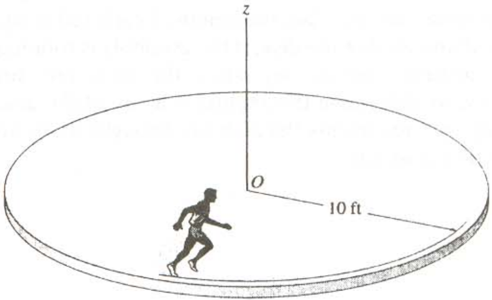
19-35. A horizontal circular platform has a weight of 300 lb and a radius of gyration $k_z = 8$ ft about the z axis passing through its center O. The platform is free to rotate about the z axis and is initially at rest. A man having a weight of 150 lb begins to run along the edge in a circular path of radius 10 ft. If he has a speed of 4 ft/s and maintains this speed relative to the platform, determine the angular velocity of the platform. Neglect friction.



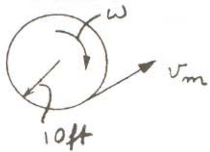
$$\mathbf{v}_m = \mathbf{v}_p + \mathbf{v}_{m/p}$$

$(\overset{+}{\rightarrow})\qquad v_m = -10\omega + 4$

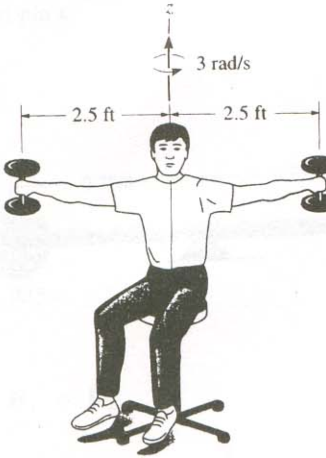
$(\circlearrowleft +)\qquad (H_z)_1 = (H_z)_2$

$$0 = -\left(\frac{300}{32.2}\right)(8)^2\,\omega + \left(\frac{150}{32.2}\right)(-10\omega + 4)(10)$$

$\omega = 0.175$ rad/s **Ans**



19-38. The man sits on the swivel chair holding two 5-lb weights with his arms outstretched. If he is turning at 3 rad/s in this position, determine his angular velocity when the weights are drawn in and held 0.3 ft from the axis of rotation. Assume he weighs 160 lb and has a radius of gyration $k_z = 0.55$ ft about the z axis. Neglect the mass of his arms and the size of the weights for the calculation.



Mass Moment of Inertia : The mass moment inertia of the man and the weights about z axis when the man arms is fully stretched is

$$(I_z)_1 = \left(\frac{160}{32.2}\right)(0.55^2) + 2\left[\frac{5}{32.2}(2.5^2)\right] = 3.444 \text{ slug}\cdot\text{ft}^2$$

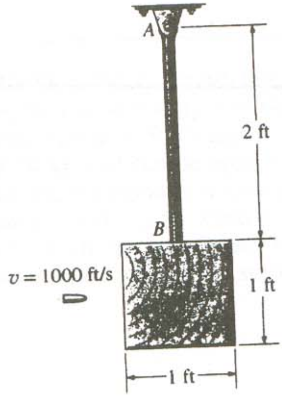
The mass moment inertia of the man and the weights about z axis when the weights are drawn in to a distance 0.3 ft from z axis

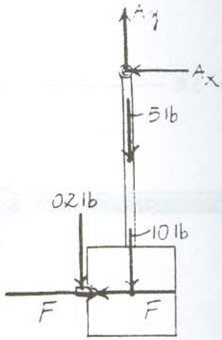
$$(I_z)_2 = \left(\frac{160}{32.2}\right)(0.55^2) + 2\left[\frac{5}{32.2}(0.3^2)\right] = 1.531 \text{ slug}\cdot\text{ft}^2$$

Conservation of Angular Momentum : Applying Eq. 19 – 17, we have

$$(H_z)_1 = (H_z)_2$$
$$3.444(3) = 1.531(\omega_z)_2$$
$$(\omega_z)_2 = 6.75 \text{ rad/s} \qquad \textbf{Ans}$$

*19-44. The pendulum consists of a 5-lb slender rod AB and a 10-lb wooden block. A projectile weighing 0.2 lb is fired into the center of the block with a velocity of 1000 ft/s. If the pendulum is initially at rest, and the projectile embeds itself into the block, determine the angular velocity of the pendulum just after the impact.





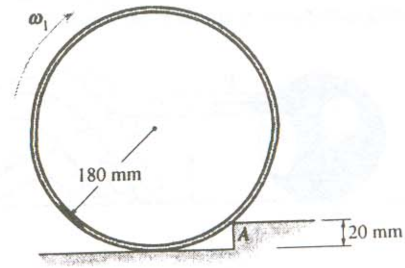
Mass Moment of Inertia : The mass moment inertia of the pendulum and the embeded bullet about point A is

$$(I_A)_2 = \frac{1}{12}\left(\frac{5}{32.2}\right)(2^2) + \frac{5}{32.2}(1^2) + \frac{1}{12}\left(\frac{10}{32.2}\right)(1^2+1^2) + \frac{10}{32.2}(2.5^2) + \frac{0.2}{32.2}(2.5^2)$$
$$= 2.239 \text{ slug}\cdot\text{ft}^2$$

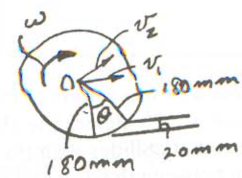
Conservation of Angular Momentum : Since force F due to the impact is *internal* to the system consisting of the pendulum and the bullet, it will cancel out. Thus, angular momentum is conserved about point A. Applying Eq. 19 – 17, we have

$$(H_A)_1 = (H_A)_2$$
$$(m_b v_b)(r_b) = (I_A)_2\,\omega_2$$
$$\left(\frac{0.2}{32.2}\right)(1000)(2.5) = 2.239\omega_2$$
$$\omega_2 = 6.94 \text{ rad/s} \qquad \textbf{Ans}$$

19-45. A thin ring having a mass of 15 kg strikes the 20-mm-high step. Determine the largest angular velocity ω_1 the ring can have so that it will not rebound off the step at A when it strikes it.



The weight is non – impulsive.

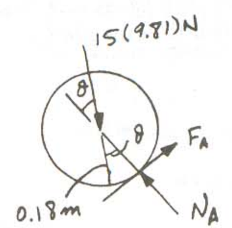


$$(H_A)_1 = (H_A)_2$$

$$15(\omega_1)(0.18)(0.18-0.02)+\left[15(0.18)^2\right](\omega_1) = \left[15(0.18)^2 + 15(0.18)^2\right]\omega_2$$

$$\omega_2 = 0.9444\omega_1$$

$$+\searrow \Sigma F_n = m(a_G)_n; \quad (15)(9.81)\cos\theta - N_A = 15\omega_2^2(0.18)$$

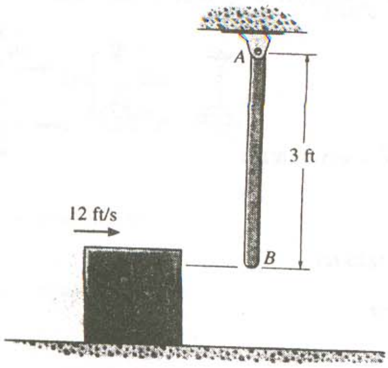


When hoop is about to rebound, $N_A \approx 0$. Also, $\cos\theta = \dfrac{160}{180}$, and so

$$\omega_2 = 6.9602 \text{ rad/s}$$

$$\omega_1 = \frac{6.9602}{0.9444} = 7.37 \text{ rad/s} \qquad \textbf{Ans}$$

*19-51. The 4-lb rod AB is hanging in the vertical position. A 2-lb block, sliding on a smooth horizontal surface with a velocity of 12 ft/s, strikes the rod at its end B. Determine the velocity of the block immediately after the collision. The coefficient of restitution between the block and the rod at B is $e = 0.8$.

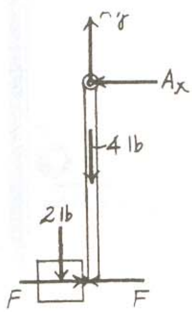


Conservation of Angular Momentum : Since force F due to the impact is *internal* to the system consisting of the slender rod and the block, it will cancel out. Thus, angular momentum is conserved about point A. The mass moment of inertia of the slender rod about point A is $I_A = \frac{1}{12}\left(\frac{4}{32.2}\right)(3^2) + \frac{4}{32.2}(1.5^2) = 0.3727 \text{ slug}\cdot\text{ft}^2$. Here, $\omega_2 = \frac{(v_B)_2}{3}$. Applying Eq. 19 – 17, we have

$$(H_A)_1 = (H_A)_2$$

$$[m_b (v_b)_1](r_b) = I_A\,\omega_2 + [m_b (v_b)_2](r_b)$$

$$\left(\frac{2}{32.2}\right)(12)(3) = 0.3727\left[\frac{(v_B)_2}{3}\right] + \left(\frac{2}{32.2}\right)(v_b)_2(3) \qquad [1]$$



Coefficient of Restitution : Applying Eq. 19 – 20, we have

$$e = \frac{(v_B)_2 - (v_b)_2}{(v_b)_1 - (v_B)_1}$$

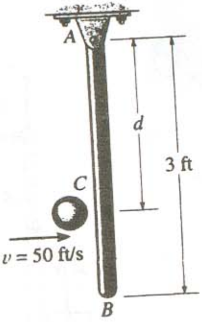
$$(\overset{+}{\rightarrow}) \qquad 0.8 = \frac{(v_B)_2 - (v_b)_2}{12 - 0} \qquad [2]$$

Solving Eqs. [1] and [2] yields

$$(v_b)_2 = 3.36 \text{ ft/s} \rightarrow \qquad \textbf{Ans}$$

$$(v_B)_2 = 12.96 \text{ ft/s} \rightarrow$$

19-53. The 6-lb slender rod AB is originally at rest, suspended in the vertical position. A 1-lb ball is thrown at the rod with a velocity $v = 50$ ft/s and strikes the rod at C. Determine the angular velocity of the rod just after the impact. Take $e = 0.7$ and $d = 2$ ft.



$$\circlearrowleft + (H_A)_1 = (H_A)_2$$

$$\left(\frac{1}{32.2}\right)(50)(2) = \left[\frac{1}{3}\left(\frac{6}{32.2}\right)(3)^2\right]\omega_2 + \frac{1}{32.2}(v_{BL})(2)$$

$$e = 0.7 = \frac{v_C - v_{BL}}{50 - 0}$$

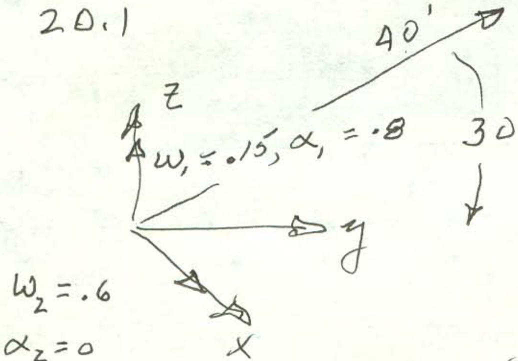
$$v_C = 2\omega_2$$

Thus,

$$\omega_2 = 7.73 \text{ rad/s} \qquad \textbf{Ans}$$

$$v_{BL} = -19.5 \text{ ft/s}$$

20.1



$$\bar v = \bar\omega \times \bar r$$

$$\bar\omega = .6\,\hat i + .15\,\hat k$$

$$\bar r = 40(\cos 30\,\hat j + \sin 30\,\hat k)$$

$$= 34.64\,\hat j + 20\,\hat k)$$

$$\bar v = (.6\,\hat i + .15\,\hat k) \times (34.64\,\hat j + 20\,\hat k)$$

$$= 20.78\,\hat k - 12\,\hat j - 5.20\,\hat i \quad \text{ft/s}$$

$$\bar\alpha_{xy} = \bar\omega_1 \times \bar\omega_2 = .15\,\hat k \times .6\,\hat i = .09\,\hat j \quad \text{rad/s}^2$$

$$\bar\alpha = .8\,\hat k + .09\,\hat j \quad \text{rad/sec}^2$$

$$a = \bar\alpha \times \bar r + \omega \times \overbrace{(\bar\omega \times \bar r)}^{\bar v}$$

$$= (.8\,\hat k + .09\,\hat j) \times (34.64\,\hat j + 20\,\hat k)$$

$$+ (.6\,\hat i + .15\,\hat k) \times (20.78\,\hat k - 12\,\hat j - 5.20\,\hat i)$$

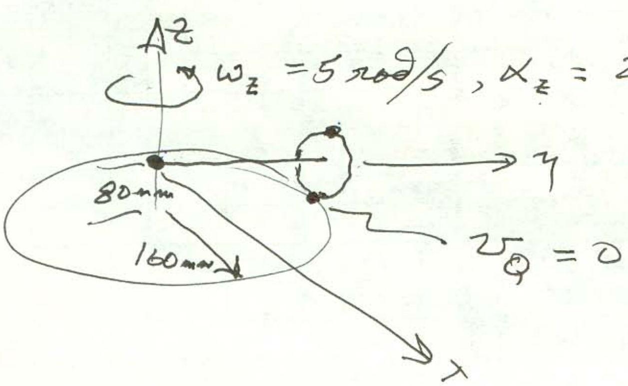
$$= -27.71\,\hat i + 1.8\,\hat i$$

$$- 12.47\,\hat j - 7.2\,\hat k + 1.8\,\hat i - .78\,\hat j$$

$$= \overset{-24.11}{\cancel{7.93}}\,\hat i - 13.25\,\hat j - 7.2\,\hat k \quad \text{ft/sec}^2$$

20.5

$\omega_z = 5\ rad/s\ ,\ \alpha_z = 2\ rad/s^2$



$$\bar{v}_Q = \bar{\omega} \times \bar{r}_Q$$

$$\bar{\omega} = \omega_y \hat{j} + \omega_z \hat{k} = \omega_y \hat{j} + 5\hat{k}$$

$$\bar{r}_Q = .16\hat{j} - .08\hat{k}$$

$$\bar{v}_Q = 0 = (\omega_y \hat{j} + 5\hat{k}) \times (.16\hat{j} - .08\hat{k})$$

$$0 = -.08\,\omega_y \hat{i} - .8\hat{i}$$

$$0 = -.08\,\omega_y - .8$$

$$\omega_y = -10\ rad/s$$

$$\bar{\alpha}_x = \bar{\omega}_z \times \bar{\omega} = 5\hat{k} \times (-10\hat{j} + 5\hat{k}) = 50\hat{i}\ rad/sec^2$$

$$\rightarrow\quad 0 = (\dot{\omega}_y \hat{j} + \dot{\omega}_z \hat{k})(.16\hat{j} - .08\hat{k})$$

$$0 = -.08\,\dot{\omega}_y \hat{i} - .16\,\dot{\omega}_z \hat{i}$$

$$\dot{\omega}_y = -\frac{.16}{.08}\dot{\omega}_z$$

$$\alpha_y = -\frac{.16}{.08}\,2 = -4\ rad/s$$

$$\bar{\alpha}_{TOTAL} = 50\hat{i} - 4\hat{j} + 2\hat{k}$$

$$\bar{v}_p = \bar{\omega} \times \bar{r}_p$$

$$= (-10\hat{j} + 5\hat{k}) \times (.16\hat{j} + .08\hat{k})$$

$$\bar{v}_p = -.8\hat{i} - .8\hat{i} = -1.6\hat{i} \text{ m/s}$$

$$\bar{a}_p = \bar{\alpha}_{TOTAL} \times \bar{r}_p + \bar{\omega} \times \underbrace{(\bar{\omega} \times \bar{r}_p)}$$

$$= (50\hat{i} - 4\hat{j} + 2\hat{k}) \times (.16\hat{j} + .08\hat{k})$$

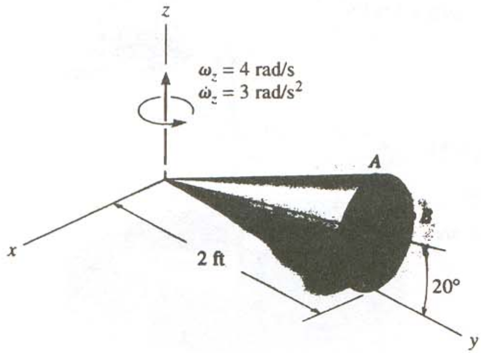
$$+ (-10\hat{j} + 5\hat{k}) \times (-1.6\hat{i})$$

$$= 8\hat{k} - 4\hat{j} - .32\hat{i} - .32\hat{i}$$

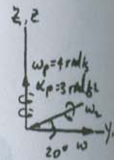
$$- 16k - 8\hat{j}$$

$$\bar{a}_p = -.64\hat{i} - 12\hat{j} - 8\hat{k} \text{ m/s}^2$$

20-9. The cone rolls without slipping such that at the instant shown $\omega_z = 4$ rad/s and $\dot{\omega}_z = 3$ rad/s^2. Determine the velocity and acceleration of point B at this instant.



Angular velocity : The resultant angular velocity $\omega = \omega_1 + \omega_2$ is always directed along the instantaneous axis of zero velocity(y axis).



$$\omega = \omega_1 + \omega_2$$
$$\omega\,\mathbf{j} = 4\mathbf{k} + (\omega_2 \cos 20^\circ \mathbf{j} + \omega_2 \sin 20^\circ \mathbf{k})$$
$$\omega\,\mathbf{j} = \omega_2 \cos 20^\circ \mathbf{j} + (4 + \omega_2 \sin 20^\circ)\,\mathbf{k}$$

Equating **j** and **k** components

$$4 + \omega_2 \sin 20^\circ = 0 \qquad \omega_2 = -11.70 \text{ rad/s}$$
$$\omega = -11.70 \cos 20^\circ = -10.99 \text{ rad/s}$$

Hemce $\quad \omega = \{-10.99\mathbf{j}\}$ rad/s

$$\omega_2 = -11.70\cos 20^\circ \mathbf{j} + (-11.70 \sin 20^\circ)\,\mathbf{k} = \{-10.99\mathbf{j} + 4\mathbf{k}\} \text{ rad/s}$$

Angular acceleration:

$$(\dot{\omega}_1)_{xyz} = \{3\mathbf{k}\} \text{ rad/s}^2$$
$$(\dot{\omega}_2)_{xyz} = \left(-\tfrac{3}{\sin 20^\circ}\right) \cos 20^\circ \mathbf{j} - 3\mathbf{k} = \{-8.2424\mathbf{j} - 3\mathbf{k}\} \text{ rad/s}^2$$

$$\dot{\omega} = \dot{\omega}_1 + \dot{\omega}_2$$
$$= \left[(\dot{\omega}_1)_{xyz} + \Omega \times \omega_1\right] + \left[(\dot{\omega}_2)_{xyz} + \Omega \times \omega_2\right]$$

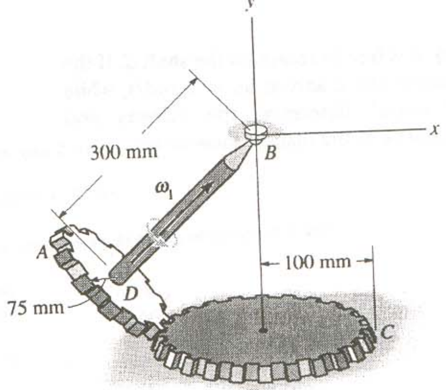
$\Omega = \omega_1 = \{4\mathbf{k}\}$ rad/s then

$$\dot{\omega} = [3\mathbf{k} + 0] + [(-8.2424\mathbf{j} - 3\mathbf{k}) + 4\mathbf{k} \times (-10.99\mathbf{j} + 4\mathbf{k})]$$
$$= \{43.9596\mathbf{i} - 8.2424\mathbf{j}\} \text{ rad/s}$$

$$\mathbf{r}_B = 2\sin 20^\circ \mathbf{i} + 2\cos 20^\circ \mathbf{j} + 2\sin 20^\circ \cos 20^\circ \mathbf{k}$$
$$= -0.68404\mathbf{i} + 1.8794\mathbf{j} + 0.64279\mathbf{k}$$
$$\mathbf{v}_B = \omega \times \mathbf{r}_B$$
$$= (-10.99\mathbf{j}) \times (-0.68404\mathbf{i} + 1.8794\mathbf{j} + 0.64279\mathbf{k})$$
$$= -7.0642\mathbf{i} - 7.5176\mathbf{k}$$
$$= \{-7.06\mathbf{i} - 7.52\mathbf{k}\} \text{ ft/s} \qquad \textbf{Ans}$$

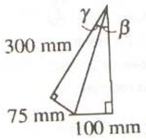
$$\mathbf{a}_B = \alpha \times \mathbf{r}_B + \omega \times \mathbf{v}_B$$
$$= (43.9596\mathbf{i} - 8.2424\mathbf{j}) \times (-0.68404\mathbf{i} + 1.8794\mathbf{j} + 0.64279\mathbf{k})$$
$$+ (-10.99\mathbf{j}) \times (-7.0642\mathbf{i} - 7.5176\mathbf{k})$$
$$= \{77.3\mathbf{i} - 28.3\mathbf{j} - 0.657\mathbf{k}\} \text{ ft/s}^2 \qquad \textbf{Ans}$$

20-13. Shaft BD is connected to a ball-and-socket joint at B, and a beveled gear A is attached to its other end. The gear is in mesh with a fixed gear C. If the shaft and gear A are *spinning* with a constant angular velocity $\omega_1 = 8$ rad/s, determine the angular velocity and angular acceleration of gear A.



$$\gamma = \tan^{-1}\frac{75}{300} = 14.04^\circ \quad \beta = \sin^{-1}\frac{100}{\sqrt{300^2 + 75^2}} = 18.87^\circ$$

The resultant angular velocity $\omega = \omega_1 + \omega_2$ is always directed along the instantaneous axis of zero velocity IA.



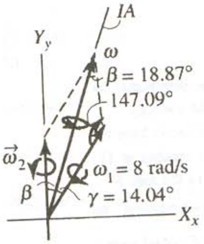
$$\frac{\omega}{\sin 147.09^\circ} = \frac{8}{\sin 18.87^\circ} \quad \omega = 13.44 \text{ rad/s}$$

$$\omega = 13.44 \sin 18.87^\circ\mathbf{i} + 13.44 \cos 18.87^\circ\mathbf{j}$$

$$= \{4.35\mathbf{i} + 12.7\mathbf{j}\} \text{ rad/s} \qquad \textbf{Ans}$$

$$\frac{\omega_2}{\sin 14.04^\circ} = \frac{8}{\sin 18.87^\circ} \quad \omega_2 = 6.00 \text{ rad/s}$$

$$\omega_2 = \{6\mathbf{j}\} \text{ rad/s}$$



$$\omega_1 = 8 \sin 32.91^\circ\mathbf{i} + 8 \cos 32.91^\circ\mathbf{j} = \{4.3466\mathbf{i} + 6.7162\mathbf{j}\} \text{ rad/s}$$

For ω_1, $\Omega = \omega_2 = \{6\mathbf{j}\}$ rad/s.

$$(\dot{\omega}_1)_{XYZ} = (\dot{\omega}_1)_{xyz} + \Omega \times \omega_1$$

$$= \mathbf{0} + (6\mathbf{j}) \times (4.3466\mathbf{i} + 6.7162\mathbf{j})$$

$$= \{-26.08\mathbf{k}\} \text{ rad/s}^2$$

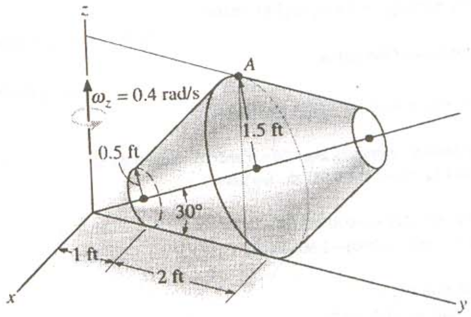
For ω_2, $\Omega = \mathbf{0}$.

$$(\dot{\omega}_2)_{XYZ} = (\dot{\omega}_2)_{xyz} + \Omega \times \omega_2 = \mathbf{0} + \mathbf{0} = \mathbf{0}$$

$$\boldsymbol{\alpha} = \dot{\omega} = (\dot{\omega}_1)_{XYZ} + (\dot{\omega}_2)_{XYZ}$$

$$\boldsymbol{\alpha} = \mathbf{0} + (-26.08\mathbf{k}) = \{-26.1\mathbf{k}\} \text{ rad/s}^2 \qquad \textbf{Ans}$$

20-14. The truncated cone rotates about the z axis at a constant rate $\omega_z = 0.4$ rad/s without slipping on the horizontal plane. Determine the velocity and acceleration of point A on the cone.



$$\theta = \sin^{-1}\left(\frac{0.5}{1}\right) = 30^\circ$$

$$\omega_s = \frac{0.4}{\sin 30^\circ} = 0.8 \text{ rad/s}$$

$$\omega = 0.8 \cos 30^\circ = 0.6928 \text{ rad/s}$$

$$\omega = \{-0.6928\mathbf{j}\} \text{ rad/s}$$

$$\Omega = 0.4\mathbf{k}$$

$$\dot{\omega} = (\dot{\omega})_{xyz} + \Omega \times \omega \qquad (1)$$

$$= 0 + (0.4\mathbf{k}) \times (-0.6928\mathbf{j})$$

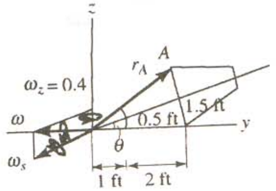
$$\dot{\omega} = \{0.2771\mathbf{i}\} \text{ rad/s}^2$$

$$\mathbf{r}_A = (3 - 3\sin 30^\circ)\mathbf{j} + 3\cos 30^\circ\mathbf{k}$$

$$= (1.5\mathbf{j} + 2.598\mathbf{k}) \text{ ft}$$

$$\mathbf{v}_A = \omega \times \mathbf{r}_A$$

$$= (-0.6928\mathbf{j}) \times (1.5\mathbf{j} + 2.598\mathbf{k})$$



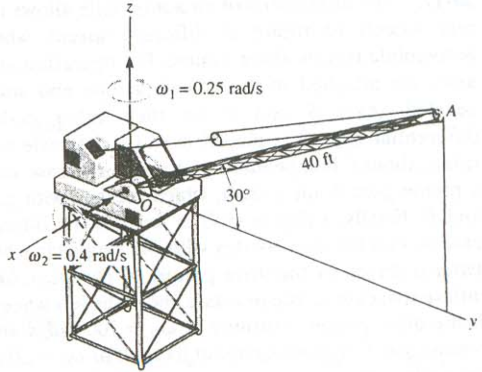
$$\mathbf{v}_A = \{-1.80\mathbf{i}\} \text{ ft/s} \qquad \textbf{Ans}$$

$$\mathbf{a}_A = \alpha \times \mathbf{r}_A + \omega \times \mathbf{v}_A$$

$$= (0.2771\mathbf{i}) \times (1.5\mathbf{j} + 2.598\mathbf{k}) + (-0.6928\mathbf{j}) \times (-1.80\mathbf{i})$$

$$\mathbf{a}_A = (-0.720\mathbf{j} - 0.831\mathbf{k}) \text{ ft/s}^2 \qquad \textbf{Ans}$$

20-15. At the instant shown, the tower crane is rotating about the z axis with an angular velocity $\omega_1 = 0.25$ rad/s, which is increasing at 0.6 rad/s^2. The boom OA is rotating downward with an angular velocity $\omega_2 = 0.4$ rad/s, which is increasing at 0.8 rad/s^2. Determine the velocity and acceleration of point A located at the top of the boom at this instant.



$$\boldsymbol{\omega} = \boldsymbol{\omega}_1 + \boldsymbol{\omega}_2 = \{-0.4\mathbf{i} + 0.25\mathbf{k}\} \text{ rad/s}$$

$$\boldsymbol{\Omega} = \{0.25\mathbf{k}\} \text{ rad/s}$$

$$\dot{\boldsymbol{\omega}} = \left(\dot{\boldsymbol{\omega}}\right)_{xyz} + \boldsymbol{\Omega} \times \boldsymbol{\omega} = (-0.8\mathbf{i} + 0.6\mathbf{k}) + (0.25\mathbf{k}) \times (-0.4\mathbf{i} + 0.25\mathbf{k})$$

$$= \{-0.8\mathbf{i} - 0.1\mathbf{j} + 0.6\mathbf{k}\} \text{ rad/s}^2$$

$$\mathbf{r}_A = 40\cos 30^\circ \mathbf{j} + 40\sin 30^\circ \mathbf{k} = \{34.64\mathbf{j} + 20\mathbf{k}\} \text{ ft}$$

$$\mathbf{v}_A = \boldsymbol{\omega} \times \mathbf{r}_A = (-0.4\mathbf{i} + 0.25\mathbf{k}) \times (34.64\mathbf{j} + 20\mathbf{k})$$

$$\mathbf{v}_A = \{-8.66\mathbf{i} + 8.00\mathbf{j} - 13.9\mathbf{k}\} \text{ ft/s} \qquad \textbf{Ans}$$

$$\mathbf{a}_A = \boldsymbol{\alpha} \times \mathbf{r}_A + \boldsymbol{\omega} \times \mathbf{v}_A = (-0.8\mathbf{i} - 0.1\mathbf{j} + 0.6\mathbf{k}) \times (34.64\mathbf{j} + 20\mathbf{k}) + (-0.4\mathbf{i} + 0.25\mathbf{k}) \times (-8.66\mathbf{i} + 8.00\mathbf{j} - 13.9\mathbf{k})$$

$$\mathbf{a}_A = \{-24.8\mathbf{i} + 8.29\mathbf{j} - 30.9\mathbf{k}\} \text{ ft/s}^2 \qquad \textbf{Ans}$$