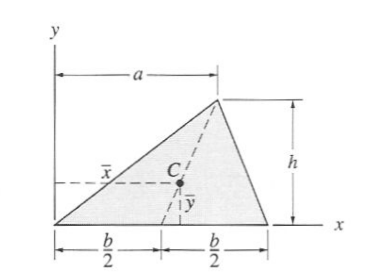
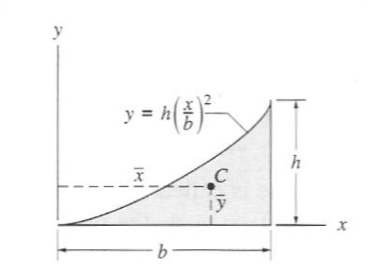
Selected Centroid and Moment of Inertia Shapes

Triangle



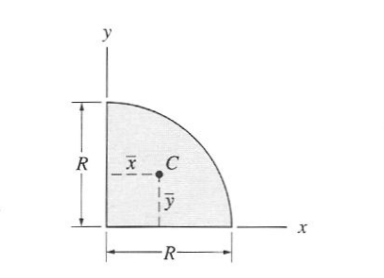
$\bar{x} = \frac{1}{3}(a + b) \quad \bar{y} = \frac{1}{3}h \quad A = \frac{1}{2}bh$

Half parabolic complement



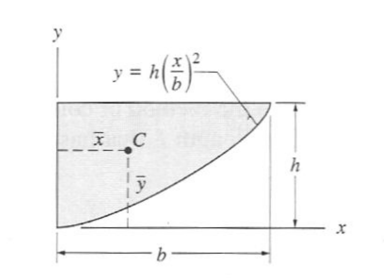
$\bar{x} = \frac{3}{4}b \quad \bar{y} = \frac{3}{10}h \quad A = \frac{1}{3}bh$

Quarter circle



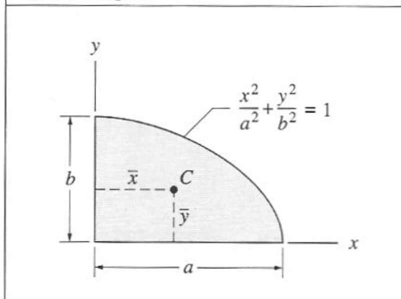
$\bar{x} = \frac{4}{3\pi}R \quad \bar{y} = \frac{4}{3\pi}R \quad A = \frac{\pi}{4}R^2$

Half parabola



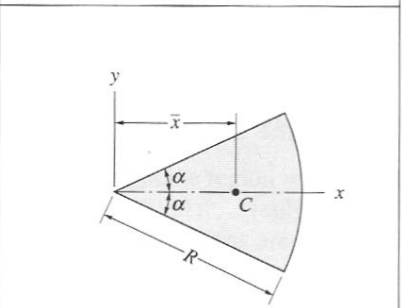
$\bar{x} = \frac{3}{8}b \quad \bar{y} = \frac{3}{5}h \quad A = \frac{2}{3}bh$

Quarter ellipse



$\bar{x} = \frac{4}{3\pi}a \quad \bar{y} = \frac{4}{3\pi}b \quad A = \frac{\pi}{4}ab$

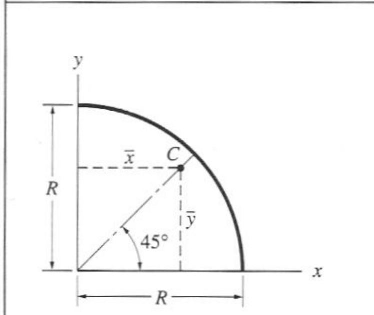
Circular sector



$\bar{x} = \frac{2R\sin\alpha}{3\alpha} \quad A = \alpha R^2$

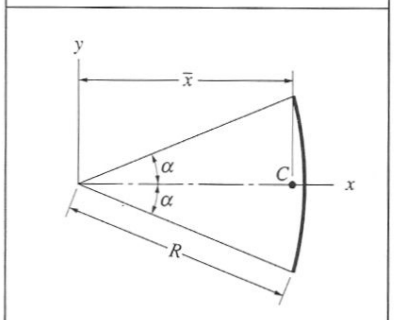
Table 8.1 *Centroids of Plane Areas*

Quarter circular arc



$x = \frac{2}{\pi}R \quad y = \frac{2}{\pi}R \quad L = \frac{\pi}{2}R$

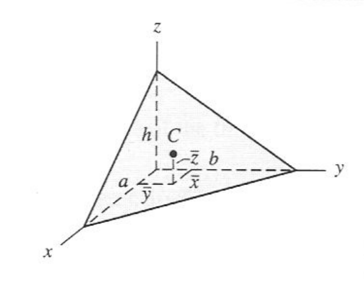
Circular arc



$\bar{x} = \frac{1}{\alpha}R\sin\alpha \quad L = 2\alpha R$

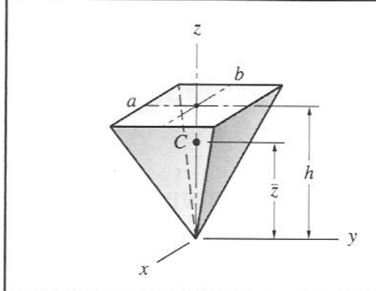
Table 8.2 *Centroids of Plane Curves*

Right tetrahedron



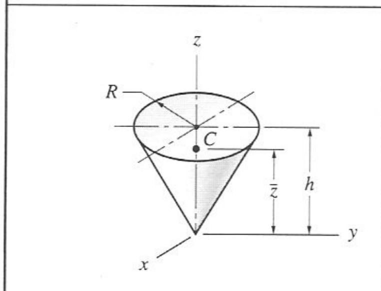
$\bar{x} = \frac{1}{4}a \qquad \bar{y} = \frac{1}{4}b \qquad \bar{z} = \frac{1}{4}h \qquad V = \frac{1}{6}abh$

Pyramid



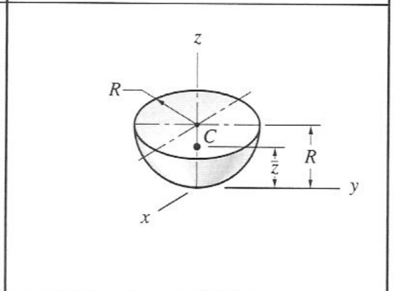
$\bar{z} = \frac{3}{4}h \qquad V = \frac{1}{3}abh$

Cone



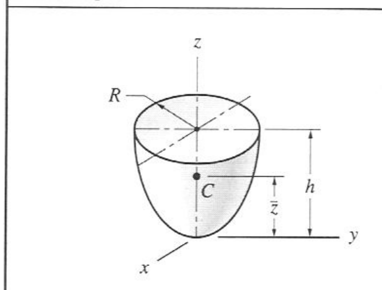
$\bar{z} = \frac{3}{4}h \qquad V = \frac{\pi}{3}R^2h$

Hemisphere



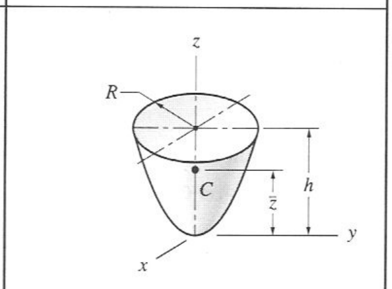
$\bar{z} = \frac{5}{8}R \qquad V = \frac{2\pi}{3}R^3$

Semi-ellipsoid of revolution



$\bar{z} = \frac{5}{8}h \qquad V = \frac{2\pi}{3}R^2h$

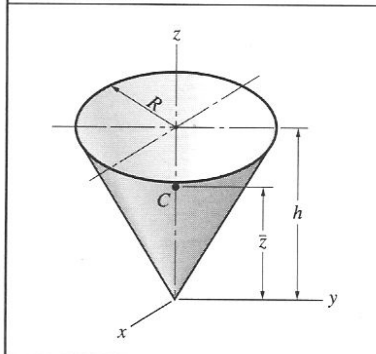
Paraboloid of revolution



$\bar{z} = \frac{2}{3}h \qquad V = \frac{\pi}{2}R^2h$

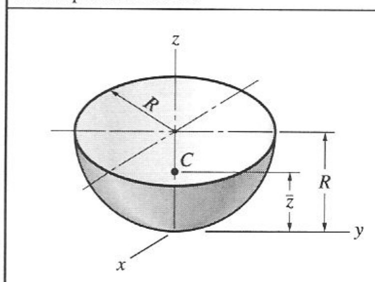
Table 8.3 *Centroids of Volumes*

Conical surface



$\bar{z} = \frac{2}{3}h \qquad A = \pi R\sqrt{R^2 + h^2}$

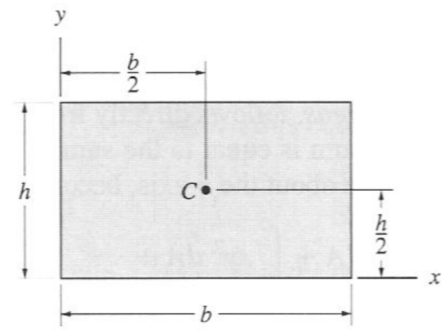
Hemispherical surface



$\bar{z} = \frac{1}{2}R \qquad A = 2\pi R^2$

Table 8.4 *Centroids of Surfaces*

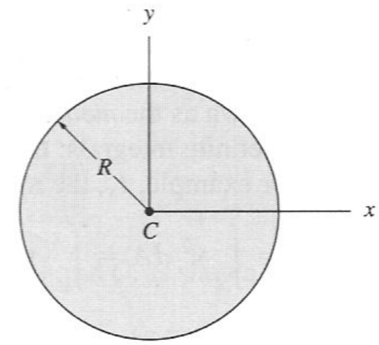
Rectangle



$\bar{I}_x = \frac{bh^3}{12}$ $\quad \bar{I}_y = \frac{b^3h}{12}$ $\quad \bar{I}_{xy} = 0$

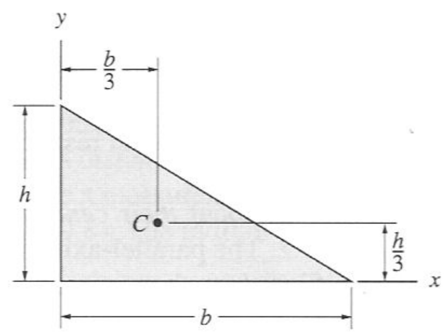
$I_x = \frac{bh^3}{3}$ $\quad I_y = \frac{b^3h}{3}$ $\quad I_{xy} = \frac{b^2h^2}{4}$

Circle



$I_x = I_y = \frac{\pi R^4}{4}$ $\quad I_{xy} = 0$

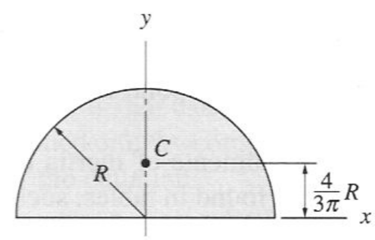
Right triangle



$\bar{I}_x = \frac{bh^3}{36}$ $\quad \bar{I}_y = \frac{b^3h}{36}$ $\quad \bar{I}_{xy} = -\frac{b^2h^2}{72}$

$I_x = \frac{bh^3}{12}$ $\quad I_y = \frac{b^3h}{12}$ $\quad I_{xy} = \frac{b^2h^2}{24}$

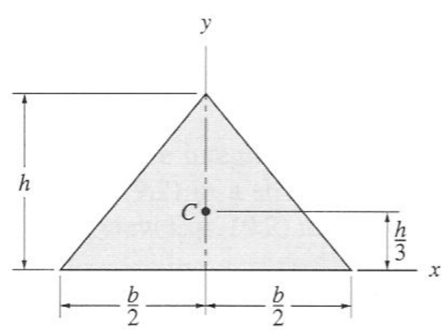
Semicircle



$\bar{I}_x = 0.1098R^4$ $\quad \bar{I}_{xy} = 0$

$I_x = I_y = \frac{\pi R^4}{8}$ $\quad I_{xy} = 0$

Isosceles triangle

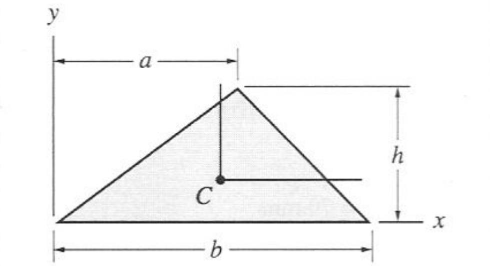


$\bar{I}_x = \frac{bh^3}{36}$ $\quad \bar{I}_y = \frac{b^3h}{48}$ $\quad \bar{I}_{xy} = 0$

$I_x = \frac{bh^3}{12}$ $\quad I_{xy} = 0$

Table 9.1 *Inertial Properties of Plane Areas: Part 1*

Triangle

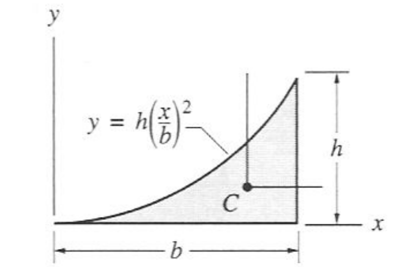


$$\bar{I}_x = \frac{bh^3}{36} \qquad I_x = \frac{bh^3}{12}$$

$$\bar{I}_y = \frac{bh}{36}(a^2 - ab + b^2) \qquad I_y = \frac{bh}{12}(a^2 + ab + b^2)$$

$$\bar{I}_{xy} = \frac{bh^2}{72}(2a - b) \qquad I_{xy} = \frac{bh^2}{24}(2a + b)$$

Half parabolic complement

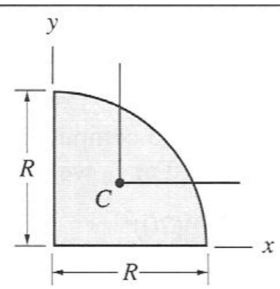


$$\bar{I}_x = \frac{37bh^3}{2100} \qquad I_x = \frac{bh^3}{21}$$

$$\bar{I}_y = \frac{b^3h}{80} \qquad I_y = \frac{b^3h}{5}$$

$$\bar{I}_{xy} = \frac{b^2h^2}{120} \qquad I_{xy} = \frac{b^2h^2}{12}$$

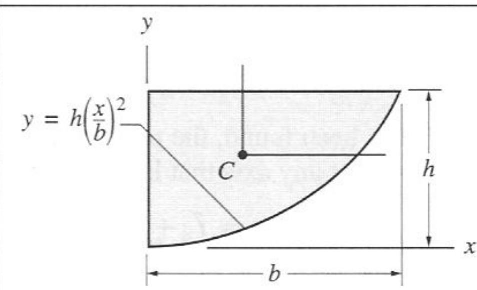
Quarter circle



$$\bar{I}_x = \bar{I}_y = 0.05488R^4 \qquad I_x = I_y = \frac{\pi R^4}{16}$$

$$\bar{I}_{xy} = -0.01647R^4 \qquad I_{xy} = \frac{R^4}{8}$$

Half parabola

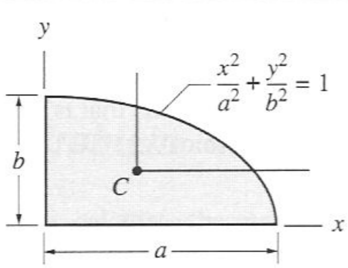


$$\bar{I}_x = \frac{8bh^3}{175} \qquad I_x = \frac{2bh^3}{7}$$

$$\bar{I}_y = \frac{19b^3h}{480} \qquad I_y = \frac{2b^3h}{15}$$

$$\bar{I}_{xy} = \frac{b^2h^2}{60} \qquad I_{xy} = \frac{b^2h^2}{6}$$

Quarter ellipse

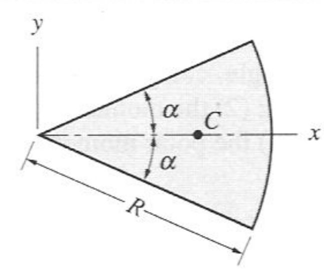


$$\bar{I}_x = 0.05488ab^3 \qquad I_x = \frac{\pi ab^3}{16}$$

$$\bar{I}_y = 0.05488a^3b \qquad I_y = \frac{\pi a^3b}{16}$$

$$\bar{I}_{xy} = -0.01647a^2b^2 \qquad I_{xy} = \frac{a^2b^2}{8}$$

Circular sector



$$I_x = \frac{R^4}{8}(2\alpha - \sin 2\alpha)$$

$$I_y = \frac{R^4}{8}(2\alpha + \sin 2\alpha)$$

$$I_{xy} = 0$$

Table 9.2 *Inertial Properties of Plane Areas: Part 2*