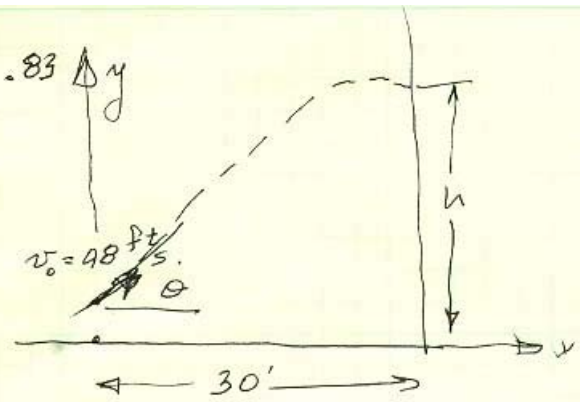
12.83



$$x = (48\cos\theta)t$$

$$y = 3 + (48\sin\theta)t - \frac{g}{2}t^2$$

no matter what θ is the time to reach the wall is $(x = 30)$

$$t_w = \frac{30}{48\cos\theta}$$

So the height at the wall is

$$y_w = 3 + (48\sin\theta)t_w - \frac{g}{2}t_w^2$$

$$= 3 + (48\sin\theta)\frac{30}{48\cos\theta} - \frac{g}{2}\left(\frac{30}{48\cos\theta}\right)^2$$

$$= 3 + 30\tan\theta - \frac{g}{2}\left(\frac{30}{48}\right)^2\frac{1}{\cos^2\theta}$$

$$\sin^2\theta + \cos^2\theta = 1$$

Divide by $\cos^2\theta$

$$\frac{\sin^2\theta}{\cos^2\theta} + 1 = \frac{1}{\cos^2\theta}$$

$$\tan^2\theta + 1 = \frac{1}{\cos^2\theta}$$

$$y_w = 3 + 30\tan\theta - \frac{g}{2}\left(\frac{30}{48}\right)^2(1+\tan^2\theta)$$

To find θ that makes y_w Max

$$\frac{\partial y_w}{\partial \theta} = 0 = 30\sec^2\theta - \frac{g}{2}\left(\frac{30}{48}\right)^2 2\tan\theta\sec\theta$$

$$0 = \sec^2\theta\left(30 - \frac{g}{2}\left(\frac{30}{48}\right)^2 2\tan\theta\right)$$

$\sec^2\theta$: Never Zero. The bracketed term: Must be Zero.

$$0 = 30 - g\left(\frac{30}{48}\right)^2\tan\theta$$

$$\tan\theta = \frac{30}{g}\frac{48^2}{30^2} = \frac{48^2}{g30} = 2.3851$$

$$\theta = \tan^{-1}(2.3851) = 67.25^\circ$$

&

$$y_w = h = 3 + 30(2.3851) - \frac{g}{2}\left(\frac{30}{48}\right)^2\left(1 + 2.3851^2\right)$$

$$= \boxed{32.49 \text{ ft}}$$

Theta =		0.00	10.00	20.00	28.50	30.00	40.00	50.00	60.00	67.25	70.00	80.00	89.00 degrees
Theta =		0.00	0.17	0.35	0.50	0.52	0.70	0.87	1.05	1.17	1.22	1.40	1.55 radians
Tan(Theta)=		0.00	0.18	0.36	0.54	0.58	0.84	1.19	1.73	2.39	2.75	5.67	57.29
x - feet													
	0	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00
	1	2.99	3.17	3.36	3.53	3.57	3.83	4.17	4.70	5.34	5.69	8.44	37.35
	2	2.97	3.32	3.70	4.05	4.12	4.63	5.32	6.35	7.58	8.26	13.42	25.81
	3	2.94	3.46	4.02	4.55	4.65	5.41	6.42	7.94	9.73	10.70	17.93	-31.61
	4	2.89	3.59	4.33	5.03	5.16	6.17	7.50	9.48	11.79	13.03	21.98	-134.91
	5	2.83	3.70	4.62	5.49	5.65	6.90	8.54	10.96	13.76	15.24	25.56	-284.10
	6	2.75	3.80	4.90	5.93	6.13	7.61	9.54	12.39	15.63	17.33	28.69	-479.18
	7	2.66	3.88	5.16	6.36	6.58	8.29	10.51	13.75	17.41	19.31	31.34	-720.13
	8	2.55	3.95	5.41	6.76	7.02	8.95	11.45	15.07	19.09	21.16	33.54	-1006.97
	9	2.43	4.00	5.63	7.15	7.44	9.59	12.36	16.32	20.68	22.89	35.27	-1339.70
	10	2.30	4.04	5.85	7.52	7.84	10.20	13.23	17.53	22.18	24.50	36.54	-1718.31
	11	2.15	4.07	6.05	7.88	8.22	10.79	14.06	18.67	23.58	25.99	37.34	-2142.80
	12	1.99	4.08	6.23	8.21	8.59	11.35	14.87	19.76	24.89	27.37	37.68	-2613.18
	13	1.82	4.07	6.39	8.53	8.93	11.90	15.63	20.79	26.11	28.62	37.56	-3129.44
	14	1.63	4.06	6.54	8.83	9.26	12.41	16.37	21.77	27.23	29.76	36.98	-3691.59
	15	1.43	4.02	6.68	9.11	9.56	12.91	17.07	22.69	28.26	30.77	35.93	-4299.62
	16	1.21	3.98	6.80	9.37	9.85	13.38	17.74	23.56	29.20	31.67	34.41	-4953.53
	17	0.98	3.92	6.90	9.62	10.12	13.82	18.37	24.37	30.04	32.44	32.44	-5653.33
	18	0.74	3.84	6.99	9.84	10.37	14.25	18.97	25.12	30.79	33.10	30.00	-6399.01
	19	0.48	3.75	7.06	10.05	10.61	14.64	19.54	25.82	31.44	33.64	27.10	-7190.58
	20	0.20	3.64	7.11	10.24	10.82	15.02	20.07	26.46	32.01	34.05	23.73	-8028.03
	21	-0.08	3.53	7.15	10.41	11.02	15.37	20.57	27.05	32.47	34.35	19.90	-8911.37
	22	-0.38	3.39	7.18	10.57	11.19	15.70	21.03	27.58	32.85	34.53	15.61	-9840.59
	23	-0.70	3.24	7.19	10.70	11.35	16.00	21.46	28.05	33.13	34.59	10.85	-10815.69
	24	-1.03	3.08	7.18	10.82	11.49	16.28	21.86	28.47	33.32	34.53	5.63	-11836.68
	25	-1.37	2.90	7.15	10.92	11.61	16.54	22.22	28.83	33.42	34.35	-0.06	-12903.55
	26	-1.72	2.71	7.11	11.00	11.71	16.77	22.55	29.14	33.42	34.05	-6.20	-14016.31
	27	-2.09	2.51	7.06	11.06	11.80	16.97	22.85	29.39	33.32	33.63	-12.81	-15174.95
	28	-2.48	2.29	6.99	11.11	11.86	17.16	23.11	29.58	33.14	33.10	-19.89	-16379.47
	29	-2.88	2.05	6.90	11.14	11.91	17.32	23.34	29.72	32.86	32.44	-27.43	-17629.88
	30	-3.29	1.81	6.80	11.15	11.94	17.46	23.53	29.81	32.49	31.66	-35.43	-18926.17
	31	-3.72	1.54	6.68	11.14	11.94	17.57	23.69	29.83	32.02	30.76	-43.89	-20268.35
	32	-4.16	1.26	6.54	11.11	11.93	17.66	23.82	29.80	31.46	29.75	-52.82	-21656.41
	33	-4.61	0.97	6.39	11.06	11.91	17.72	23.91	29.72	30.81	28.61	-62.21	-23090.35
	34	-5.08	0.67	6.23	11.00	11.86	17.76	23.97	29.58	30.06	27.36	-72.07	-24570.18
	35	-5.56	0.35	6.04	10.92	11.79	17.78	23.99	29.38	29.22	25.98	-82.39	-26095.90
	36	-6.06	0.01	5.85	10.82	11.71	17.77	23.98	29.13	28.29	24.49	-93.17	-27667.50

12-86 $x = (v_0 \cos 60)\, t$

$$y = (v_0 \sin 60)\, t - \frac{g}{2} t^2$$

$$x = 20 = (v_0 \cos 60)\, t$$

$$t = \frac{20}{v_0 \cos 60}$$

$$y = 0 = (v_0 \sin 60) \frac{20}{v_0 \cos 60} - \frac{g}{2} \left(\frac{20}{v_0 \cos 60} \right)^2$$

$$0 = 20 \tan 60 - \frac{g}{2} \frac{20^2}{v_0^2 \cos^2 60}$$

$$20 \tan 60 = \frac{g}{2} \frac{400}{v_0^2 \cos^2 60}$$

$$v^2 = \frac{g}{2} \frac{400}{20 \tan 60 \cos^2 60}$$

$$v = \sqrt{\frac{10\, g}{\tan 60 \cos^2 60}}$$

$$= \boxed{27.26 \ \text{ft/s}}$$

12-93 (Modify problem so ground is at $y = -.001x^2$ The original problem does not give a physical solution.)

$$x = (80\cos 30)t$$

$$y = (80\sin 30)t - \frac{g}{2}t^2$$

Ground is at

$$y = -.001x^2$$

and the ball is at the same point

$$(80\sin 30)t - \frac{g}{2}t^2 = -.001(80\cos 30)^2 t^2$$

$$\Rightarrow \left((80\cos 30)^2 .001 - \frac{g}{2}\right)t^2 + (80\sin 30)t = 0$$

$$\Rightarrow \underbrace{\left[\left((80\cos 30)^2 .001 - \frac{g}{2}\right)t + (80\sin 30)\right]}_{t_1 \text{ This}} t = 0$$

So either $t = 0$ or $t_1 = 0$

$t = 0$ is when it is kicked

$$t = \frac{-80\sin 30}{(80\cos 30)^2 .001 - \frac{g}{2}} = \frac{80\sin 30}{\frac{g}{2} - (80\cos 30)^2 .001}$$

$t = 3.540$ sec. Time the ball hits the ground.

$x = (80\cos 30)(3.540) = \boxed{245.25 \text{ ft.}}$

$y = (80\sin 30)(3.540) - \frac{g}{2}(3.540)^2$

$= \boxed{-60.14 \text{ ft.}}$

to check.

$-.001(245.25)^2 = -60.14$ ft ✓

12-111 $v = 20\ m/s$

$|\bar{a}| = 14\ m/s^2$ at $75°$ to $\hat{u}_t$

$$\bar{a} = \frac{dv}{dt}\hat{u}_t + \frac{v^2}{\rho}\hat{u}_n = 14\cos 75\,\hat{u}_t + 14\sin 75\,\hat{u}_n$$

$$\frac{dv}{dt}\hat{u}_t + \frac{v^2}{\rho}u_n = 3.6235\,\hat{u}_t + 13.523\,\hat{u}_n$$

$\hat{u}_t$: $\quad \frac{dv}{dt} = 3.6235\ m/s^2$

$\hat{u}_n$: $\quad \frac{v^2}{\rho} = 13.523\ m/s^2$

$$\rho = \frac{v^2}{13.523} = \frac{(20)^2}{13.523} = 29.579\ m$$

12-139 $r = 300\ ft$ CIRCULAR $\Rightarrow r = \text{CONST}$

$\dot{\theta} = .4\ rad/sec$

$\ddot{\theta} = .2\ rad/sec^2$

$$\bar{v} = \dot{r}\hat{u}_r + r\dot{\theta}\hat{u}_\theta$$

$$= 0\hat{u}_r + 300(.4)\hat{u}_\theta$$

$$= 120\ ft/s\ \hat{u}_\theta$$

$$|\bar{v}| = 120\ ft/sec$$

12-139 Continued

$$\bar{a} = (\ddot{r} - r\dot{\theta}^2)\hat{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{e}_\theta$$

$$= (0 - 300(.4)^2)\hat{e}_r + (300(.2) + 2\dot{r}\dot{\theta})\hat{e}_\theta$$

$$= -48\,\hat{e}_r + 60\,\hat{e}_\theta \ \text{ft/s}^2$$

$$|\bar{a}| = \sqrt{(48)^2 + (60)^2} = 76.8375 \ \text{ft/s}^2.$$

12-144

$r = 60$ m CIRCULAR $\Rightarrow r = \text{CONST}$

$v = 20$ m/s, $= \text{CONST}$

$$\bar{v} = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta \qquad |\bar{v}| = v = r\dot{\theta} = 20$$

$$\dot{\theta} = \frac{20}{60} = .33 \ \text{rad/s}$$

$$\bar{a} = (\ddot{r} - r\dot{\theta}^2)\hat{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{e}_\theta$$

$v = \text{const} = r\dot{\theta}$

$\dot{v} = 0 = r\ddot{\theta}$ since $r = \text{Const}$

$\Rightarrow \ddot{\theta} = 0$

$$\bar{a} = -r\dot{\theta}^2\hat{e}_r$$

$$= -60(.33)^2\hat{e}_r = -6.67\,\hat{e}_r$$

$$|\bar{a}| = 6.67 \ \text{m/s}^2$$

12-150

$\dot{r} = 4t \ m/s.$

$\dot{\theta} = 6 \ rad/s = Const.$

$\Rightarrow \ddot{\theta} = 0$

$\dot{r} = \frac{\partial r}{\partial t} = 4t$

$\int \partial r = \int 4t \, \partial t$

$r = 2t^2 + K_1$

at $t = 0 \quad r = 0 \quad \Rightarrow K_1 = 0$

$r = 2t^2 \ m$

$\ddot{r} = 4 \ \frac{m}{s^2}$

when $t = 1 \ sec.$

$r = 2 \ m \qquad \dot{\theta} = 6 \ rad/s$

$\dot{r} = 4 \ m/s. \qquad \ddot{\theta} = 0$

$\ddot{r} = 4 \ m/s^2$

$\bar{v} = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta = 4\hat{e}_r + 2(6)\hat{e}_\theta$

$= 4\hat{e}_r + 12\hat{e}_\theta$

$|\bar{v}| = \sqrt{4^2 + 12^2} = 12.649 \ m/s.$

$\bar{a} = (\ddot{r} - r\dot{\theta}^2)\hat{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{e}_\theta$

$= (4 - 2(6)^2)\hat{e}_r + (2(0) + 2(4)6)\hat{e}_\theta$

$= -68\hat{e}_r + 48\hat{e}_\theta \ \frac{m}{s^2} \quad |\bar{a}| = \sqrt{68^2 + 48^2} = 83.235 \ m/s^2$