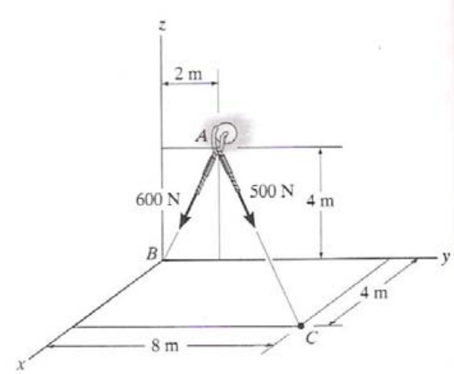
Homework Set #3

2–90. Determine the magnitude and coordinate direction angles of the resultant force.



$\mathbf{r}_{AB} = \{-2\,\mathbf{j} - 4\,\mathbf{k}\}\text{m}; \quad r_{AB} = 4.472\text{ m}$

$\mathbf{u}_{AB} = \left(\frac{\mathbf{r}_{AB}}{r_{AB}}\right) = -0.447\,\mathbf{j} - 0.894\,\mathbf{k}$

$\mathbf{F}_{AB} = 600\,\mathbf{u}_{AB} = \{-268.33\,\mathbf{j} - 536.66\,\mathbf{k}\}\text{N}$

$\mathbf{r}_{AC} = \{4\,\mathbf{i} + 6\,\mathbf{j} - 4\,\mathbf{k}\}\text{m}; \quad r_{AC} = 8.246\text{ m}$

$\mathbf{u}_{AC} = \left(\frac{\mathbf{r}_{AC}}{r_{AC}}\right) = 0.485\,\mathbf{i} + 0.728\,\mathbf{j} - 0.485\,\mathbf{k}$

$\mathbf{F}_{AC} = 500\,\mathbf{u}_{AC} = \{242.54\,\mathbf{i} + 363.80\,\mathbf{j} - 242.54\,\mathbf{k}\}\text{N}$

$\mathbf{F}_R = \mathbf{F}_{AB} + \mathbf{F}_{AC}$

$\mathbf{F}_R = \{242.54\,\mathbf{i} + 95.47\,\mathbf{j} - 779.20\,\mathbf{k}\}$

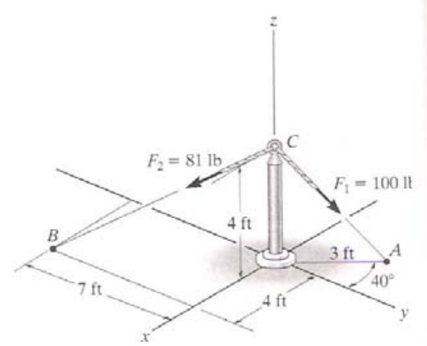
$F_R = \sqrt{(242.54)^2 + (95.47)^2 + (-779.20)^2} = 821.64 = 822\text{ N}$ **Ans**

$\alpha = \cos^{-1}\left(\frac{242.54}{821.64}\right) = 72.8°$ **Ans**

$\beta = \cos^{-1}\left(\frac{95.47}{821.64}\right) = 83.3°$ **Ans**

$\gamma = \cos^{-1}\left(\frac{-779.20}{821.64}\right) = 162°$ **Ans**

***2–92.** Determine the magnitude and coordinate direction angles of the resultant force.



$\mathbf{F}_1 = -100\left(\frac{3}{5}\right)\sin 40°\,\mathbf{i} + 100\left(\frac{3}{5}\right)\cos 40°\,\mathbf{j} - 100\left(\frac{4}{5}\right)\mathbf{k}$

$= \{-38.567\,\mathbf{i} + 45.963\,\mathbf{j} - 80\,\mathbf{k}\}\text{ lb}$

$\mathbf{F}_2 = 81\text{ lb}\left(\frac{4}{9}\,\mathbf{i} - \frac{7}{9}\,\mathbf{j} - \frac{4}{9}\,\mathbf{k}\right)$

$= \{36\,\mathbf{i} - 63\,\mathbf{j} - 36\,\mathbf{k}\}\text{lb}$

$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 = \{-2.567\,\mathbf{i} - 17.04\,\mathbf{j} - 116.0\,\mathbf{k}\}\text{ lb}$

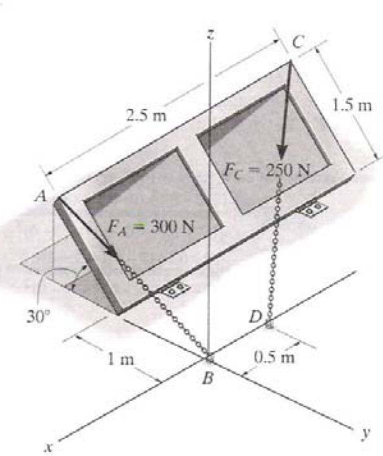
$F_R = \sqrt{(-2.567)^2 + (-17.04)^2 + (-116.0)^2} = 117.27\text{ lb} = 117\text{ lb}$ **Ans**

$\alpha = \cos^{-1}\left(\frac{-2.567}{117.27}\right) = 91.3°$ **Ans**

$\beta = \cos^{-1}\left(\frac{-17.04}{117.27}\right) = 98.4°$ **Ans**

$\gamma = \cos^{-1}\left(\frac{-116.0}{117.27}\right) = 172°$ **Ans**

•2–97. The door is held opened by means of two chains. If the tension in AB and CD is $F_A = 300$ N and $F_C = 250$ N, respectively, express each of these forces in Cartesian vector form.



Unit Vector : First determine the position vector $\mathbf{r}_{AB}$ and $\mathbf{r}_{CD}$. The coordinates of points A and C are

$$A[0,\ -(1+1.5\cos 30°),\ 1.5\sin 30°]\ \text{m} = A(0,\ -2.299,\ 0.750)\ \text{m}$$
$$C[-2.50,\ -(1+1.5\cos 30°),\ 1.5\sin 300°]\ \text{m} = C(-2.50,\ -2.299,\ 0.750)\ \text{m}$$

Then

$$\mathbf{r}_{AB} = \{(0-0)\mathbf{i}+[0-(-2.299)]\mathbf{j}+(0-0.750)\mathbf{k}\}\ \text{m}$$
$$= \{2.299\mathbf{j}-0.750\mathbf{k}\}\ \text{m}$$
$$r_{AB} = \sqrt{2.299^2+(-0.750)^2} = 2.418\ \text{m}$$
$$\mathbf{u}_{AB} = \frac{\mathbf{r}_{AB}}{r_{AB}} = \frac{2.299\mathbf{j}-0.750\mathbf{k}}{2.418} = 0.9507\mathbf{j}-0.3101\mathbf{k}$$

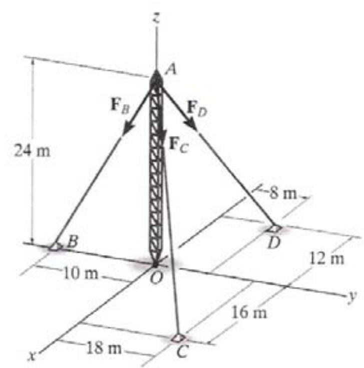
$$\mathbf{r}_{CD} = \{[-0.5-(-2.5)]\mathbf{i}+[0-(-2.299)]\mathbf{j}+(0-0.750)\mathbf{k}\}\ \text{m}$$
$$= \{2.00\mathbf{i}+2.299\mathbf{j}-0.750\mathbf{k}\}\ \text{m}$$
$$r_{CD} = \sqrt{2.00^2+2.299^2+(-0.750)^2} = 3.138\ \text{m}$$
$$\mathbf{u}_{CD} = \frac{\mathbf{r}_{CD}}{r_{CD}} = \frac{2.00\mathbf{i}+2.299\mathbf{j}-0.750\mathbf{k}}{3.138} = 0.6373\mathbf{i}+0.7326\mathbf{j}-0.2390\mathbf{k}$$

Force Vector :

$$\mathbf{F}_A = F_A\mathbf{u}_{AB} = 300\{0.9507\mathbf{j}-0.3101\mathbf{k}\}\ \text{N}$$
$$= \{285.21\mathbf{j}-93.04\mathbf{k}\}\ \text{N}$$
$$= \{285\mathbf{j}-93.0\mathbf{k}\}\ \text{N} \qquad \textbf{Ans}$$

$$\mathbf{F}_C = F_C\mathbf{u}_{CD} = 250\{0.6373\mathbf{i}+0.7326\mathbf{j}-0.2390\mathbf{k}\}\ \text{N}$$
$$= \{159.33\mathbf{i}+183.15\mathbf{j}-59.75\mathbf{k}\}\ \text{N}$$
$$= \{159\mathbf{i}+183\mathbf{j}-59.7\mathbf{k}\}\ \text{N} \qquad \textbf{Ans}$$

*2–104. The antenna tower is supported by three cables. If the forces of these cables acting on the antenna are $F_B = 520$ N, $F_C = 680$ N, and $F_D = 560$ N, determine the magnitude and coordinate direction angles of the resultant force acting at A.



$$\mathbf{F}_B = 520\left(\frac{\mathbf{r}_{AB}}{r_{AB}}\right) = 520\left(-\frac{10}{26}\mathbf{j} - \frac{24}{26}\mathbf{k}\right) = -200\,\mathbf{j} - 480\,\mathbf{k}$$

$$\mathbf{F}_C = 680\left(\frac{\mathbf{r}_{AC}}{r_{AC}}\right) = 680\left(\frac{16}{34}\mathbf{i} + \frac{18}{34}\mathbf{j} - \frac{24}{34}\mathbf{k}\right) = 320\,\mathbf{i} + 360\,\mathbf{j} - 480\,\mathbf{k}$$

$$\mathbf{F}_D = 560\left(\frac{\mathbf{r}_{AD}}{r_{AD}}\right) = 560\left(-\frac{12}{28}\mathbf{i} + \frac{8}{28}\mathbf{j} - \frac{24}{28}\mathbf{k}\right) = -240\,\mathbf{i} + 160\,\mathbf{j} - 480\,\mathbf{k}$$

$$\mathbf{F}_R = \Sigma\mathbf{F} = \{80\,\mathbf{i} + 320\,\mathbf{j} - 1440\,\mathbf{k}\}\ \text{N}$$

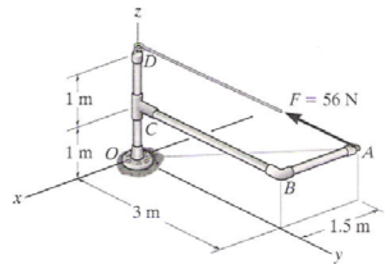
$$F_R = \sqrt{(80)^2 + (320)^2 + (-1440)^2} = 1477.3 = 1.48\ \text{kN} \qquad \textbf{Ans}$$

$$\alpha = \cos^{-1}\left(\frac{80}{1477.3}\right) = 86.9^\circ \qquad \textbf{Ans}$$

$$\beta = \cos^{-1}\left(\frac{320}{1477.3}\right) = 77.5^\circ \qquad \textbf{Ans}$$

$$\gamma = \cos^{-1}\left(\frac{-1440}{1477.3}\right) = 167^\circ \qquad \textbf{Ans}$$

•2–113. Determine the magnitudes of the components of force $F = 56$ N acting along and perpendicular to line AO.



Unit Vectors: The unit vectors $\mathbf{u}_{AD}$ and $\mathbf{u}_{AO}$ must be determined first. From Fig. *a*,

$$\mathbf{u}_{AD} = \frac{\mathbf{r}_{AD}}{r_{AD}} = \frac{[0-(-1.5)]\mathbf{i}+(0-3)\mathbf{j}+(2-1)\mathbf{k}}{\sqrt{[0-(-1.5)]^2+(0-3)^2+(2-1)^2}} = \frac{3}{7}\mathbf{i}-\frac{6}{7}\mathbf{j}+\frac{2}{7}\mathbf{k}$$

$$\mathbf{u}_{AO} = \frac{\mathbf{r}_{AO}}{r_{AO}} = \frac{[0-(-1.5)]\mathbf{i}+(0-3)\mathbf{j}+(0-1)\mathbf{k}}{\sqrt{[0-(-1.5)]^2+(0-3)^2+(0-1)^2}} = \frac{3}{7}\mathbf{i}-\frac{6}{7}\mathbf{j}-\frac{2}{7}\mathbf{k}$$

Thus, the force vector **F** is given by

$$\mathbf{F} = F\mathbf{u}_{AD} = 56\left(\frac{3}{7}\mathbf{i}-\frac{6}{7}\mathbf{j}+\frac{2}{7}\mathbf{k}\right) = [24\mathbf{i}-48\mathbf{j}+16\mathbf{k}]\text{N}$$

Vector Dot Product: The magnitude of the projected component of **F** parallel to line AO is

$$(F_{AO})_{paral} = \mathbf{F}\cdot\mathbf{u}_{AO} = (24\mathbf{i}-48\mathbf{j}+16\mathbf{k})\cdot\left(\frac{3}{7}\mathbf{i}-\frac{6}{7}\mathbf{j}-\frac{2}{7}\mathbf{k}\right)$$

$$= (24)\left(\frac{3}{7}\right)+(-48)\left(-\frac{6}{7}\right)+(16)\left(-\frac{2}{7}\right)$$

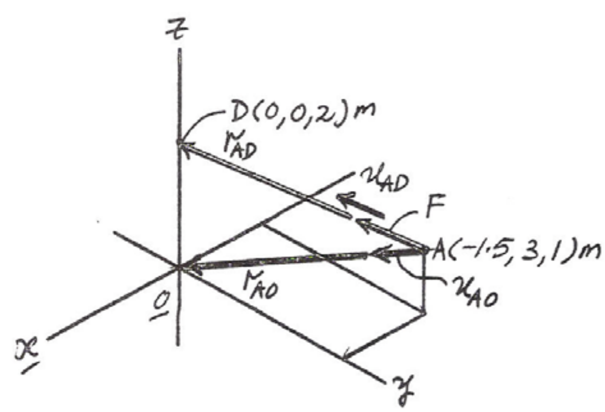
$$= 46.86\text{ N} = 46.9\text{ N}$$ **Ans.**

The component of **F** perpendicular to line AO is

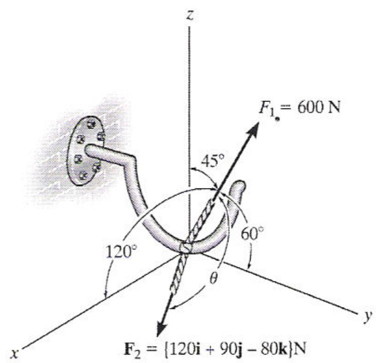
$$(F_{AO})_{per} = \sqrt{F^2-(F_{AO})_{paral}}$$

$$= \sqrt{56^2-46.86^2}$$

$$= 30.7\text{N}$$ **Ans.**



*2–116. Two forces act on the hook. Determine the angle θ between them. Also, what are the projections of $\mathbf{F}_1$ and $\mathbf{F}_2$ along the y axis?



$$\mathbf{F}_1 = 600 \cos 120° \, \mathbf{i} + 600 \cos 60° \, \mathbf{j} + 600 \cos 45° \, \mathbf{k}$$

$$= -300\,\mathbf{i} + 300\,\mathbf{j} + 424.3\,\mathbf{k}; \quad F_1 = 600 \text{ N}$$

$$\mathbf{F}_2 = 120\,\mathbf{i} + 90\,\mathbf{j} - 80\,\mathbf{k}; \quad F_2 = 170 \text{ N}$$

$$\mathbf{F}_1 \cdot \mathbf{F}_2 = (-300)(120) + (300)(90) + (424.3)(-80) = -42\,944$$

$$\theta = \cos^{-1}\left(\frac{-42\,944}{(170)(600)}\right) = 115° \quad \text{Ans}$$

$$\mathbf{u} = \mathbf{j}$$

So,

$$F_{1y} = \mathbf{F}_1 \cdot \mathbf{j} = (300)(1) = 300 \text{ N} \quad \text{Ans}$$

$$F_{2y} = \mathbf{F}_2 \cdot \mathbf{j} = (90)(1) = 90 \text{ N} \quad \text{Ans}$$