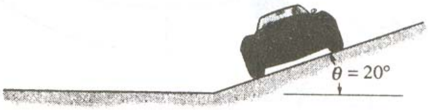
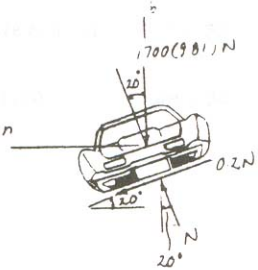
13-53. The sports car, having a mass of 1700 kg, is traveling horizontally along a 20° banked track which is circular and has a radius of curvature of $\rho = 100$ m. If the coefficient of static friction between the tires and the road is $\mu_s = 0.2$, determine the *maximum constant speed* at which the car can travel without sliding up the slope. Neglect the size of the car.





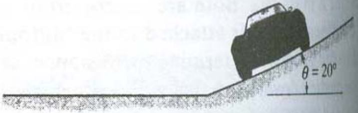
$+\uparrow \Sigma F_b = 0;\quad N\cos 20° - 0.2N\sin 20° - 1700(9.81) = 0$

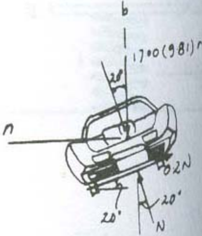
$N = 19\,140.6 \text{ N}$

$\leftarrow \Sigma F_n = ma_n;\quad 19\,140.6\sin 20° + 0.2(19\,140.6)\cos 20° = 1700\left(\frac{v_{max}^2}{100}\right)$

$v_{max} = 24.4 \text{ m/s}$ **Ans**

13-54. Using the data in Prob. 13-53, determine the *minimum speed* at which the car can travel around the track without sliding down the slope.





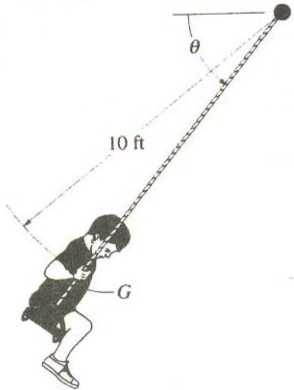
$$+\uparrow \Sigma F_b = 0; \qquad N\cos 20^\circ + 0.2N\sin 20^\circ - 1700(9.81) = 0$$

$$N = 16543.1 \text{ N}$$

$$\overset{+}{\leftarrow} \Sigma F_n = ma_n; \qquad 16543.1\sin 20^\circ - 0.2(16543.1)\cos 20^\circ = 1700\left(\frac{v_{\min}^2}{100}\right)$$

$$v_{\min} = 12.2 \text{ m/s} \qquad \textbf{Ans}$$

***13-60.** At the instant $\theta = 60°$, the boy's center of mass G is momentarily at rest. Determine his speed and the tension in each of the two supporting cords of the swing when $\theta = 90°$. The boy has a weight of 60 lb. Neglect his size and the mass of the seat and cords.



$+\searrow \Sigma F_t = ma_t;\qquad 60\cos\theta = \frac{60}{32.2}a_t \qquad a_t = 32.2\cos\theta$

$\nearrow + \Sigma F_n = ma_n;\qquad 2T - 60\sin\theta = \frac{60}{32.2}\left(\frac{v^2}{10}\right) \qquad [1]$

$v\,dv = a\,ds \qquad$ however $ds = 10d\theta$

$\int_0^v v\,dv = \int_{60°}^{90°} 322\cos\theta\,d\theta$

$v = 9.289$ ft/s

From Eq.[1] $\qquad 2T - 60\sin 90° = \frac{60}{32.2}\left(\frac{9.289^2}{10}\right) \qquad T = 38.0$ lb $\qquad$ An

13-83. A particle, having a mass of 1.5 kg, moves along a path defined by the equations $r = (4 + 3t)$ m, $\theta = (t^2 + 2)$ rad, and $z = (6 - t^3)$ m, where t is in seconds. Determine the r, θ, and z components of force which the path exerts on the particle when $t = 2$ s.

$r = 4 + 3t|_{t=2\,s} = 10$ m $\qquad \dot{r} = 3$ m/s $\qquad \ddot{r} = 0$

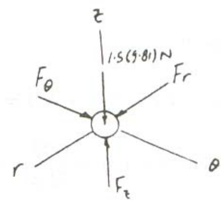
$\theta = t^2 + 2 \qquad \dot{\theta} = 2t|_{t=2\,s} = 4$ rad/s $\qquad \ddot{\theta} = 2$ rad/s^2

$z = 6 - t^3 \qquad \dot{z} = -3t^2 \qquad \ddot{z} = -6t|_{t=2\,s} = -12$ m/s^2

$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 10(4)^2 = -160$ m/s^2

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 10(2) + 2(3)(4) = 44$ m/s^2

$a_z = \ddot{z} = -12$ m/s^2

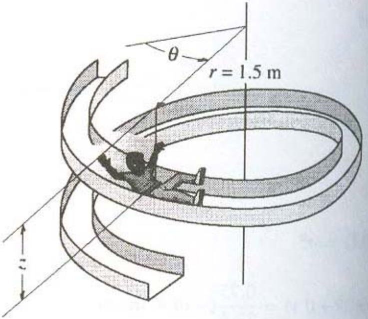


$\Sigma F_r = ma_r; \quad F_r = 1.5(-160) = -240$ N **Ans**

$\Sigma F_\theta = ma_\theta; \quad F_\theta = 1.5(44) = 66$ N **Ans**

$\Sigma F_z = ma_z; \quad F_z - 1.5(9.81) = 1.5(-12) \quad F_z = -3.28$ N **Ans**

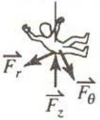
***13-88.** The boy of mass 40 kg is sliding down the spiral slide at a constant speed such that his position, measured from the top of the chute, has components $r = 1.5$ m, $\theta = (0.7t)$ rad, and $z = (-0.5t)$ m, where t is in seconds. Determine the components of force $\mathbf{F}_r$, $\mathbf{F}_\theta$, and $\mathbf{F}_z$ which the slide exerts on him at the instant $t = 2$ s. Neglect the size of the boy.



$r = 1.5 \qquad \theta = 0.7t \qquad z = -0.5t$

$\dot{r} = \ddot{r} = 0 \qquad \dot{\theta} = 0.7 \qquad \dot{z} = -0.5$

$\ddot{\theta} = 0 \qquad \ddot{z} = 0$



$a_r = \ddot{r} - r(\dot{\theta})^2 = 0 - 1.5(0.7)^2 = -0.735$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0$

$a_z = \ddot{z} = 0$

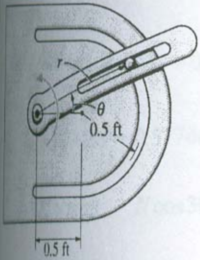
$\sum F_r = ma_r; \quad F_r = 40(-0.735) = -29.4 \text{ N}$ **Ans**

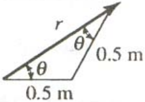
$\sum F_\theta = ma_\theta; \quad F_\theta = 0$ **Ans**

$\sum F_z = ma_z; \quad F_z - 40(9.81) = 0$

$F_z = 392 \text{ N}$ **Ans**

13-90. The 0.5-lb particle is guided along the circular path using the slotted arm guide. If the arm has an angular velocity $\ddot{\theta} = 4$ rad/s and an angular acceleration $\ddot{\theta} = 8\ \text{rad/s}^2$ at the instant $\theta = 30°$, determine the force of the guide on the particle. Motion occurs in the *horizontal* plane.

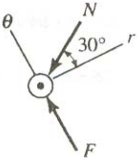




$$r = 2(0.5\cos\theta) = 1\cos\theta$$

$$\dot{r} = -\sin\theta\dot{\theta}$$

$$\ddot{r} = -\cos\theta\dot{\theta}^2 - \sin\theta\ddot{\theta}$$



At $\theta = 30°$, $\dot{\theta} = 4$ rad/s and $\ddot{\theta} = 8\ \text{rad/s}^2$

$$r = 1\cos 30° = 0.8660\ \text{ft}$$

$$\dot{r} = -\sin 30°(4) = -2\ \text{ft/s}$$

$$\ddot{r} = -\cos 30°(4)^2 - \sin 30°(8) = -17.856\ \text{ft/s}^2$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = -17.856 - 0.8660(4)^2 = -31.713\ \text{ft/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0.8660(8) + 2(-2)(4) = -9.072\ \text{ft/s}^2$$

$$\nearrow + \sum F_r = ma_r: \quad -N\cos 30° = \frac{0.5}{32.2}(-31.713) \quad N = 0.5686\ \text{lb}$$

$$+\nwarrow \sum F_\theta = ma_\theta; \quad F - 0.5686\sin 30° = \frac{0.5}{32.2}(-9.072)$$

$$F = 0.143\ \text{lb} \qquad \textbf{Ans}$$

22-1. When a 20-lb weight is suspended from a spring, the spring is stretched a distance of 4 in. Determine the natural frequency and the period of vibration for a 10-lb weight attached to the same spring.

$$k = \frac{20}{\frac{4}{12}} = 60 \text{ lb/ft}$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{60}{\frac{10}{32.2}}} = 13.90 \text{ rad/s}$$

$$\tau = \frac{2\pi}{\omega_n} = 0.452 \text{ s}$$ **Ans**

$$f = \frac{1}{\tau} = 2.21 \text{ Hz}$$ **Ans**

22-5. A 2-lb weight is suspended from a spring having a stiffness $k = 2$ lb/in. If the weight is pushed 1 in. upward from its equilibrium position and then released from rest, determine the equation which describes the motion. What is the amplitude and the natural frequency of the vibration?

$k = 2(12) = 24$ lb/ft

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{24}{\frac{2}{32.2}}} = 19.66 \text{ rad/s}$$

$$f = \frac{\omega_n}{2\pi} = 3.13 \text{ Hz}$$ **Ans**

$$y = -\frac{1}{12}, \quad v = 0 \text{ at } t = 0$$

From Eqs. 22-3 and 22-4,

$$-\frac{1}{12} = 0 + B$$

$B = -0.0833$

$0 = A\omega_n + 0$

$A = 0$

$C = \sqrt{A^2 + B^2} = 0.0833$ ft $= 1$ in. **Ans**

Position equation,

$y = (0.0833 \cos 19.7t)$ ft **Ans**

22-10. A pendulum has a 0.4-m-long cord and is given a tangential velocity of 0.2 m/s toward the vertical from a position $\theta = 0.3$ rad. Determine the equation which describes the angular motion.

See Example 22-1.

$$\omega_n = \sqrt{\frac{g}{l}} = \sqrt{\frac{9.81}{0.4}} = 4.95$$

$$\theta = A \sin \omega_n t + B \cos \omega_n t$$

$\theta = 0.3$ rad when $t = 0$.

$$0.3 = 0 + B; \quad B = 0.3$$

$$\dot{\theta} = A\omega_n \cos \omega_n t - B\omega_n \sin \omega_n t$$

Since $s = \theta l$, $\dot{v} = \dot{\theta} l$. Hence,

$-0.2 = \dot{\theta}(0.4)$, $\dot{\theta} = -0.5$ when $t = 0$,

$$-0.5 = A(4.95); \quad A = -0.101$$

Thus,

$$\theta = -0.101 \sin(4.95t) + 0.3 \cos(4.95t)$$ **Ans**