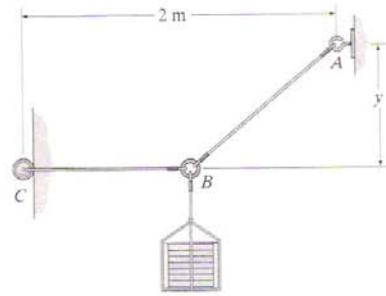
Homework Set #4

3–2. If the 1.5-m-long cord AB can withstand a maximum force of 3500 N, determine the force in cord BC and the distance y so that the 200-kg crate can be supported.

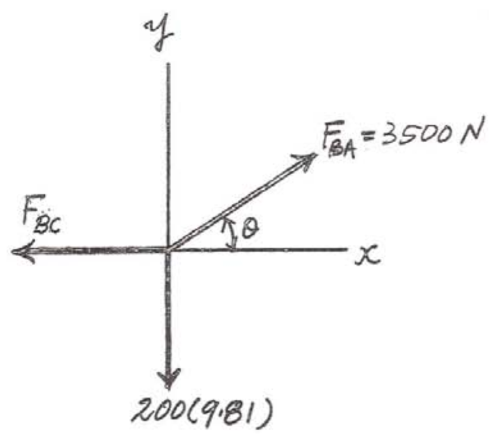


Equations of Equilibrium: Applying the equations of equilibrium along the x and y axes to the free - body diagram in Fig. (a),

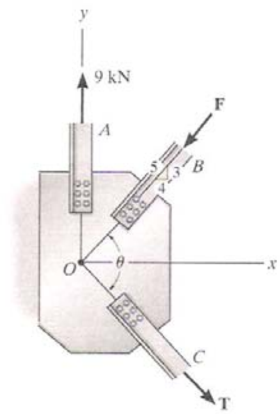
$+\uparrow \Sigma F_y = 0;$ $\quad 3500\sin\theta - 200(9.81) = 0 \quad \theta = 34.10^\circ$

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad 3500\cos 34.10^\circ - F_{BC} = 0 \quad F_{BC} = 2898.37\text{ N} = 2.90\text{ kN}$ **Ans.**

$$y = 1.5\sin 34.10^\circ = 0.841\text{ m} = 841\text{ mm}$$ **Ans.**

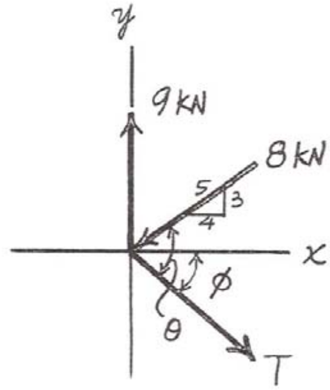


3–11. The gusset plate is subjected to the forces of three members. Determine the tension force in member C and its angle θ for equilibrium. The forces are concurrent at point O. Take $F = 8$ kN.



$\xrightarrow{+} \Sigma F_x = 0; \quad T\cos\phi - 8\left(\frac{4}{5}\right) = 0 \quad (1)$

$+\uparrow \Sigma F_y = 0; \quad 9 - 8\left(\frac{3}{5}\right) - T\sin\phi = 0 \quad (2)$



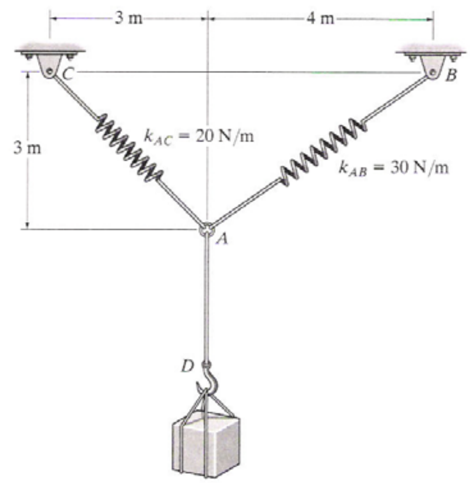
Rearrange then divide Eq. (1) into Eq. (2) :

$\tan\phi = 0.656, \quad \phi = 33.27°$

$T = 7.66$ kN **Ans**

$\theta = \phi + \tan^{-1}\left(\frac{3}{4}\right) = 70.1°$ **Ans**

3–15. The unstretched length of spring AB is 3 m. If the block is held in the equilibrium position shown, determine the mass of the block at D.



$F = kx = 30(5 - 3) = 60\text{ N}$

$\xrightarrow{+} \Sigma F_x = 0; \quad T\cos 45° - 60\left(\frac{4}{5}\right) = 0$

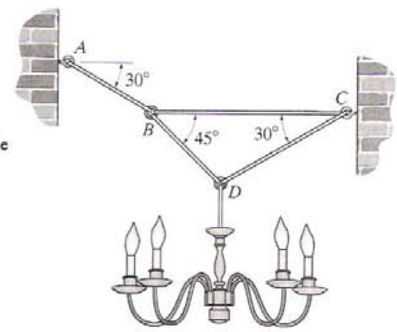
$T = 67.88\text{ N}$

$+\uparrow \Sigma F_y = 0; \quad -W + 67.88\sin 45° + 60\left(\frac{3}{5}\right) = 0$

$W = 84\text{ N}$

$m = \frac{84}{9.81} = 8.56\text{ kg}$ **Ans.**

•3–20. Determine the tension developed in each wire used to support the 50-kg chandelier.



Equations of Equilibrium: First, we will apply the equations of equilibrium along the x and y axes to the free-body diagram of joint D shown in Fig. (a).

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad F_{CD}\cos 30^\circ - F_{BD}\cos 45^\circ = 0 \qquad (1)$$

$$+\uparrow \Sigma F_y = 0; \qquad F_{CD}\sin 30^\circ + F_{BD}\sin 45^\circ - 50(9.81) = 0 \qquad (2)$$

Solving Eqs. (1) and (2), yields

$F_{CD} = 359\text{ N}$ $\qquad F_{BD} = 439.77\text{ N} = 440\text{ N}$ **Ans.**

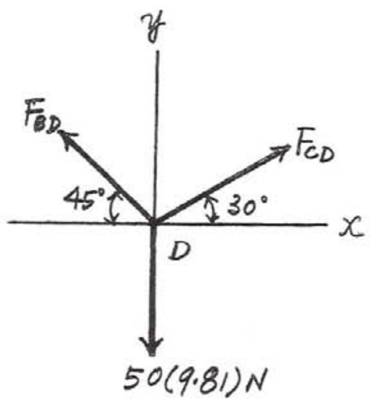
Using the result $F_{BD} = 439.77$ N and applying the equations of equilibrium along the x and y axes to the free-body diagram of joint B shown in Fig. (b),

$$+\uparrow \Sigma F_y = 0; \qquad F_{AB}\sin 30^\circ - 439.77\sin 45^\circ = 0$$

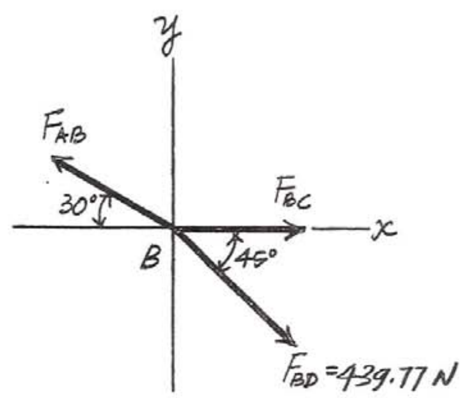
$F_{AB} = 621.93\text{N} = 622\text{ N}$ **Ans.**

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad F_{BC} + 439.77\cos 45^\circ - 621.93\cos 30^\circ = 0$$

$F_{BC} = 228\text{ N}$ **Ans.**

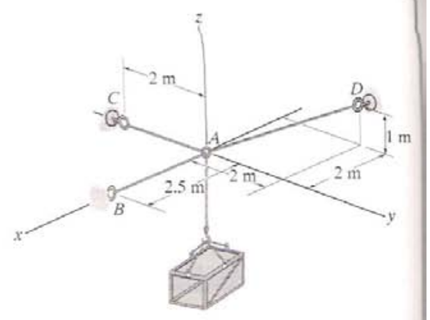


(a)



(b)

3–46. Determine the maximum mass of the crate so that the tension developed in any cable does not exceeded 3 kN.



Force Vectors: We can express each of the forces on the free-body diagram shown in Fig. (*a*) in Cartesian vector form as

$$\mathbf{F}_{AB} = F_{AB}\,\mathbf{i}$$

$$\mathbf{F}_{AC} = -F_{AC}\,\mathbf{j}$$

$$\mathbf{F}_{AD} = F_{AD}\left[\frac{(-2-0)\mathbf{i}+(2-0)\mathbf{j}+(1-0)\mathbf{k}}{\sqrt{(-2-0)^2+(2-0)^2+(1-0)^2}}\right] = -\frac{2}{3}F_{AD}\mathbf{i}+\frac{2}{3}F_{AD}\mathbf{j}+\frac{1}{3}F_{AD}\mathbf{k}$$

$$\mathbf{W} = [-m(9.81)\mathbf{k}]$$

Equations of Equilibrium: Equilibrium requires

$$\Sigma\mathbf{F} = \mathbf{0}; \quad \mathbf{F}_{AB}+\mathbf{F}_{AC}+\mathbf{F}_{AD}+\mathbf{W} = \mathbf{0}$$

$$F_{AB}\,\mathbf{i}+(-F_{AC}\,\mathbf{j})+\left(-\frac{2}{3}F_{AD}\mathbf{i}+\frac{2}{3}F_{AD}\mathbf{j}+\frac{1}{3}F_{AD}\mathbf{k}\right)+[-m(9.81)\mathbf{k}] = \mathbf{0}$$

$$\left(F_{AB}-\frac{2}{3}F_{AD}\right)\mathbf{i}+\left(-F_{AC}+\frac{2}{3}F_{AD}\right)\mathbf{j}+\left(\frac{1}{3}F_{AD}-9.81m\right)\mathbf{k} = 0$$

Equating the **i**, **j**, and **k** components yields

$$F_{AB}-\frac{2}{3}F_{AD} = 0 \qquad (1)$$

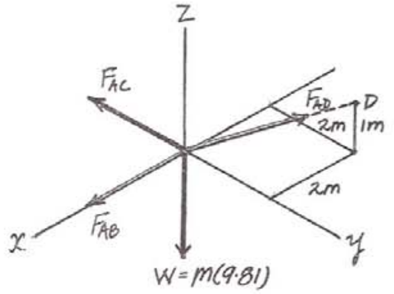
$$-F_{AC}+\frac{2}{3}F_{AD} = 0 \qquad (2)$$

$$\frac{1}{3}F_{AD}-9.81m = 0 \qquad (3)$$

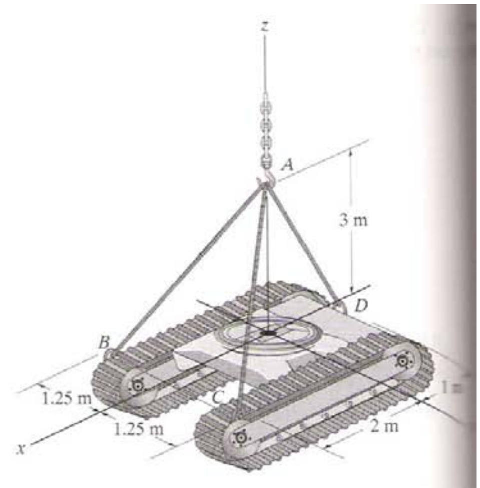
When cable *AD* is subjected to maximum tension, $F_{AD} = 3000$ N. Thus, by substituting this value into Eqs. (1) through (3), we have

$F_{AB} = F_{AC} = 2000$ N

$m = 102$ kg **Ans.**



*3–52. Determine the force in each of the three cables needed to lift the tractor which has a mass of 8 Mg.



Cartesian Vector Notation :

$$\mathbf{F}_{AB} = F_{AB}\left(\frac{2\mathbf{i} - 1.25\mathbf{j} - 3\mathbf{k}}{\sqrt{2^2 + (-1.25)^2 + (-3)^2}}\right) = 0.5241F_{AB}\mathbf{i} - 0.3276F_{AB}\mathbf{j} - 0.7861F_{AB}\mathbf{k}$$

$$\mathbf{F}_{AC} = F_{AC}\left(\frac{2\mathbf{i} + 1.25\mathbf{j} - 3\mathbf{k}}{\sqrt{2^2 + 1.25^2 + (-3)^2}}\right) = 0.5241F_{AC}\mathbf{i} + 0.3276F_{AC}\mathbf{j} - 0.7861F_{AC}\mathbf{k}$$

$$\mathbf{F}_{AD} = F_{AD}\left(\frac{-1\mathbf{i} - 3\mathbf{k}}{\sqrt{(-1)^2 + (-3)^2}}\right) = -0.3162F_{AD}\mathbf{i} - 0.9487F_{AD}\mathbf{k}$$

$$\mathbf{F} = \{78.48\mathbf{k}\}\ \text{kN}$$

Equations of Equilibrium :

$$\Sigma\mathbf{F} = 0; \qquad \mathbf{F}_{AB} + \mathbf{F}_{AC} + \mathbf{F}_{AD} + \mathbf{F} = \mathbf{0}$$

$$(0.5241F_{AB} + 0.5241F_{AC} - 0.3162F_{AD})\mathbf{i} + (-0.3276F_{AB} + 0.3276F_{AC})\mathbf{j} + (-0.7861F_{AB} - 0.7861F_{AC} - 0.9487F_{AD} + 78.48)\mathbf{k} = \mathbf{0}$$

Equating **i**, **j** and **k** components, we have

$$0.5241F_{AB} + 0.5241F_{AC} - 0.3162F_{AD} = 0 \qquad [1]$$
$$-0.3276F_{AB} + 0.3276F_{AC} = 0 \qquad [2]$$
$$-0.7861F_{AB} - 0.7861F_{AC} - 0.9487F_{AD} + 78.48 = 0 \qquad [3]$$

Solving Eqs. [1], [2] and [3] yields

$$F_{AB} = F_{AC} = 16.6\ \text{kN} \qquad F_{AD} = 55.2\ \text{kN} \qquad \textbf{Ans}$$