12-2. A car starts from rest and reaches a speed of 80 ft/s after traveling 500 ft along a straight road. Determine its constant acceleration and the time of travel.

$v_2^2 = v_1^2 + 2a_c(s_2 - s_1)$

$(80)^2 = 0 + 2a_c(500 - 0)$

$a_c = 6.40 \text{ ft/s}^2$ **Ans**

$v_2 = v_1 + a_c t$

$80 = 0 + 6.4(t)$

$t = 12.5 \text{ s}$ **Ans**

12-11. The acceleration of a particle as it moves along a straight line is given by $a = (2t - 1) \text{ m/s}^2$, where t is in seconds. If $s = 1$ m and $v = 2$ m/s when $t = 0$, determine the particle's velocity and position when $t = 6$ s. Also, determine the total distance the particle travels during this time period.

$\int_2^v dv = \int_0^t (2t - 1)\, dt$

$v = t^2 - t + 2$

$\int_1^s ds = \int_0^t (t^2 - t + 2)\, dt$

$s = \frac{1}{3}t^3 - \frac{1}{2}t^2 + 2t + 1$

When $t = 6$ s,

$v = 32 \text{ m/s}$ **Ans**

$s = 67 \text{ m}$ **Ans**

Since $v \neq 0$ then

$d = 67 - 1 = 66 \text{ m}$ **Ans**

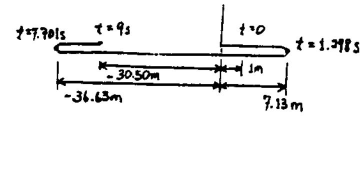
***12-28.** The acceleration of a particle along a straight line is defined by $a = (2t - 9) \text{ m/s}^2$, where t is in seconds. At $t = 0$, $s = 1$ m and $v = 10$ m/s. When $t = 9$ s, determine (a) the particle's position, (b) the total distance traveled, and (c) the velocity.

$a = 2t - 9$

$\int_{10}^v dv = \int_0^t (2t - 9)\, dt$

$v - 10 = t^2 - 9t$

$v = t^2 - 9t + 10$



$\int_1^s ds = \int_0^t (t^2 - 9t + 10)\, dt$

$s - 1 = \frac{1}{3}t^3 - 4.5t^2 + 10t$

$s = \frac{1}{3}t^3 - 4.5t^2 + 10t + 1$

Note when $v = 0$ at $t^2 - 9t + 10 = 0$

$t = 1.298$ s and $t = 7.701$ s

When $t = 1.298$ s, $s = 7.13$ m

When $t = 7.701$ s, $s = -36.63$ m

When $t = 9$ s, $s = -30.50$ m

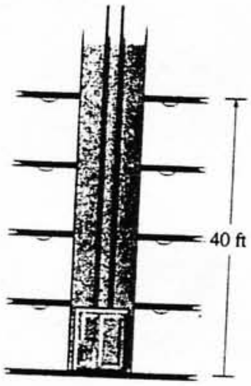
(a) $s = -30.5 \text{ m}$ **Ans**

(b) $s_{Tot} = (7.13 - 1) + 7.13 + 36.63 + (36.63 - 30.50)$

$s_{Tot} = 56.0 \text{ m}$ **Ans**

(c) $v = 10 \text{ m/s}$ **Ans**

12-38. The elevator starts from rest at the first floor of the building. It can accelerate at $5\ \text{ft/s}^2$ and then decelerate at $2\ \text{ft/s}^2$. Determine the shortest time it takes to reach a floor 40 ft above the ground. The elevator starts from rest and then stops. Draw the a–t, v–t, and s–t graphs for the motion.



$+\uparrow\ v_2 = v_1 + a_c t_1$

$v_{max} = 0 + 5t_1$

$+\uparrow\ v_3 = v_2 + a_c t$

$0 = v_{max} - 2t_2$

Thus

$t_1 = 0.4\, t_2$

$+\uparrow\ s_2 = s_1 + v_1 t + \frac{1}{2} a_c t_1^2$

$h = 0 + 0 + \frac{1}{2}(5)(t_1^2) = 2.5\, t_1^2$

$+\uparrow\ 40 - h = 0 + v_{max} t_2 - \frac{1}{2}(2)\, t_2^2$

$+\uparrow\ v^2 = v_1^2 + 2\, a_c (s - s_1)$

$v_{max}^2 = 0 + 2(5)(h - 0)$

$v_{max}^2 = 10\, h$

$0 = v_{max}^2 + 2(-2)(40 - h)$

$v_{max}^2 = 160 - 4\, h$

Thus,

$10\, h = 160 - 4h$

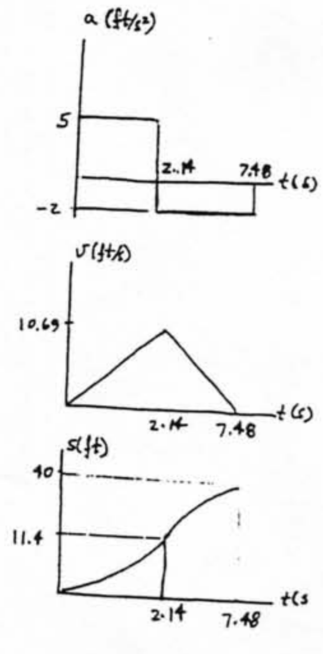
$h = 11.429\ \text{ft}$

$v_{max} = 10.69\ \text{ft/s}$

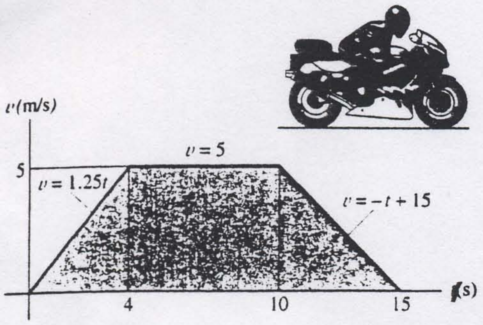
$t_1 = 2.138\ \text{s}$

$t_2 = 5.345\ \text{s}$

$t = t_1 + t_2 = 7.48\ \text{s}$ Ans



•12-44. A motorcycle starts from rest at $s = 0$ and travels along a straight road with the speed shown by the v–t graph. Determine the motorcycle's acceleration and position when $t = 8$ s and $t = 12$ s.



At $t = 8$ s

$$a = \frac{dv}{dt} = 0 \qquad \textbf{Ans}$$

$$\Delta s = \int v\,dt$$

$$s - 0 = \frac{1}{2}(4)(5) + (8-4)(5) = 30$$

$$s = 30 \text{ m} \qquad \textbf{Ans}$$

At $t = 12$ s

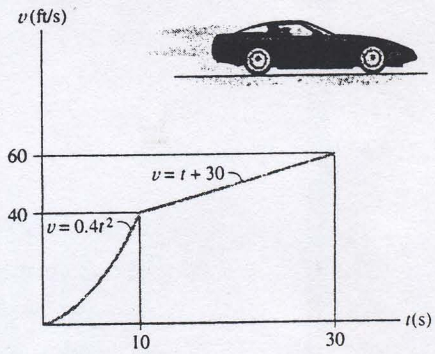
$$a = \frac{dv}{dt} = \frac{-5}{5} = -1 \text{ m/s}^2 \qquad \textbf{Ans}$$

$$\Delta s = \int v\,dt$$

$$s - 0 = \frac{1}{2}(4)(5) + (10-4)(5) + \frac{1}{2}(15-10)(5) - \frac{1}{2}\left(\frac{3}{5}\right)(5)\left(\frac{3}{5}\right)(5)$$

$$s = 48 \text{ m} \qquad \textbf{Ans}$$

12-50. The v–t graph for the motion of a car as it moves along a straight road is shown. Draw the s–t graph and determine the average speed and the distance traveled for the 30-s time interval. The car starts from rest at $s = 0$.



For $t < 10$ s,

$$v = 0.4t^2$$

$$ds = v\,dt$$

$$\int_0^s ds = \int_0^t 0.4t^2\,dt$$

$$s = 0.1333t^3$$

At $t = 10$ s,

$$s = 133.3 \text{ ft}$$

For $10 < t < 30$ s,

$$v = t + 30$$

$$ds = v\,dt$$

$$\int_{133.3}^{s} ds = \int_{10}^{t} (t+30)\,dt$$

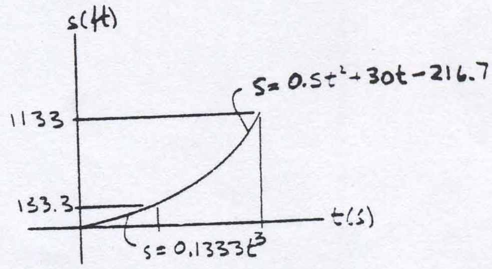
$$s = 0.5t^2 + 30t - 216.7$$

At $t = 30$ s,

$$s = 1133 \text{ ft}$$

$$(v_{sp})_{Avg} = \frac{\Delta s}{\Delta t} = \frac{1133}{30} = 37.8 \text{ ft/s} \qquad \textbf{Ans}$$

$$s_T = 1133 \text{ ft} = 1.13(10^3) \text{ ft} \qquad \textbf{Ans}$$



12-66. A particle, originally at rest and located at point (3 ft, 2 ft, 5 ft), is subjected to an acceleration of $\mathbf{a} = \{6t\mathbf{i} + 12t^2\mathbf{k}\}$ ft/s^2. Determine the particle's position (x, y, z) at $t = 1$ s.

***Velocity*:** The velocity express in Cartesian vector form can be obtained by applying Eq. 12 – 9.

$$d\mathbf{v} = \mathbf{a}dt$$

$$\int_0^{\mathbf{v}} d\mathbf{v} = \int_0^t \left(6t\mathbf{i} + 12t^2\mathbf{k}\right) dt$$

$$\mathbf{v} = \{3t^2\mathbf{i} + 4t^3\mathbf{k}\} \text{ ft/s}$$

***Position*:** The position express in Cartesian vector form can be obtained by applying Eq. 12 – 7.

$$d\mathbf{r} = \mathbf{v}\,dt$$

$$\int_{\mathbf{r}_1}^{\mathbf{r}} d\mathbf{r} = \int_0^t \left(3t^2\mathbf{i} + 4t^3\mathbf{k}\right) dt$$

$$\mathbf{r} - (3\mathbf{i} + 2\mathbf{j} + 5\mathbf{k}) = t^3\mathbf{i} + t^4\mathbf{k}$$

$$\mathbf{r} = \left\{\left(t^3 + 3\right)\mathbf{i} + 2\mathbf{j} + \left(t^4 + 5\right)\mathbf{k}\right\} \text{ ft}$$

When $t = 1$ s, $\mathbf{r} = \left(1^3 + 3\right)\mathbf{i} + 2\mathbf{j} + \left(1^4 + 5\right)\mathbf{k} = \{4\mathbf{i} + 2\mathbf{j} + 6\mathbf{k}\}$ ft.
The coordinates of the particle are

(4, 2, 6) ft **Ans**

12-69. The position of a particle is defined by $r = \{5(\cos 2t)\,\mathbf{i} + 4(\sin 2t)\mathbf{j}\}$ m, where t is in seconds and the arguments for the sine and cosine are given in radians. Determine the magnitudes of the velocity and acceleration of the particle when $t = 1$ s. Also, prove that the path of the particle is elliptical.

***Velocity*:** The velocity expressed in Cartesian vector form can be obtained by applying Eq. 12 – 7.

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = \{-10\sin 2t\mathbf{i} + 8\cos 2t\mathbf{j}\} \text{ m/s}$$

When $t = 1$ s, $\mathbf{v} = -10\sin 2(1)\mathbf{i} + 8\cos 2(1)\mathbf{j} = \{-9.093\mathbf{i} - 3.329\mathbf{j}\}$ m/s. Thus, the magnitude of the velocity is

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(-9.093)^2 + (-3.329)^2} = 9.68 \text{ m/s}$$ **Ans**

***Acceleration*:** The acceleration express in Cartesian vector form can be obtained by applying Eq. 12 – 9.

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = \{-20\cos 2t\mathbf{i} - 16\sin 2t\mathbf{j}\} \text{ m/s}^2$$

When $t = 1$ s, $\mathbf{a} = -20\cos 2(1)\mathbf{i} - 16\sin 2(1)\mathbf{j} = \{8.323\mathbf{i} - 14.549\mathbf{j}\}$ m/s^2. Thus, the magnitude of the acceleration is

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{8.323^2 + (-14.549)^2} = 16.8 \text{ m/s}^2$$ **Ans**

***Travelling Path*:** Here, $x = 5\cos 2t$ and $y = 4\sin 2t$. Then,

$$\frac{x^2}{25} = \cos^2 2t \qquad [1]$$

$$\frac{y^2}{16} = \sin^2 2t \qquad [2]$$

Adding Eqs[1] and [2] yields

$$\frac{x^2}{25} + \frac{y^2}{16} = \cos^2 2t + \sin^2 2t$$

However, $\cos^2 2t + \sin^2 2t = 1$. Thus,

$$\frac{x^2}{25} + \frac{y^2}{16} = 1$$ **(Equation of an Ellipse)** ***(Q.E.D.)***

***12-72.** A car travels east 2 km for 5 minutes, then north 3 km for 8 minutes. and then west 4 km for 10 minutes. Determine the total distance traveled and the magnitude of displacement of the car. Also, what is the magnitude of the average velocity and the average speed?

Total Distance Traveled and Displacement : The total distance traveled is

$$s = 2 + 3 + 4 = 9 \text{ km} \qquad \textbf{Ans}$$

and the magnitude of the displacement is

$$\Delta r = \sqrt{2^2 + 3^2} = 3.606 \text{ km} = 3.61 \text{ km} \qquad \textbf{Ans}$$

Average Velocity and Speed : The total time is $\Delta t = 5 + 8 + 10 = 23 \text{ min} = 1380 \text{ s}$. The magnitude of average velocity is

$$v_{avg} = \frac{\Delta r}{\Delta t} = \frac{3.606(10^3)}{1380} = 2.61 \text{ m/s} \qquad \textbf{Ans}$$

and the average speed is

$$(v_{sp})_{avg} = \frac{s}{\Delta t} = \frac{9(10^3)}{1380} = 6.52 \text{ m/s} \qquad \textbf{Ans}$$