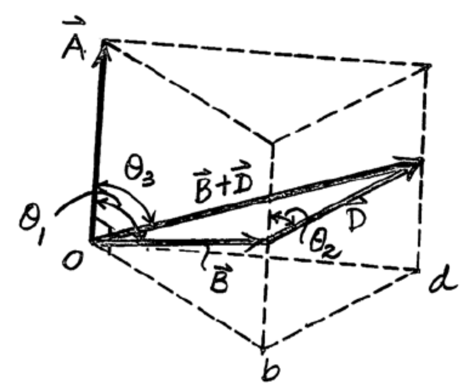
Homework Set # 5

•4–1. If **A**, **B**, and **D** are given vectors, prove the distributive law for the vector cross product, i.e., $\mathbf{A} \times (\mathbf{B} + \mathbf{D}) = (\mathbf{A} \times \mathbf{B}) + (\mathbf{A} \times \mathbf{D})$.



Consider the three vectors; with **A** vertical.

Note *obd* is perpendicular to **A**.

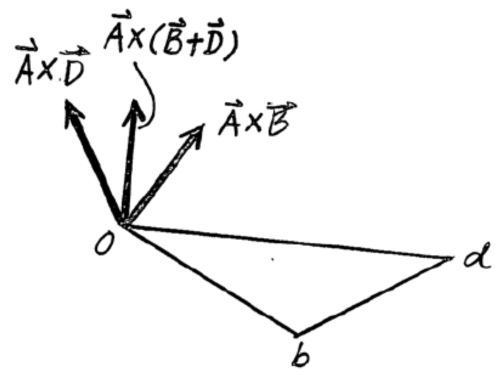
$od = |\mathbf{A} \times (\mathbf{B} + \mathbf{D})| = |\mathbf{A}|(|\mathbf{B} + \mathbf{D}|) \sin\theta_3$

$ob = |\mathbf{A} \times \mathbf{B}| = |\mathbf{A}||\mathbf{B}| \sin\theta_1$

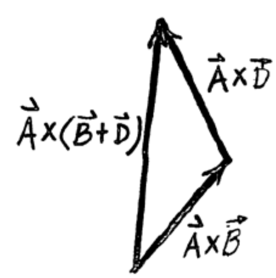
$bd = |\mathbf{A} \times \mathbf{D}| = |\mathbf{A}||\mathbf{D}| \sin\theta_2$

Also, these three cross products all lie in the plane *obd* since they are all perpendicular to **A**. As noted the magnitude of each cross product is proportional to the length of each side of the triangle.

The three vector cross-products also form a closed triangle $o'b'd'$ which is similar to triangle *obd*. Thus from the figure,



$$\mathbf{A} \times (\mathbf{B} + \mathbf{D}) = \mathbf{A} \times \mathbf{B} + \mathbf{A} \times \mathbf{D} \qquad \text{(QED)}$$



Note also,

$$\mathbf{A} = A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}$$

$$\mathbf{B} = B_x\mathbf{i} + B_y\mathbf{j} + B_z\mathbf{k}$$

$$\mathbf{D} = D_x\mathbf{i} + D_y\mathbf{j} + D_z\mathbf{k}$$

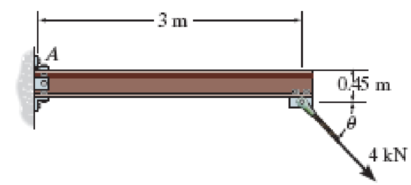
$$\mathbf{A} \times (\mathbf{B} + \mathbf{D}) = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x + D_x & B_y + D_y & B_z + D_z \end{vmatrix}$$

$$= [A_y(B_z + D_z) - A_z(B_y + D_y)]\mathbf{i}$$
$$-[A_x(B_z + D_z) - A_z(B_x + D_x)]\mathbf{j}$$
$$+[A_x(B_y + D_y) - A_y(B_x + D_x)]\mathbf{k}$$
$$= [(A_yB_z - A_zB_y)\mathbf{i} - (A_xB_z - A_zB_x)\mathbf{j} + (A_xB_y - A_yB_x)\mathbf{k}]$$
$$+ [(A_yD_z - A_zD_y)\mathbf{i} - (A_xD_z - A_zD_x)\mathbf{j} + (A_xD_y - A_yD_x)\mathbf{k}]$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ D_x & D_y & D_z \end{vmatrix}$$

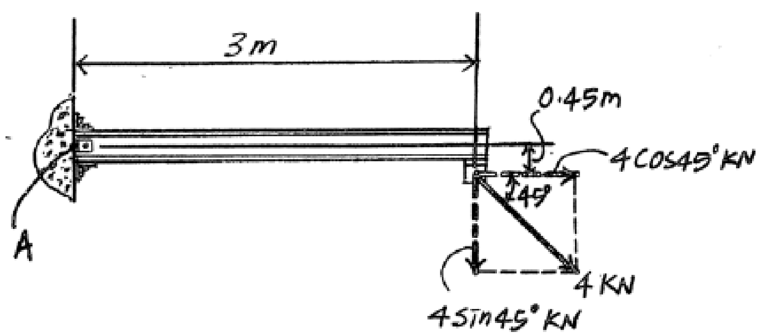
$$= (\mathbf{A} \times \mathbf{B}) + (\mathbf{A} \times \mathbf{D}) \qquad \text{(QED)}$$

4-6. If $\theta = 45°$, determine the moment produced by the 4-kN force about point A.



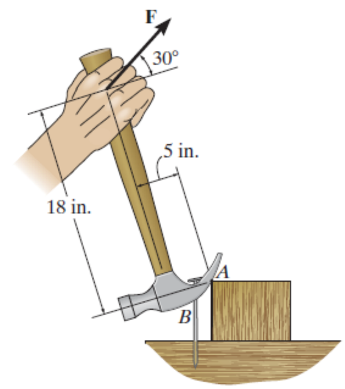
Resolving the 4 - kN force into its horizontal and vertical components, Fig. a, and applying the principle of moments,

$$\circlearrowleft +M_A = 4\cos 45°(0.45) - 4\sin 45°(3)$$
$$= -7.21\ \text{kN}\cdot\text{m} = 7.21\ \text{kN}\cdot\text{m}\quad \text{(clockwise)}$$ **Ans.**



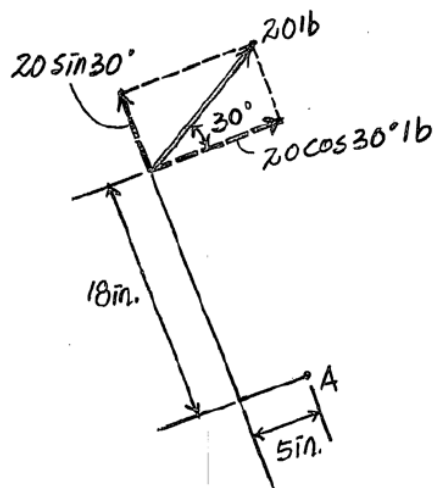
(a)

*4–8. The handle of the hammer is subjected to the force of $F = 20$ lb. Determine the moment of this force about the point A.



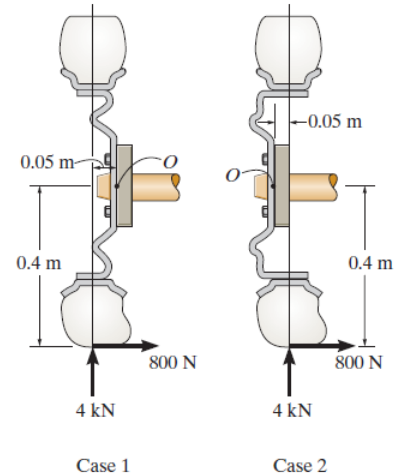
Resolving the 20 - lb force into components parallel and perpendicular to the hammer, Fig. *a*, and applying the principle of moments,

$$\circlearrowleft + M_A = -20\cos 30°(18) - 20\sin 30°(5)$$
$$= -361.77\ \text{lb}\cdot\text{in} = 362\ \text{lb}\cdot\text{in} \quad \text{(clockwise)}$$ **Ans.**



(a)

4–10. The hub of the wheel can be attached to the axle either with negative offset (left) or with positive offset (right). If the tire is subjected to both a normal and radial load as shown, determine the resultant moment of these loads about point O on the axle for both cases.



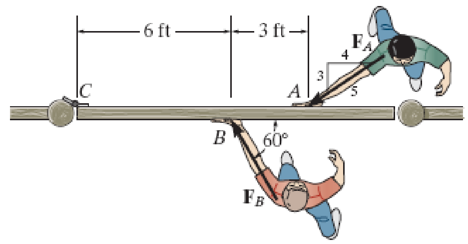
For case 1 with negative offset, we have

$$\circlearrowleft + \; M_O = 800(0.4) - 4000(0.05)$$
$$= 120 \text{ N} \cdot \text{m} \quad (\textit{Counterclockwise}) \qquad \textbf{Ans}$$

For case 2 with positive offset, we have

$$\circlearrowleft + \; M_O = 800(0.4) + 4000(0.05)$$
$$= 520 \text{ N} \cdot \text{m} \quad (\textit{Counterclockwise}) \qquad \textbf{Ans}$$

•4–17. The two boys push on the gate with forces of $F_A = 30$ lb and as shown. Determine the moment of each force about C. Which way will the gate rotate, clockwise or counterclockwise? Neglect the thickness of the gate.



$$\circlearrowleft + \ (M_{F_A})_C = -30\left(\frac{3}{5}\right)(9)$$

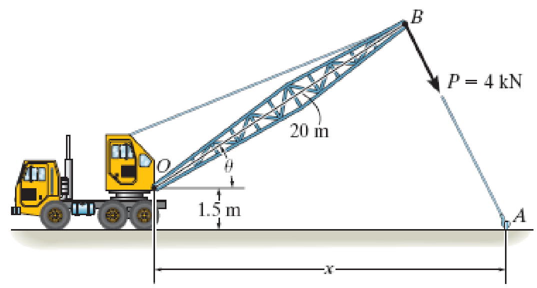
$$= -162 \text{ lb}\cdot\text{ft} = 162 \text{ lb}\cdot\text{ft} \quad \textbf{(Clockwise)} \qquad \textbf{Ans}$$

$$\circlearrowleft + \ (M_{F_B})_C = 50(\sin 60°)(6)$$

$$= 260 \text{ lb}\cdot\text{ft} \quad \textbf{(Counterclockwise)} \qquad \textbf{Ans}$$

Since $(M_{F_B})_C > (M_{F_A})_C$, the gate will rotate ***Counterclockwise.*** **Ans**

*4–32. The towline exerts a force of $P = 4$ kN at the end of the 20-m-long crane boom. If $\theta = 30°$, determine the placement x of the hook at A so that this force creates a maximum moment about point O. What is this moment?



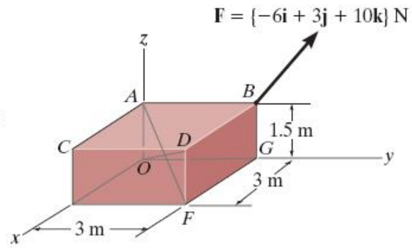
Maximum moment, $OB \perp BA$

$$\circlearrowright + (M_O)_{max} = 4 \text{ kN}(20) = 80 \text{ kN}\cdot\text{m} \circlearrowright \qquad \textbf{Ans}$$

$$4 \text{ kN} \sin 60°(x) - 4 \text{ kN} \cos 60°(1.5) = 80 \text{ kN}\cdot\text{m}$$

$$x = 24.0 \text{ m} \qquad \textbf{Ans}$$

4–51. Determine the moment produced by force **F** about the diagonal AF of the rectangular block. Express the result as a Cartesian vector.



Moment About Diagonal AF: Either position vector $\mathbf{r}_{AB}$ or $\mathbf{r}_{FB}$, Fig. *a*, can be used to find the moment of **F** about diagonal AF.

$$\mathbf{r}_{AB} = (0-0)\mathbf{i} + (3-0)\mathbf{j} + (1.5-1.5)\mathbf{k} = \{3\mathbf{j}\}\text{ m}$$

$$\mathbf{r}_{FB} = (0-3)\mathbf{i} + (3-3)\mathbf{j} + (1.5-0)\mathbf{k} = \{-3\mathbf{i} + 1.5\mathbf{k}\}\text{ m}$$

The unit vector $\mathbf{u}_{AF}$, Fig. *a*, that specifies the direction of diagonal AF is given by

$$\mathbf{u}_{AF} = \frac{(3-0)\mathbf{i} + (3-0)\mathbf{j} + (0-1.5)\mathbf{k}}{\sqrt{(3-0)^2 + (3-0)^2 + (0-1.5)^2}} = \frac{2}{3}\mathbf{i} + \frac{2}{3}\mathbf{j} - \frac{1}{3}\mathbf{k}$$

The magnitude of the moment of **F** about diagonal AF axis is

$$M_{AF} = \mathbf{u}_{AF} \cdot \mathbf{r}_{AB} \times \mathbf{F} = \begin{vmatrix} \frac{2}{3} & \frac{2}{3} & -\frac{1}{3} \\ 0 & 3 & 0 \\ -6 & 3 & 10 \end{vmatrix}$$

$$= \frac{2}{3}[3(10) - (3)(0)] - \frac{2}{3}[0(10) - (-6)(0)] + \left(-\frac{1}{3}\right)[0(3) - (-6)(3)]$$

$$= 14\text{ N}\cdot\text{m}$$

or

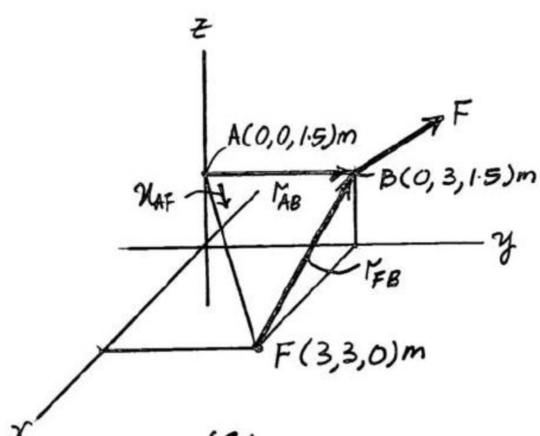
$$M_{AF} = \mathbf{u}_{AF} \cdot \mathbf{r}_{FB} \times \mathbf{F} = \begin{vmatrix} \frac{2}{3} & \frac{2}{3} & -\frac{1}{3} \\ -3 & 0 & 1.5 \\ -6 & 3 & 10 \end{vmatrix}$$

$$= \frac{2}{3}[0(10) - (3)(1.5)] - \frac{2}{3}[(-3)(10) - (-6)(1.5)] + \left(-\frac{1}{3}\right)[(-3)(3) - (-6)(0)]$$

$$= 14\text{ N}\cdot\text{m}$$

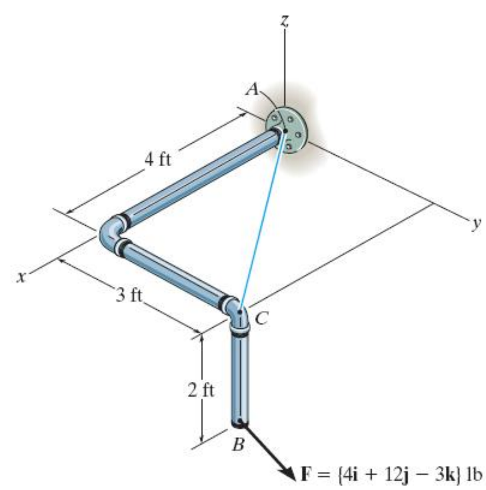
Thus, $\mathbf{M}_{AF}$ can be expressed in Cartesian vector form as

$$\mathbf{M}_{AF} = M_{AF}\mathbf{u}_{AF} = 14\left(\frac{2}{3}\mathbf{i} + \frac{2}{3}\mathbf{j} - \frac{1}{3}\mathbf{k}\right) = \{9.33\mathbf{i} + 9.33\mathbf{j} - 4.67\mathbf{k}\}\text{N}\cdot\text{m}$$ **Ans.**



(a)

4–54. Determine the magnitude of the moments of the force **F** about the x, y, and z axes. Solve the problem (a) using a Cartesian vector approach and (b) using a scalar approach.



a) *Vector Analysis*

Position Vector :

$$\mathbf{r}_{AB} = \{(4-0)\mathbf{i} + (3-0)\mathbf{j} + (-2-0)\mathbf{k}\}\ \text{ft} = \{4\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}\}\ \text{ft}$$

Moment of Force* F *About x, y and z Axes : The unit vectors along x, y and z axes are **i**, **j** and **k** respectively. Applying Eq. 4–[illegible], we have

$$M_x = \mathbf{i}\cdot(\mathbf{r}_{AB}\times\mathbf{F}) = \begin{vmatrix} 1 & 0 & 0 \\ 4 & 3 & -2 \\ 4 & 12 & -3 \end{vmatrix}$$

$$= 1[3(-3) - (12)(-2)] - 0 + 0 = 15.0\ \text{lb}\cdot\text{ft} \qquad \textbf{Ans}$$

$$M_y = \mathbf{j}\cdot(\mathbf{r}_{AB}\times\mathbf{F}) = \begin{vmatrix} 0 & 1 & 0 \\ 4 & 3 & -2 \\ 4 & 12 & -3 \end{vmatrix}$$

$$= 0 - 1[4(-3) - (4)(-2)] + 0 = 4.00\ \text{lb}\cdot\text{ft} \qquad \textbf{Ans}$$

$$M_z = \mathbf{k}\cdot(\mathbf{r}_{AB}\times\mathbf{F}) = \begin{vmatrix} 0 & 0 & 1 \\ 4 & 3 & -2 \\ 4 & 12 & -3 \end{vmatrix}$$

$$= 0 - 0 + 1[4(12) - 4(3)] = 36.0\ \text{lb}\cdot\text{ft} \qquad \textbf{Ans}$$

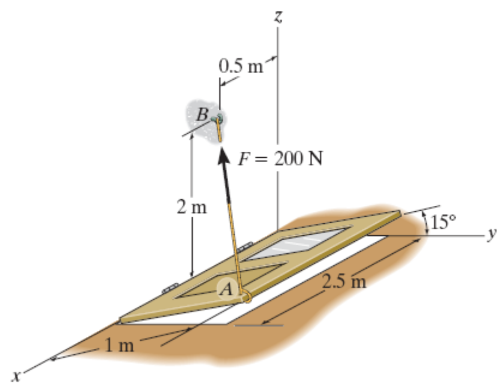
b) *Scalar Analysis*

$M_x = \Sigma M_x;$ $\quad M_x = 12(2) - 3(3) = 15.0\ \text{lb}\cdot\text{ft}$ $\qquad$ **Ans**

$M_y = \Sigma M_y;$ $\quad M_y = -4(2) + 3(4) = 4.00\ \text{lb}\cdot\text{ft}$ $\qquad$ **Ans**

$M_z = \Sigma M_z;$ $\quad M_z = -4(3) + 12(4) = 36.0\ \text{lb}\cdot\text{ft}$ $\qquad$ **Ans**

*4–60. Determine the magnitude of the moment produced by the force of $F = 200$ N about the hinged axis (the x axis) of the door.



Moment About the x axis: Either position vector $\mathbf{r}_{OB}$ or $\mathbf{r}_{CA}$ can be used to determine the moment of **F** about the x axis.

$$\mathbf{r}_{CA} = (2.5 - 2.5)\mathbf{i} + (0.9659 - 0)\mathbf{j} + (0.2588 - 0)\mathbf{k} = [0.9659\mathbf{j} + 0.2588\mathbf{k}]\text{ m}$$

$$\mathbf{r}_{OB} = (0.5 - 0)\mathbf{i} + (0 - 0)\mathbf{j} + (2 - 0)\mathbf{k} = [0.5\mathbf{i} + 2\mathbf{k}]\text{ m}$$

The force vector **F** is given by

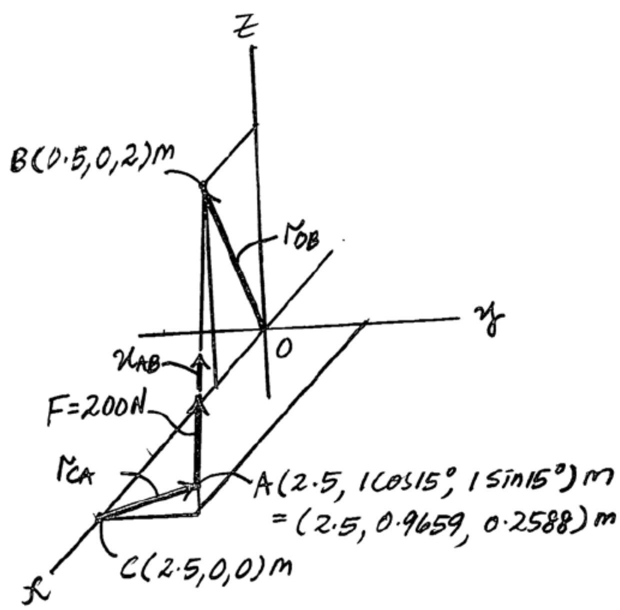
$$\mathbf{F} = F\mathbf{u}_{AB} = 200\left[\frac{(0.5-2.5)\mathbf{i} + (0-0.9659)\mathbf{j} + (2-0.2588)\mathbf{k}}{\sqrt{(0.5-2.5)^2 + (0-0.9659)^2 + (2-0.2588)^2}}\right] = [-141.73\mathbf{i} - 68.45\mathbf{j} + 123.39\mathbf{k}]\text{ N}$$

Knowing that the unit vector of the x axis is **i**, the magnitude of the moment of **F** about the x axis is given by

$$M_x = \mathbf{i}\cdot\mathbf{r}_{CA}\times\mathbf{F} = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 0.9659 & 0.2588 \\ -141.73 & -68.45 & 123.39 \end{vmatrix} = 137\text{ N}\cdot\text{m}$$ **Ans.**

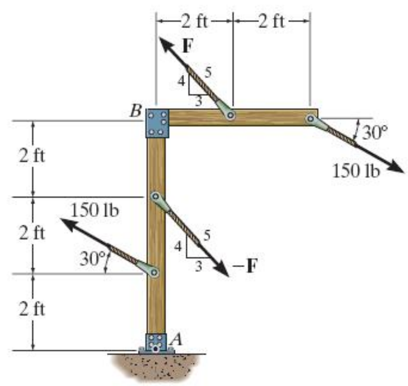
or

$$M_x = \mathbf{i}\cdot\mathbf{r}_{OB}\times\mathbf{F} = \begin{vmatrix} 1 & 0 & 0 \\ 0.5 & 0 & 2 \\ -141.73 & -68.45 & 123.39 \end{vmatrix} = 137\text{ N}\cdot\text{m}$$ **Ans.**



(a)

4–75. If $F = 200$ lb, determine the resultant couple moment.



a) By resolving the 150 - lb and 200 - lb couples into their x and y components, Fig. a, the couple moments $(M_C)_1$ and $(M_C)_2$ produced by the 150 - lb and 200 - lb couples, respectively, are given by

$$\circlearrowleft +(M_c)_1 = -150\cos 30°(4) - 150\sin 30°(4) = -819.62 \text{ lb}\cdot\text{ft} = 819.62 \text{ lb}\cdot\text{ft}$$

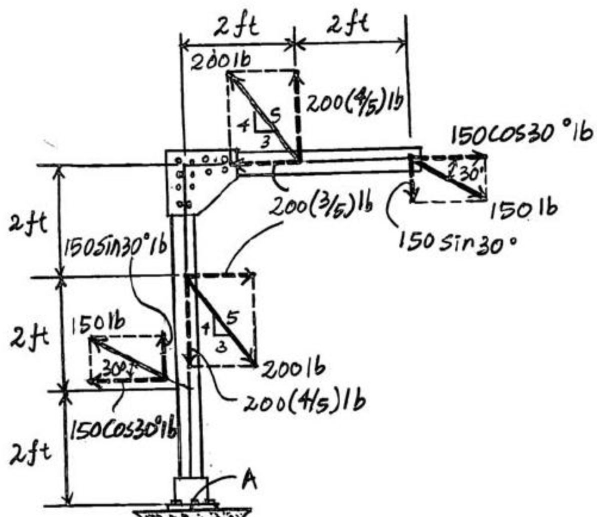
$$\circlearrowleft +(M_c)_2 = 200\left(\frac{4}{5}\right)(2) + 200\left(\frac{3}{5}\right)(2) = 560 \text{ lb}\cdot\text{ft}$$

Thus, the resultant couple moment can be determined from

$$\circlearrowleft +(M_c)_R = (M_c)_1 + (M_c)_2$$
$$= -819.62 + 560 = -259.62 \text{ lb}\cdot\text{ft} = 260 \text{ lb}\cdot\text{ft (clockwise)}$$ **Ans.**

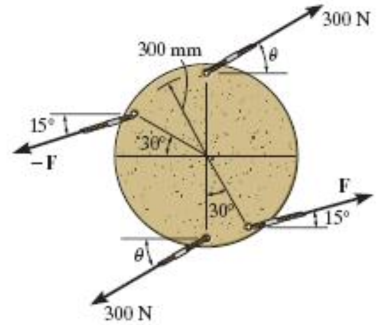
b) By resolving the 150 - lb and 200 - lb couples into their x and y components, Fig. a, and summing the moments of these force components algebraically about point A,

$$\circlearrowleft +(M_C)_R = \Sigma M_A;\ (M_C)_R = -150\sin 30°(4) - 150\cos 30°(6) + 200\left(\frac{4}{5}\right)(2) + 200\left(\frac{3}{5}\right)(6)$$
$$-200\left(\frac{3}{5}\right)(4) + 200\left(\frac{4}{5}\right)(0) + 150\cos 30°(2) + 150\sin 30°(0)$$
$$= -259.62 \text{ lb}\cdot\text{ft} = 260 \text{ lb}\cdot\text{ft (clockwise)}$$ **Ans.**



(a)

4–78. If $\theta = 30°$, determine the magnitude of force **F** so that the resultant couple moment is 100 N · m, clockwise.

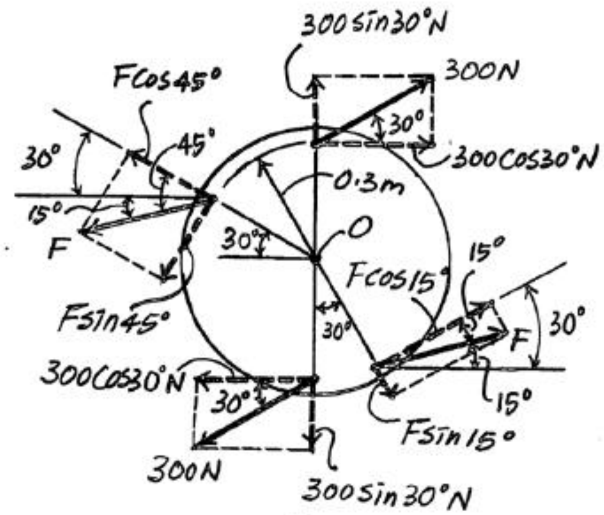


By resolving **F** and the 300 -N couple into their radial and tangential components, Fig. *a*, and summing the moment of these two force components about point *O*,

$$\curvearrowleft +(M_c)_R = \Sigma M_O; \quad -100 = F\sin 45°(0.3) + F\cos 15°(0.3) - 2(300\cos 30°)(0.3)$$

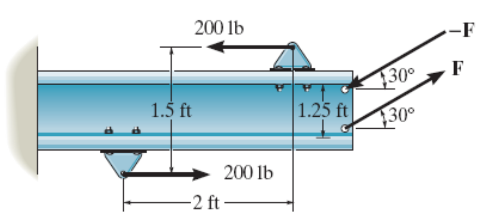
$$F = 111\text{ N} \qquad \textbf{Ans.}$$

Note: Since the line of action of the radial component of the forces pass through point *O*, no moment is produced about this point.



(a)

***4–80.** Two couples act on the beam. Determine the magnitude of **F** so that the resultant couple moment is 450 lb · ft, counterclockwise. Where on the beam does the resultant couple moment act?



$$\curvearrowleft + M_R = \Sigma M; \quad 450 = 200(1.5) + F\cos 30°(1.25)$$

$$F = 139\text{ lb} \qquad \textbf{Ans}$$

The resultant couple moment is a free vector. It can act at any point on the beam.