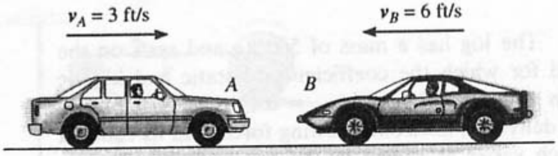
15-33. The car *A* has a weight of 4500 lb and is traveling to the right at 3 ft/s. Meanwhile a 3000-lb car *B* is traveling at 6 ft/s to the left. If the cars crash head-on and become entangled, determine their common velocity just after the collision. Assume that the brakes are not applied during collision.

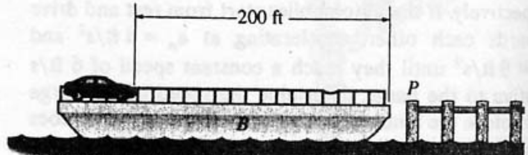


$$(\overset{+}{\rightarrow}) \qquad m_A(v_A)_1 + m_B(v_B)_1 = (m_A + m_B)v_2$$

$$\frac{4500}{32.2}(3) - \frac{3000}{32.2}(6) = \frac{7500}{32.2}v_2$$

$$v_2 = -0.600 \text{ ft/s} = 0.600 \text{ ft/s} \leftarrow \qquad \textbf{Ans}$$

15-39. The barge B weighs 30 000 lb and supports an automobile weighing 3000 lb. If the barge is not tied to the pier P and someone drives the automobile to the other side of the barge for unloading, determine how far the barge moves away from the pier. Neglect the resistance of the water.



Relative Velocity : The relative velocity of the car with respect to the barge is $v_{c/b}$. Thus, the velocity of the car is

$$(\overset{+}{\rightarrow}) \qquad v_c = -v_b + v_{c/b} \qquad [1]$$

Conservation of Linear Momentum : If we consider the car and the barge as a system, then the impulsive force caused by the traction of the tires is *internal* to the system. Therefore, it will cancel out. As the result, the linear momentum is conserved along the x axis.

$$0 = m_c v_c + m_b v_b$$

$$(\overset{+}{\rightarrow}) \qquad 0 + 0 = \left(\frac{3000}{32.2}\right) v_c + \left(\frac{30\,000}{32.2}\right) v_b \qquad [2]$$

Substituting Eq.[1] into [2] yields

$$11 v_b - v_{c/b} = 0 \qquad [3]$$

Integrating Eq.[3] becomes

$$(\overset{+}{\rightarrow}) \qquad 11 s_b - s_{c/b} = 0 \qquad [4]$$

Here, $s_{c/b} = 200$ ft. Then, from Eq.[4]

$$11 s_b - 200 = 0 \qquad s_b = 18.2 \text{ ft} \qquad \textbf{Ans}$$

***15-43.** The man M weighs 150 lb and jumps onto the boat B which is originally at rest. If he has a horizontal component of velocity of 3 ft/s just before he enters the boat, determine the weight of the boat if it has a velocity of 2 ft/s once the man enters it.

$(\xrightarrow{+})\quad v_M = v_B + v_{M/B}$

$v_M = 0+3$

$v_M = 3 \text{ ft/s}$

$(\xrightarrow{+})\quad \Sigma m(v_1) = \Sigma m(v_2)$

$$\frac{150}{32.2}(3) + \frac{W_B}{32.2}(0) = \frac{(W_B + 150)}{32.2}(2)$$

$W_B = 75 \text{ lb}$ **Ans**

15-55. An ivory ball having a mass of 200 g is released from rest at a height of 400 mm above a very large fixed metal surface. If the ball rebounds to a height of 325 mm above the surface, determine the coefficient of restitution between the ball and the surface.

Before impact

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 0.2(9.81)(0.4) = \frac{1}{2}(0.2)v_1^2 + 0$$

$$v_1 = 2.801 \text{ m/s}$$

After the impact

$$\frac{1}{2}(0.2)v_2^2 = 0 + 0.2(9.81)(0.325)$$

$$v_2 = 2.525 \text{ m/s}$$

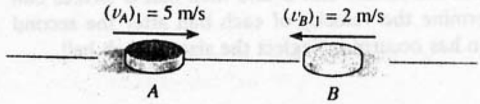
Coefficient of restitution:

$$(+\downarrow) \qquad e = \frac{(v_A)_2 - (v_B)_2}{(v_B)_1 - (v_A)_1}$$

$$= \frac{0 - (-2.525)}{2.801 - 0}$$

$$= 0.901 \qquad \textbf{Ans}$$

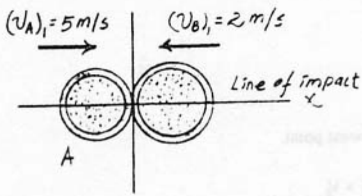
15-57. Disk A has a mass of 2 kg and is sliding forward on the *smooth* surface with a velocity $(v_A)_1 = 5$ m/s when it strikes the 4-kg disk B, which is sliding towards A at $(v_B)_1 = 2$ m/s, with direct central impact. If the coefficient of restitution between the disks is $e = 0.4$, compute the velocities of A and B just after collision.



Conservation of Momentum :

$$m_A(v_A)_1 + m_B(v_B)_1 = m_A(v_A)_2 + m_B(v_B)_2$$

$$(\overset{+}{\rightarrow}) \qquad 2(5) + 4(-2) = 2(v_A)_2 + 4(v_B)_2 \qquad [1]$$



Coefficient of Restitution :

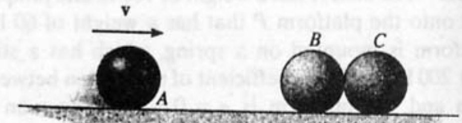
$$e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$$

$$(\overset{+}{\rightarrow}) \qquad 0.4 = \frac{(v_B)_2 - (v_A)_2}{5 - (-2)} \qquad [2]$$

Solving Eqs. [1] and [2] yields

$(v_A)_2 = -1.53$ m/s $= 1.53$ m/s $\leftarrow$ $\qquad (v_B)_2 = 1.27$ m/s $\rightarrow$ $\qquad$ **Ans**

15-63. The three balls each have the same mass m. If A has a speed v just before a direct collision with B, determine the speed of C after collision. The coefficient of restitution between each ball is e. Neglect the size of each ball.



Conservation of Momentum : When ball A strikes ball B, we have

$$m_A(v_A)_1 + m_B(v_B)_1 = m_A(v_A)_2 + m_B(v_B)_2$$

$$(\overset{+}{\rightarrow}) \qquad mv + 0 = m(v_A)_2 + m(v_B)_2 \qquad [1]$$

Coefficient of Restitution :

$$e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$$

$$(\overset{+}{\rightarrow}) \qquad e = \frac{(v_B)_2 - (v_A)_2}{v - 0} \qquad [2]$$

Solving Eqs.[1] and [2] yields

$$(v_A)_2 = \frac{v(1-e)}{2} \qquad (v_B)_2 = \frac{v(1+e)}{2}$$

Conservation of Momentum : When ball B strikes ball C, we have

$$m_B(v_B)_2 + m_C(v_C)_1 = m_B(v_B)_3 + m_C(v_C)_2$$

$$(\overset{+}{\rightarrow}) \qquad m\left[\frac{v(1+e)}{2}\right] + 0 = m(v_B)_3 + m(v_C)_2 \qquad [3]$$

Coefficient of Restitution :

$$e = \frac{(v_C)_2 - (v_B)_3}{(v_B)_2 - (v_C)_1}$$

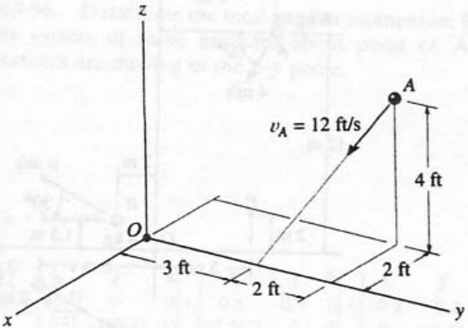
$$(\overset{+}{\rightarrow}) \qquad e = \frac{(v_C)_2 - (v_B)_3}{\frac{v(1+e)}{2} - 0} \qquad [4]$$

Solving Eqs.[3] and [4] yields

$$(v_C)_2 = \frac{v(1+e)^2}{4} \qquad \textbf{Ans}$$

$$(v_B)_3 = \frac{v(1-e^2)}{4}$$

15-90. Determine the angular momentum of the 2-lb particle A about point O. Use a Cartesian vector solution.



$$m\mathbf{v}_A = \frac{2}{32.2}(12)\left(\frac{2}{\sqrt{24}}\mathbf{i} - \frac{2}{\sqrt{24}}\mathbf{j} - \frac{4}{\sqrt{24}}\mathbf{k}\right)$$

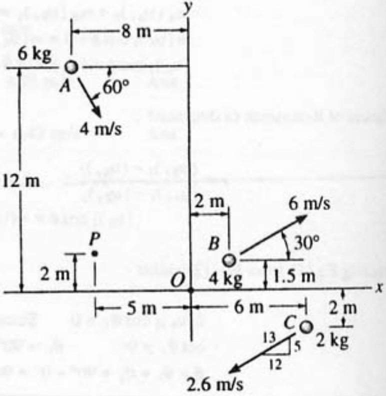
$$= \{0.3043\mathbf{i} - 0.3043\mathbf{j} - 0.6086\mathbf{k}\}\ \text{slug}\cdot\text{ft/s}$$

$$(\mathbf{H}_A)_0 = \mathbf{r}_A \times m\mathbf{v}_A$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -2 & 5 & 4 \\ 0.3043 & -0.3043 & -0.6086 \end{vmatrix}$$

$$= \{-1.83\mathbf{i} - 0.913\mathbf{k}\}\ \text{slug}\cdot\text{ft}^2/\text{s} \qquad \textbf{Ans}$$

***15-92.** Determine the angular momentum $\mathbf{H}_O$ of each of the particles about point O.

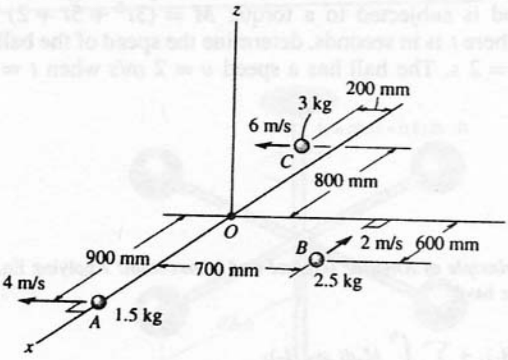


$(H_A)_O = 8(6)(4\sin 60°) - 12(6)(4\cos 60°) = 22.3 \text{ kg}\cdot\text{m}^2/\text{s}$ **Ans**

$(H_B)_O = -1.5(4)(6\cos 30°) + 2(4)(6\sin 30°) = -7.18 \text{ kg}\cdot\text{m}^2/\text{s}$ **Ans**

$(H_C)_O = -2(2)\left(\frac{12}{13}\right)(2.6) - 6(2)\left(\frac{5}{13}\right)(2.6) = -21.6 \text{ kg}\cdot\text{m}^2/\text{s}$ **Ans**

***15-96.** Determine the total angular momentum $\mathbf{H}_O$ for the system of three particles about point O. All the particles are moving in the x–y plane.

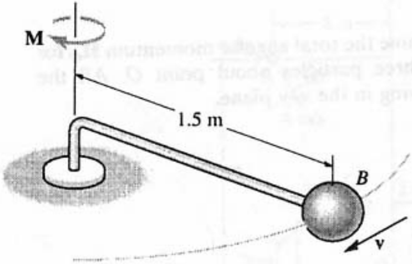


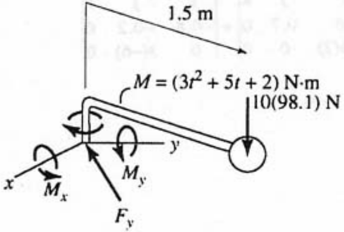
$$\mathbf{H}_O = \Sigma \mathbf{r} \times m\mathbf{v}$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.9 & 0 & 0 \\ 0 & -1.5(4) & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.6 & 0.7 & 0 \\ -2.5(2) & 0 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.8 & -0.2 & 0 \\ 0 & 3(-6) & 0 \end{vmatrix}$$

$$= \{12.5\mathbf{k}\}\ \text{kg} \cdot \text{m}^2/\text{s}$$ **Ans**

15-99. The ball B has a mass of 10 kg and is attached to the end of a rod whose mass may be neglected. If the rod is subjected to a torque $M = (3t^2 + 5t + 2)$ N · m, where t is in seconds, determine the speed of the ball when $t = 2$ s. The ball has a speed $v = 2$ m/s when $t = 0$.





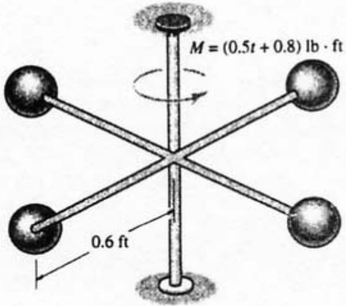
Principle of Angular Impluse and Momentum: Applying Eq. 15-22, we have

$$(H_z)_1 + \sum \int_{t_1}^{t_2} M_z dt = (H_z)_2$$

$$1.5(10)(2) + \int_0^{2s} (3t^2 + 5t + 2)dt = 1.5(10)v$$

$v = 3.47$ m/s **Ans**

15-102. The four 5-lb spheres are rigidly attached to the crossbar frame having a negligible weight. If a couple moment $M = (0.5t + 0.8)$ lb·ft, where t is in seconds, is applied as shown, determine the speed of each of the spheres in 4 seconds starting from rest. Neglect the size of the spheres.



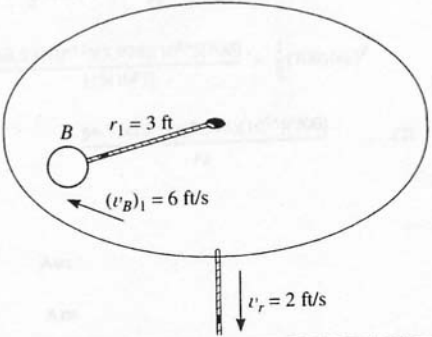
$$(H_z)_1 + \Sigma \int M_z \, dt = (H_z)_2$$

$$0 + \int_0^4 (0.5t + 0.8) \, dt = 4\left[\left(\frac{5}{32.2}\right)(0.6v_2)\right]$$

$$7.2 = 0.37267 \, v_2$$

$v_2 = 19.3$ ft/s **Ans**

15-106. A 4-lb ball B is traveling around in a circle of radius $r_1 = 3$ ft with a speed $(v_B)_1 = 6$ ft/s. If the attached cord is pulled down through the hole with a constant speed $v_r = 2$ ft/s, determine how much time is required for the ball to reach a speed of 12 ft/s. How far r_2 is the ball from the hole when this occurs? Neglect friction and the size of the ball.



$$v = \sqrt{(v_\theta)^2 + (2)^2}$$

$$12 = \sqrt{(v_\theta)^2 + (2)^2}$$

$$v_\theta = 11.832 \text{ ft/s}$$

$$H_1 = H_2$$

$$\frac{4}{32.2}(6)(3) = \frac{4}{32.2}(11.832)(r_2)$$

$$r_2 = 1.5213 = 1.52 \text{ ft} \qquad \textbf{Ans}$$

$$\Delta r = v_r t$$

$$(3 - 1.5213) = 2t$$

$$t = 0.739 \text{ s} \qquad \textbf{Ans}$$