22-41. The block shown in Fig. 22-16 has a mass of 20 kg, and the spring has a stiffness $k = 600$ N/m. When the block is displaced and released, two successive amplitudes are measured as $x_1 = 150$ mm and $x_2 = 87$ mm. Determine the coefficient of viscous damping, c.

Assuming that the system is underdamped.

$$x_1 = De^{-\left(\frac{c}{2m}\right)t_1} \quad (1)$$

$$x_2 = De^{-\left(\frac{c}{2m}\right)t_2} \quad (2)$$

Divide Eq. (1) by Eq. (2) $\quad \dfrac{x_1}{x_2} = \dfrac{e^{-\left(\frac{c}{2m}\right)t_1}}{e^{-\left(\frac{c}{2m}\right)t_2}}$

$$\ln\left(\frac{x_1}{x_2}\right) = \left(\frac{c}{2m}\right)(t_2 - t_1) \quad (3)$$

However, $\quad t_2 - t_1 = \tau_d = \dfrac{2\pi}{\omega_d} = \dfrac{2\pi}{\omega_n\sqrt{1-\left(\frac{c}{c_c}\right)^2}} \quad$ and $\quad \omega_n = \dfrac{C_c}{2m}$

$$t_2 - t_1 = \frac{4m\pi}{C_c\sqrt{1-\left(\frac{c}{C_c}\right)^2}} \quad (4)$$

Substitute Eq. (4) into Eq. (3) yields:

$$\ln\left(\frac{x_1}{x_2}\right) = \left(\frac{c}{2m}\right)\frac{4m\pi}{C_c\sqrt{1-\left(\frac{C}{C_c}\right)^2}}$$

$$\ln\left(\frac{x_1}{x_2}\right) = \frac{2\pi\left(\frac{C}{C_c}\right)}{\sqrt{1-\left(\frac{C}{C_c}\right)^2}} \quad (5)$$

From Eq. (5)

$x_1 = 0.15$ m $\quad x_2 = 0.087$ m $\quad \omega_n = \sqrt{\dfrac{k}{m}} = \sqrt{\dfrac{600}{20}} = 5.477$ rad/s

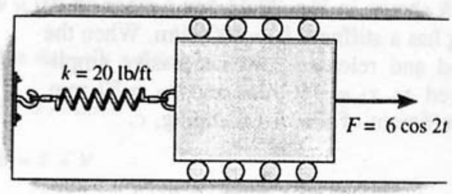
$$C_c = 2m\omega_n = 2(20)(5.477) = 219.09 \text{ N}\cdot\text{s/m}$$

$$\ln\left(\frac{0.15}{0.087}\right) = \frac{2\pi\left(\frac{c}{219.09}\right)}{\sqrt{1-\left(\frac{c}{219.09}\right)^2}}$$

$c = 18.9$ N · s/m **Ans**

Since $C < C_c$, the system is underdamped. Therefore, the assumption is OK!

22-42. The 20-lb block is attached to a spring having a stiffness of 20 lb/ft. A force $F = (6\cos 2t)$ lb, where t is in seconds, is applied to the block. Determine the maximum speed of the block after frictional forces cause the free vibrations to dampen out.



$$C = \frac{\frac{F_0}{k}}{1 - \left(\frac{\omega}{\omega_n}\right)^2}$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{20}{\frac{20}{32.2}}} = 5.6745 \text{ rad/s}$$

$$C = \frac{\frac{6}{20}}{1 - \left(\frac{2}{5.6745}\right)^2} = 0.343 \text{ ft}$$

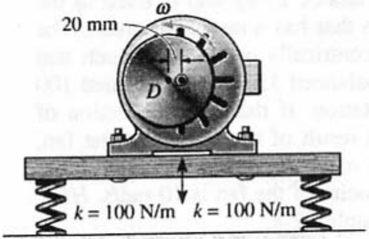
$$x_p = C\cos 2t$$

$$\dot{x}_p = -C(2)\sin 2t$$

Maximum velocity is

$$v_{max} = C(2) = 0.343(2) = 0.685 \text{ ft/s} \quad \textbf{Ans}$$

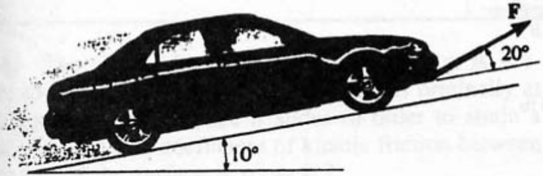
***22-48.** The electric motor has a mass of 50 kg and is supported by *four springs*, each spring having a stiffness of 100 N/m. If the motor turns a disk D which is mounted eccentrically, 20 mm from the disk's center, determine the angular rotation ω at which resonance occurs. Assume that the motor only vibrates in the vertical direction.

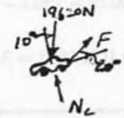


Resonance occurs when $\omega = \omega_n$.

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{4(100)}{50}} = 2.83 \text{ rad/s}$$

14-2. The car having a mass of 2 Mg is originally traveling at 2 m/s. Determine the distance it must be towed by a force $F = 4$ kN in order to attain a speed of 5 m/s. Neglect friction and the mass of the wheels.



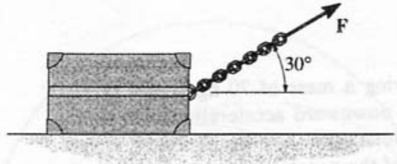


$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}(2000)(2)^2 + [\,4000\cos 20°(s) - 19\,620\sin 10°(s)\,] = \frac{1}{2}(2000)(5)^2$$

$s = 59.7$ m **Ans**

14-3. The 20-kg crate is subjected to a force having a constant direction and a magnitude $F = 100$ N, where s is measured in meters. When $s = 15$ m, the crate is moving to the right with a speed of 8 m/s. Determine its speed when $s = 25$ m. The coefficient of kinetic friction between the crate and the ground is $\mu_k = 0.25$.

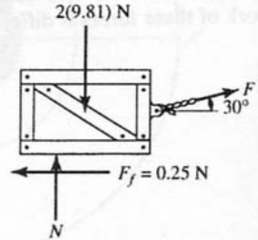


Equation of Motion: Since the crate slides, the friction force developed between the crate and its contact surface is $F_f = \mu_k N = 0.25$ N. Applying Eq. 13-7, we have

$$+\uparrow \sum F_y = ma_y; \quad N + \left(\frac{300}{1+s}\right)\sin 30^\circ - 2(9.81) = 2(0)$$

$$N = \left(-\frac{150}{1+s} + 19.62\right) \text{N}$$

Principle of Work and Energy: The horizontal component of force F which acts in the direction of displacement does *positive* work, whereas the friction force $F_f = 0.25\left(-\frac{150}{1+s} + 19.62\right) = \left(-\frac{37.5}{1+s} + 4.905\right)$ N does *negative* work since it acts in the opposite direction to that of displacement. The normal reaction N, the vertical component of force F and the weight of the crate do not displace hence do no work. Applying Eq. 14-7, we have



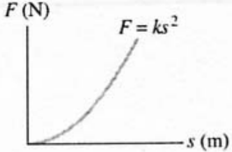
$$T_1 + \sum U_{1-2} = T_2$$

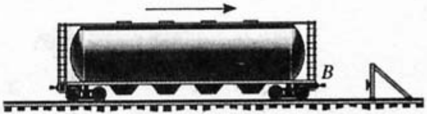
$$\frac{1}{2}(2)(8^2) + \int_{15\text{ m}}^{25\text{ m}} \left(\frac{300}{1+s}\right)\cos 30^\circ \, ds$$

$$- \int_{15\text{ m}}^{25\text{ m}} \left(-\frac{37.5}{1+s} + 4.905\right) ds = \frac{1}{2}(2)v^2$$

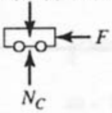
$v = 12.6$ m/s **Ans**

14-7. Design considerations for the bumper B on the 5-Mg train car require use of a nonlinear spring having the load-deflection characteristics shown in the graph. Select the proper value of k so that the maximum deflection of the spring is limited to 0.2 m when the car, traveling at 4 m/s, strikes the rigid stop. Neglect the mass of the car wheels.





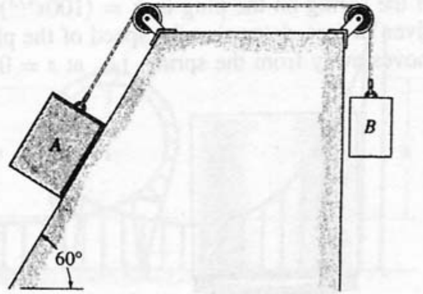
$$\frac{1}{2}(5000)(4)^2 - \int_0^{0.2} ks^2\, ds = 0$$



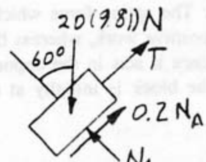
$$40000 - k\frac{(0.2)^3}{3} = 0$$

$k = 15.0\ \text{MN/m}^2$ **Ans**

14-14. Determine the velocity of the 20-kg block A after it is released from rest and moves 2 m down the plane. Block B has a mass of 10 kg and the coefficient of kinetic friction between the plane and block A is $\mu_k = 0.2$. Also, what is the tension in the cord?



Block A :



$\nwarrow \Sigma F_y = 0; \quad N_A - 20(9.81)\cos 60° = 0$

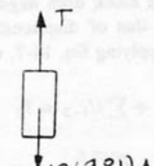
$N_A = 98.1 \text{ N}$

System :

$T_1 + \Sigma U_{1-2} = T_2$

$$(0+0) + 20(9.81)(\sin 60°)2 - 0.2(98.1)(2) - 10(9.81)(2) = \frac{1}{2}(20)v^2 + \frac{1}{2}(10)v^2$$

$v = 2.638 = 2.64 \text{ m/s}$ **Ans**



Block B :

$T_1 + \Sigma U_{1-2} = T_2$

$$0 + T(2) - 10(9.81)(2) = \frac{1}{2}(10)(2.638)^2$$

$T = 115 \text{ N}$ **Ans**

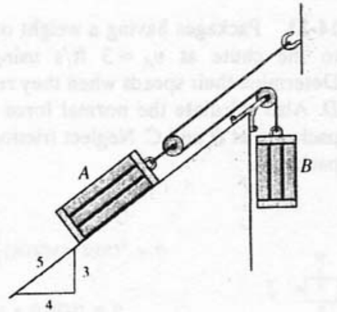
Also, block A :

$T_1 + \Sigma U_{1-2} = T_2$

$$0 + 20(9.81)(\sin 60°)(2) - T(2) - 0.2(98.1)(2) = \frac{1}{2}(20)(2.638)^2$$

$T = 115 \text{ N}$ **Ans**

4-22. The two blocks A and B have weights $W_A = 60$ lb nd $W_B = 10$ lb. If the kinetic coefficient of friction etween the incline and block A is $\mu_k = 0.2$, determine he speed of A after it moves 3 ft down the plane starting rom rest. Neglect the mass of the cord and pulleys.

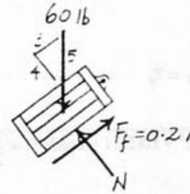


nematics : The speed of the block A and B can be related by using position ordinate equation.

$$s_A + (s_A - s_B) = l \qquad 2s_A - s_B = l$$

$$2\Delta s_A - \Delta s_B = 0 \qquad \Delta s_B = 2\Delta s_A = 2(3) = 6 \text{ ft}$$

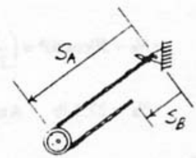
$$2v_A - v_B = 0 \qquad [1]$$



uation of Motion : Applying Eq. 13 – 7, we have

$$+\Sigma F_{y'} = ma_{y'}; \qquad N - 60\left(\frac{4}{5}\right) = \frac{60}{32.2}(0) \qquad N = 48.0 \text{ lb}$$

inciple of Work and Energy : By considering the whole system, W_A ich acts in the direction of the displacement does *positive* work. W_B and the ction force $F_f = \mu_k N = 0.2(48.0) = 9.60$ lb does *negative* work since they act he opposite direction to that of displacement. Here, W_A is being displaced rtically (downward) $\frac{3}{5}\Delta s_A$ and W_B is being displaced vertically (upward) $_B$. Since blocks A and B are at rest initially, $T_1 = 0$. Applying Eq. 14 – 7, have



$$T_1 + \sum U_{1-2} = T_2$$

$$0 + W_A\left(\frac{3}{5}\Delta s_A\right) - F_f \Delta s_A - W_B \Delta s_B = \frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2$$

$$60\left[\frac{3}{5}(3)\right] - 9.60(3) - 10(6) = \frac{1}{2}\left(\frac{60}{32.2}\right)v_A^2 + \frac{1}{2}\left(\frac{10}{32.2}\right)v_B^2$$

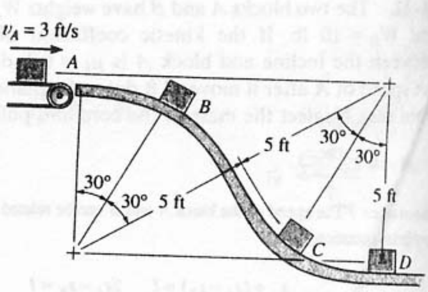
$$1236.48 = 60v_A^2 + 10v_B^2 \qquad [2]$$

qs. [1] and [2] yields

$$v_A = 3.52 \text{ ft/s} \qquad \textbf{Ans}$$

$$v_B = 7.033 \text{ ft/s}$$

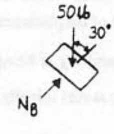
14-23. Packages having a weight of 50 lb are delivered to the chute at $v_A = 3$ ft/s using a conveyor belt. Determine their speeds when they reach points B, C, and D. Also calculate the normal force of the chute on the packages at B and C. Neglect friction and the size of the packages.



$$T_A + \Sigma U_{A-B} = T_B$$

$$\frac{1}{2}\left(\frac{50}{32.2}\right)(3)^2 + 50(5)(1 - \cos 30°) = \frac{1}{2}\left(\frac{50}{32.2}\right)v_B^2$$

$v_B = 7.221 = 7.22$ ft/s **Ans**



$$+\swarrow \Sigma F_n = ma_n; \quad -N_B + 50\cos 30° = \left(\frac{50}{32.2}\right)\left[\frac{(7.221)^2}{5}\right]$$

$N_B = 27.1$ lb **Ans**

$$T_A + \Sigma U_{A-C} = T_C$$

$$\frac{1}{2}\left(\frac{50}{32.2}\right)(3)^2 + 50(5\cos 30°) = \frac{1}{2}\left(\frac{50}{32.2}\right)v_C^2$$

$v_C = 16.97 = 17.0$ ft/s **Ans**

$$+\nearrow \Sigma F_n = ma_n; \quad N_C - 50\cos 30° = \left(\frac{50}{32.2}\right)\left[\frac{(16.97)^2}{5}\right]$$

$N_C = 133$ lb **Ans**

$$T_A + \Sigma U_{A-D} = T_D$$

$$\frac{1}{2}\left(\frac{50}{32.2}\right)(3)^2 + 50(5) = \frac{1}{2}\left(\frac{50}{32.2}\right)v_D^2$$

$v_D = 18.2$ ft/s **Ans**

14-41. The diesel engine of a 400-Mg train increases the train's speed uniformly from rest to 10 m/s in 100 s along a horizontal track. Determine the average power developed.

$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + U_{1-2} = \frac{1}{2}(400)(10^3)(10)^2$$

$$U_{1-2} = 20(10^6)\ \text{J}$$

$$P_{avg} = \frac{U_{1-2}}{t} = \frac{20(10^6)}{100} = 200\ \text{kW} \qquad \textbf{Ans}$$

Also,

$$v = v_0 + a_c t$$

$$10 = 0 + a_c(100)$$

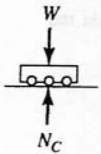
$$a_c = 0.1\ \text{m/s}^2$$

$$\xrightarrow{+} \Sigma F_x = m a_x; \qquad F = 400(10^3)(0.1) = 40(10^3)\ \text{N}$$

$$P_{avg} = \mathbf{F} \cdot \mathbf{v}_{avg} = 40(10^3)\left(\frac{10}{2}\right) = 200\ \text{kW} \qquad \textbf{Ans}$$

14-47. An electric streetcar has a weight of 15 000 lb and accelerates along a horizontal straight road from rest such that the power is always 100 hp. Determine how far it must travel to reach a speed of 40 ft/s.

$$F = ma = \frac{W}{g}\left(\frac{v\,dv}{ds}\right)$$



$$P = Fv = \left(\frac{W}{g}\frac{v\,dv}{ds}\right)v$$

$$\int_0^s P\,ds = \int_0^v \frac{W}{g}v^2 dv$$

P = constant

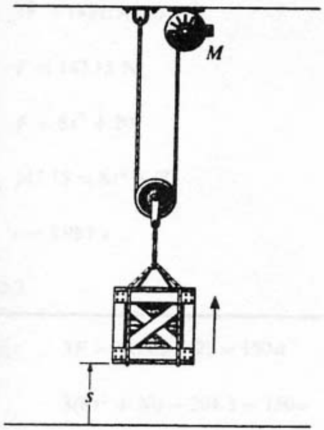
$$Ps = \frac{W}{g}\left(\frac{1}{3}\right)v^3$$

$$s = \frac{W}{3gP}v^3$$

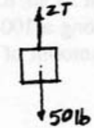
$$s = \frac{15\,000(40)^3}{3(32.2)(100)(550)}$$

$s = 181$ ft **Ans**

14-49. The 50-lb crate is given a speed of 10 ft/s in $t = 4$ s starting from rest. If the acceleration is constant, determine the power that must be supplied to the motor when $t = 2$ s. The motor has an efficiency $\epsilon = 0.76$. Neglect the mass of the pulley and cable.



$+\uparrow \Sigma F_y = m\,a_y; \qquad 2T - 50 = \frac{50}{32.2}a$



$(+\uparrow)\ v = v_0 + a_c t$

$10 = 0 + a(4)$

$a = 2.5 \text{ ft/s}^2$

$T = 26.94 \text{ lb}$

In $t = 2$ s,

$(+\uparrow)\quad v = v_0 + a_c t$

$v = 0 + 2.5(2) = 5 \text{ ft/s}$

$s_C + (s_C - s_P) = l$

$2v_C = v_P$

$2(5) = v_P = 10 \text{ ft/s}$

$P_O = 26.94(10) = 269.2$

$P_{in} = \frac{269.2}{0.76} = 354.49 \text{ ft}\cdot\text{lb/s}$

$P_{in} = 0.644 \text{ hp} \qquad$ **Ans**