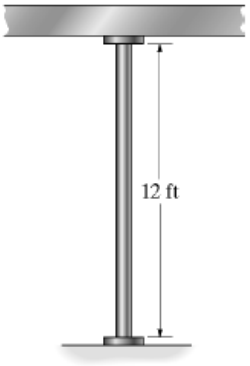
HW 26 Solutions

13–18. The 12-ft A-36 steel pipe column has an outer diameter of 3 in. and a thickness of 0.25 in. Determine the critical load if the ends are assumed to be pin connected.



$$A = \pi(1.5^2 - 1.25^2) = 2.1598 \text{ in}^2$$

$$I = \frac{\pi}{4}(1.5^4 - 1.25^4) = 2.0586 \text{ in}^4$$

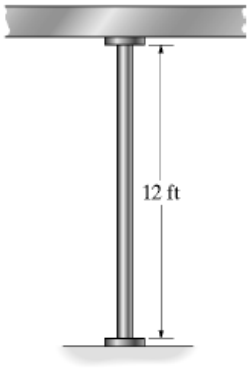
$$K = 1.0$$

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2(29)(10^3)(2.0586)}{[(1.0)(12)(12)]^2} = 28.4 \text{ kip} \quad \textbf{Ans}$$

Check:

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{28.4}{2.1598} = 13.1 \text{ ksi} < \sigma_Y \quad \text{OK}$$

13–19. The 12-ft A-36 steel column has an outer diameter of 3 in. and a thickness of 0.25 in. Determine the critical load if the bottom is fixed and the top is pinned.



$$A = \pi(1.5^2 - 1.25^2) = 2.1598 \text{ in}^2$$

$$I = \frac{\pi}{4}(1.5^4 - 1.25^4) = 2.0586 \text{ in}^4$$

$$K = 0.7$$

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2(29)(10^3)(2.0586)}{[(0.7)(12)(12)]^2} = 58.0 \text{ kip} \quad \textbf{Ans}$$

Check:

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{58.0}{2.1598} = 26.8 \text{ ksi} < \sigma_Y \quad \text{OK}$$

***13–80.** Determine the largest length of a W8 × 31 structural A-36 steel section if it is pin supported and is subjected to an axial load of 80 kip. Use the AISC equations.

For a W8 x 31 $A = 9.13\ \text{in}^2$ $r_y = 2.02$ in.

$$\sigma = \frac{P}{A} = \frac{80}{9.13} = 8.762 \text{ ksi}$$

Assume a long column:

$$\sigma_{allow} = \frac{12\pi^2 E}{23(KL/r)^2}$$

$$\left(\frac{KL}{r}\right) = \sqrt{\frac{12\,\pi^2 E}{23\,\sigma_{allow}}} = \sqrt{\frac{12\pi^2(29)(10^3)}{23(8.762)}} = 130.54$$

$$\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2(29)(10^3)}{36}} = 126.1$$

$$\frac{KL}{r} > \left(\frac{KL}{r}\right)_c \quad \text{(Assumption OK)}$$

$$\frac{KL}{r} = 130.54$$

$$L = 130.54\left(\frac{2.02}{1.0}\right) = 263.7 \text{ in.} = 22.0 \text{ ft} \quad \textbf{Ans}$$

13–81. Using the AISC equations, check if a W6 × 9 structural A-36 steel column that is 10 ft long can support an axial load of 40 kip. The ends are fixed.

Section Properties: For a W6 × 9 wide flange section,

$$A = 2.68\ \text{in}^2 \qquad r_y = 0.905 \text{ in}$$

Slenderness Ratio: For a column fixed at both ends, $K = 0.5$. Thus,

$$\left(\frac{KL}{r}\right)_y = \frac{0.5(10)(12)}{0.905} = 66.30$$

AISC Column Formula: For A–36 steel, $\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}}$

$= \sqrt{\frac{2\pi^2[29(10^3)]}{36}} = 126.1$. Since $\frac{KL}{r} < \left(\frac{KL}{r}\right)_c$, the column is an *intermediate* column. Applying Eq. 13 – 23,

$$\sigma_{allow} = \frac{\left[1 - \frac{(KL/r)^2}{2(KL/r)_c^2}\right]\sigma_Y}{\frac{5}{3} + \frac{3(KL/r)}{8(KL/r)_c} - \frac{(KL/r)^3}{8(KL/r)_c^3}}$$

$$= \frac{\left[1 - \frac{(66.30^2)}{2(126.1^2)}\right](36)}{\frac{5}{3} + \frac{3(66.30)}{8(126.1)} - \frac{(66.30^3)}{8(126.1^3)}}$$

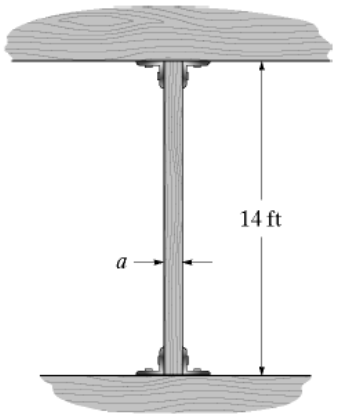
$$= 16.809 \text{ ksi}$$

The allowable load is

$$P_{allow} = \sigma_{allow} A$$
$$= 16.809(2.68)$$
$$= 45.05 \text{ kip} > P = 40 \text{ kip} \quad (O.K.!)$$

Thus, the column is **adequate.** Ans

13–99. The timber column has a square cross section and is assumed to be pin connected at its top and bottom. If it supports an axial load of 50 kip, determine its side dimensions a to the nearest $\frac{1}{2}$ in. Use the NFPA formulas.



Section properties:

$$A = a^2 \qquad \sigma_{allow} = \sigma = \frac{P}{A} = \frac{50}{a^2}$$

Assume long column:

$$\sigma_{allow} = \frac{540}{(\frac{KL}{d})^2}$$

$$\frac{50}{a^2} = \frac{540}{[\frac{(1.0)(14)(12)}{a}]^2}$$

$$a = 7.15 \text{ in.}$$

$$\frac{KL}{d} = \frac{(1.0)(14)(12)}{7.15} = 23.5, \quad \frac{KL}{d} < 26 \qquad \text{Assumption NG}$$

Assume intermediate column:

$$\sigma_{allow} = 1.20[1 - \frac{1}{3}(\frac{KL/d}{26.0})^2$$

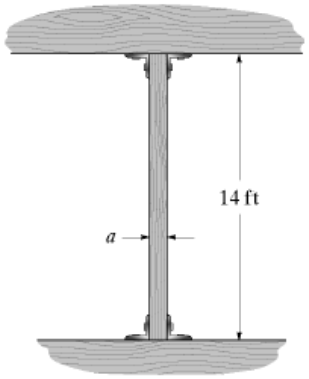
$$\frac{50}{a^2} = 1.20[1 - \frac{1}{3}(\frac{\frac{1.0(14)(12)}{a}}{26.0})^2$$

$$a = 7.45 \text{ in.} \qquad \textbf{Ans}$$

$$\frac{KL}{d} = \frac{1.0(14)(12)}{7.45} = 22.53, \quad 11 < \frac{KL}{d} < 26 \qquad \text{Assumption OK}$$

Use $a = 7\frac{1}{2}$ in. **Ans**

***13–100.** Solve Prob. 13–99 if the column is assumed to be fixed connected at its top and bottom.



$$\sigma_{allow} = \sigma = \frac{P}{A} = \frac{50}{a^2}$$

Assume long column:

$$\sigma_{allow} = \frac{540}{(KL/d)^2}$$

$$\frac{50}{a^2} = \frac{540}{(\frac{0.5(14)(12)}{a})^2}$$

$$a = 5.056 \text{ in.}$$

$$\frac{KL}{d} = \frac{0.5(14)(12)}{5.056} = 16.615, \qquad \frac{KL}{d} < 26 \qquad \text{Assumption N.G.}$$

Assume intermediate column:

$$\sigma_{allow} = 1.20[1 - \frac{1}{3}(\frac{KL/d}{26.0})^2]$$

$$\frac{50}{a^2} = 1.20[1 - \frac{1}{3}(\frac{\frac{0.5(14)(12)}{a}}{26.0})^2]$$

$$a = 6.72 \text{ in.}$$

$$\frac{KL}{d} = \frac{0.5(14)(12)}{6.72} = 12.5, \qquad 11 < \frac{KL}{d} < 26 \qquad \text{Assumption OK}$$

Use $a = 7.00$ in. **Ans**