ME 3210 Mechatronics Modeling
Free-Body Diagrams

Getting the Differential Equations for Systems

The main purpose of modelling systems is to obtain the differential equation describing the system. The differential equation will tell how the actual system will behave. Two techniques will be used to obtain the differential equations, free body diagrams and linear graphs. There are many other techniques available to the engineer that will not be discussed such as bond graphs. All of the techniques are methods of showing connections of different elements to each other and to handle the bookkeeping of keeping track of directions of forces and velocities and other variables.

First free body diagrams will be reviewed.

Free Body Diagrams

The idea of a free body diagram is to show the forces of acting on a mass. These forces can be forces due to passive elements such as springs and friction as well as active forces due to sources. To illustrate the use of free body diagrams, the following example shown in figure 1 will be used.

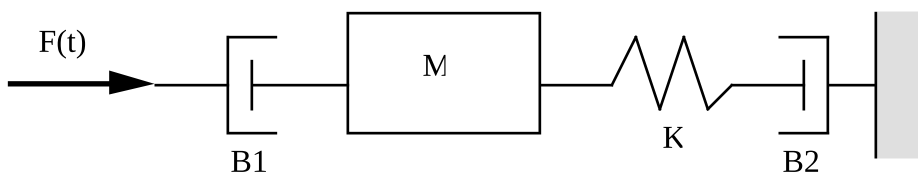

Figure 1 Example mechanical system

The first step is to assign positive directions to all of the displacements (and velocities and accelerations). There are four displacements needed for this example. These are shown in figure 2.

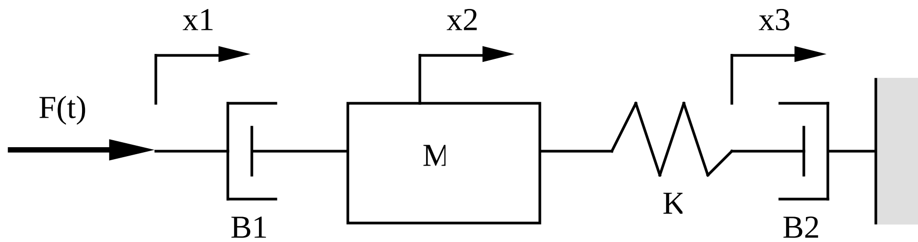

Figure 2 System with displacements assigned

Three displacements are indicated by x1, x2, and x3. The fourth is ground.

The next step is to show each displacement as a free body diagram. The mass is drawn in figure 3 showing the forces due to B1 and K as they attach to the mass.

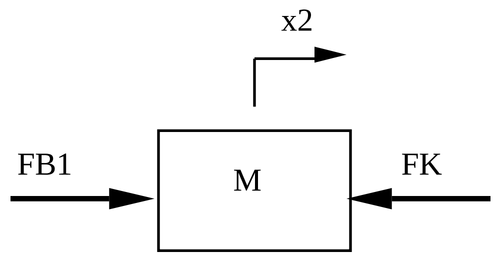

Figure 3 Free body diagram of the mass of the system of figure 1

Note that the direction for positive displace (and velocity and acceleration) is indicated. Note, also, that the directions of the forces of the damper, B1, and the spring,K, are in opposite directions. The directions of forces of the passive elements are arbitrary. Summing the forces acting on the mass and equating that to the acceleration of the mass results in equation (1).

$$FB1 - Fk = M \frac{dx2^2}{d^2t} \tag{1}$$

FB1 is positive because it will cause a positive acceleration of the mass. FK is negative because that force will cause a deceleration of the mass. These signs result from the (arbitrary) assignment of the force directions. These directions must be kept consistent for the rest of the derivation.

Next, the forces for the damper and the spring are derived. Free body diagrams for x1 and x3 are used to help here. These are shown in figure 4.

Figure 4 Free body diagrams for x1 and x3

Note that these free body diagrams do not involve any masses. The equations are derived in exactly the same manner, however. The directions for the forces sue to damper, B1, and the spring, K, are not arbitrary for these diagrams. They were defined in the previous part. Also, the direction of the force of the source, F(t), is not arbitrary. Sources must always correspond to the actual system. Equations (2) and (3) result from summing the forces from these diagrams.

$$F(t) - FB1 = 0 \qquad (2)$$

$$FK - FB2 = 0 \qquad (3)$$

Obviously FB1 is equal to F(t). This can be put into equation (1). The force FK is due to the relative displacement of each end of the spring. The easiest method to determine this is to imagine holding one end still, say, the x2 end, and displacing the other end, x3, in the positive direction. When this is done for the example system, the resultant force will be to the right. This is in the opposite direction as indicated in figure 3.

The imagined holding and stretching can be reversed with the same result. The result of the imagined holding and stretching results in the definition of the spring force as given in equation (4).

$$FK = K(x2 - x3) \qquad (4)$$

Putting this result along with the result for FB1 into equation (1) results in equation (5).

$$F(t) - K(x2 - x3) = M\frac{dx2^2}{d^2t} \qquad (5)$$

There is now one equation and two unknowns, x2 and x3. The other equation is derived from the free body diagram for x3. Using the same technique that was used for the spring for the dashpot, B2, results in equation (6).

$$K(x2 - x3) = B2\frac{dx3}{dt} \qquad (6)$$

Finally, equations (5) and (6) are put into the Laplace domain, resulting in the final form that will be found.

$$(Ms^2 + K)X2(s) = F(s) \qquad (7)$$

$$(Bs + K)X3(s) = KX2(s) \qquad (8)$$