 x 34 , $S = 48.6 \text{ in}^3$, $t_w = 0.285$ in., $d = 13.98$ in., $I = 340 \text{ in}^4$

$$\tau_{allow} = \frac{V}{A_{web}}$$

$$14 \geq \frac{24}{(13.98)(0.285)} = 6.02 \text{ ksi} \quad \text{OK}$$

Deflection criterion;

Maximum is at center.

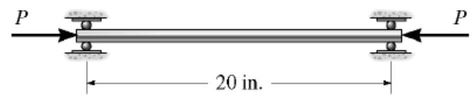
$$v_{max} = \frac{5wL^4}{384EI} + (2)\frac{P(4)(8)}{6EI(16)}[(16)^2 - (4)^2 - (8)^2](12)^3$$

$$= [\frac{5(2)(16)^4}{384EI} + \frac{117.33(8)}{EI}](12)^3$$

$$= \frac{4.571(10^6)}{29(10^6)(340)} = 0.000464 \text{ in.} < \frac{1}{360}(16)(12) = 0.533 \text{ in.} \quad \text{OK}$$

Use W14 x34 **Ans**

13–7. The rod is made from a 1-in.-diameter steel rod. Determine the critical buckling load if the ends are roller supported. $E_{st} = 29(10^3)$ ksi, $\sigma_Y = 50$ ksi.



Critical Buckling Load: $I = \frac{\pi}{4}(0.5^4) = 0.015625\pi \text{ in}^4$

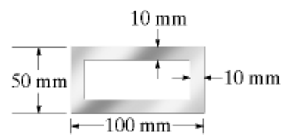
and $K = 1$ for roller supported ends column. Applying *Euler's* formula,

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2(29)(10^3)(0.015625\pi)}{[1(20)]^2} = 35.12 \text{ kip} = 35.1 \text{ kip} \quad \textbf{Ans}$$

Critical Stress: *Euler's* formula is only valid if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{35.12}{\frac{\pi}{4}(1^2)} = 44.72 \text{ ksi} < \sigma_Y = 50 \text{ ksi } (O.K!)$$

***13–8.** An A-36 steel column has a length of 5 m and is fixed at both ends. If the cross-sectional area has the dimensions shown, determine the critical load.



$$I = \frac{1}{12}(0.1)(0.05^3) - \frac{1}{12}(0.08)(0.03^3) = 0.86167\ (10^{-6})\text{ m}^4$$

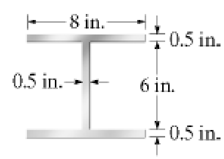
$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2(200)(10^9)(0.86167)(10^{-6})}{[(0.5)(5)]^2} = 272\ 138 \text{ N} = 272 \text{ kN} \quad \textbf{Ans}$$

$$\sigma_{cr} = \frac{P_{cr}}{A}; \qquad A = (0.1)(0.05) - (0.08)(0.03) = 2.6\,(10^{-3})\text{ m}^2$$

$$= \frac{272\ 138}{2.6\,(10^{-3})} = 105 \text{ MPa} < \sigma_Y$$

Therefore, Euler's formula is valid.

13–9. An A-36 steel column has a length of 15 ft and is pinned at both ends. If the cross-sectional area has the dimensions shown, determine the critical load.



$$I_x = \frac{1}{12}(8)(7^3) - \frac{1}{12}(7.5)(6^3) = 93.67 \text{ in}^4$$

$$I_y = 2\left(\frac{1}{12}\right)(0.5)(8^3) + \frac{1}{12}(6)(0.5^3) = 42.729 \text{ in}^4 \text{ (controls)}$$

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2(29)(10^3)(42.729)}{[(1.0)(15)(12)]^2}$$

$$= 377 \text{ kip} \quad \textbf{Ans}$$

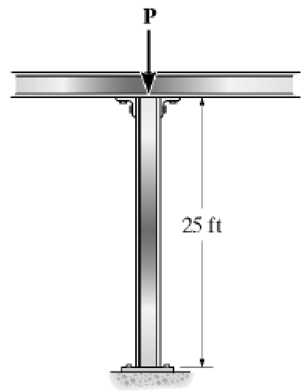
Check:

$$A = (2)(8)(0.5) + 6(0.5) = 11 \text{ in}^2$$

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{377}{11} = 34.3 \text{ ksi} < \sigma_Y$$

Therefore, Euler's formula is valid

13–14. The W8 × 67 is used as a structural A-36 steel column that can be assumed fixed at its base and pinned at its top. Determine the largest axial force P that can be applied without causing it to buckle.



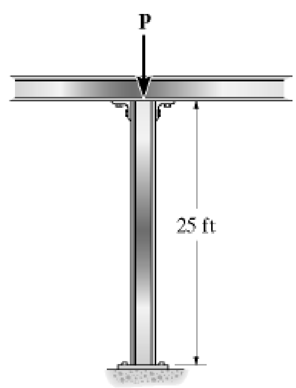
Critical Buckling Load: $I_y = 88.6\ \text{in}^4$ for a W8 × 67 wide flange section and $K = 0.7$ for one end fixed and the other end pinned. Applying *Euler's* formula,

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$
$$= \frac{\pi^2(29)(10^3)(88.6)}{[0.7(25)(12)]^2}$$
$$= 575\ \text{kip} \qquad \textbf{Ans}$$

Critical Stress: *Euler's* formula is only valid if $\sigma_{cr} < \sigma_Y$.
$A = 19.7\ \text{in}^2$ for a W8 × 67 wide flange section.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{575.03}{19.7} = 29.19\ \text{ksi} < \sigma_Y = 36\ \text{ksi}\ (O.K!)$$

13–15. Solve Prob. 13–14 if the column is assumed fixed at its bottom and free at its top.



Critical Buckling Load: $I_y = 88.6\ \text{in}^4$ for a W8 × 67 wide flange section and $K = 2$ for one end fixed and the other end free. Applying *Euler's* formula,

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$
$$= \frac{\pi^2(29)(10^3)(88.6)}{[2(25)(12)]^2}$$
$$= 70.4\ \text{kip} \qquad \textbf{Ans}$$

Critical Stress: *Euler's* formula is only valid if $\sigma_{cr} < \sigma_Y$.
$A = 19.7\ \text{in}^2$ for a W8 × 67 wide flange section.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{70.44}{19.7} = 3.58\ \text{ksi} < \sigma_Y = 36\ \text{ksi}\ (O.K!)$$