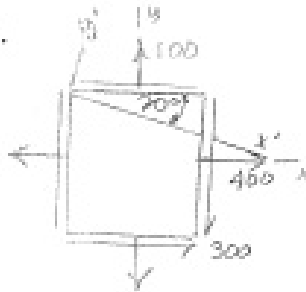


To receive full credit you must show all work and insert your "final" answers in the indicated spots.

1.



The stresses on an element are as shown in the figure. Determine the principle stresses, the absolute maximum shear stress and the normal and shear stresses on the plane that makes an angle of 20° with the y-face, as shown? Complete the sketch of the element below (i.e. put directions on arrows).



$$\sigma_1 = 250 + 335.4 = 585.4$$

$$\sigma_2 = 250 - 335.4 = -85.4$$

$$\tau_{\max \text{ in plane}} = \tau_{\max \text{ abs}} = R = 335.4$$

$$\tan 2\theta_p = \frac{300}{150} = 2$$

$$2\theta_p = 63.43^\circ$$

$$\theta_p = 31.7^\circ$$

$$\text{So } \sigma_{x'} = C - R =$$

$$= 250 - 335.4 \cos 23.43^\circ = -57.7 \text{ MPa}$$

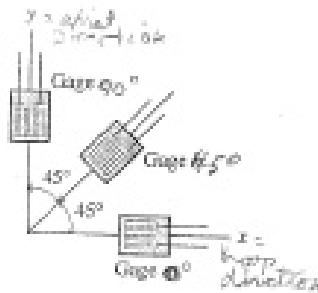
$$\tau_{x'y'} = 133 \text{ Such as to cause a CCW rotation}$$

$$C = \frac{400 + 100}{2} = 250$$

$$R = \sqrt{\left(\frac{400 - 100}{2}\right)^2 + 300^2} = 335.4$$

$$\sigma_1 = 585 \text{ MPa}, \sigma_2 = -85 \text{ MPa}, \tau_{\max} = 335.4, \sigma_r = -57.7 \text{ MPa} \text{ \& } \tau_{xy} = 133 \text{ CCW}$$

2.



A strain rosette is attached to a pressure vessel that not only is exposed to an internal pressure but also to an axial force and a torsion. If the strains indicated in the rosette shown are $\epsilon_0 = 800\mu$, $\epsilon_{45} = 100\mu$, and $\epsilon_{90} = -300\mu$, determine the strains ϵ_x , ϵ_y , & the shear strain γ_{xy} . If the vessel is of metal ($E = 100 \text{ GPa}$ & $\nu = 0.3$) determine the shear stress τ_{xy} .

$$\epsilon_x = \epsilon_0 = 800\mu$$

$$\epsilon_y = \epsilon_{90} = -300\mu$$

$$\epsilon_{45} = \left[\frac{800 - 300}{2} + \frac{800 + 300}{2} \cos 90^\circ + \frac{\gamma_{xy} \sin 90^\circ}{2} \right] \mu = 100\mu$$

$$\epsilon_{45} = 100\mu = (250 + 0 + \frac{\gamma_{xy}}{2}) \mu$$

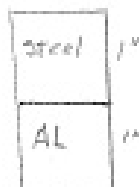
$$\text{So } \gamma_{xy} = -750\mu \Rightarrow \gamma_{xy} = -300\mu$$

$$G = \frac{E}{2(1+\nu)} = \frac{100 \times 10^9}{2(1+0.3)} = 3.84 \times 10^{10} \text{ Pa}$$

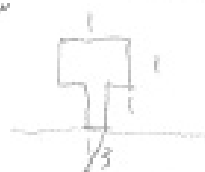
$$\tau_{xy} = \gamma_{xy} G = (-500 \times 10^{-6})(3.84 \times 10^{10}) = -19.2 \text{ MPa}$$

$$\epsilon_0 = 800\mu, \epsilon_y = -300, \gamma_{xy} = -300\mu \text{ \& } \tau_{xy} = -19.2 \text{ MPa}$$

3.



A 20 inch long beam is constructed by bonding together an aluminum bar and a steel bar both with square cross-sections 1 in x 1 in. If a moment $M_{xx} = 10,000$ in-lb is applied, determine the maximum normal stress in each material. Use $E_{AL} = 10 \times 10^6$ psi, $E_{steel} = 30 \times 10^6$ psi and $\nu = 0.3$ for both.



$$\bar{y} = \frac{1/2(1)(1) + (1)(1)(1)}{1/2 + 1} = 1.25 \text{ in}$$

$$\bar{I}_{AL} = \frac{1}{3}(1)(1)^3 + (1/2)(1)(1.25 - 0.5)^2 + \frac{1}{3}(1)(1)^3 + (1)(1)(1.25 - 1)^2$$

$$\bar{I}_{AL} = 0.3611 \text{ in}^4$$

$$\sigma_{max, steel} = \frac{M_{xx} \bar{y}}{\bar{I}_{AL}} = \frac{10,000(1.25)}{0.3611} = 34,337 \text{ psi}$$

$$\sigma_{max, AL} = \frac{M_{xx} \bar{y}}{\bar{I}_{AL}} = \frac{10,000(0.5)}{0.3611} = 13,847 \text{ psi}$$

$$\sigma_{max, aluminum} = 13.8 \text{ ksi (Tension)}$$

$$\sigma_{max, steel} = 34.3 \text{ ksi (Compression)}$$

4.



A "lam" beam is made up of three 1 in. x 3 in. (full size) pieces of wood bonded together to form the beam cross-section shown. If the beam is used in an application where $M_{max} = 22,000$ in-lbs and $V_{max} = 4000$ lbs, Determine (a) the maximum tensile stress, (b) the maximum compressive stress and (c) the shear stress in the adhesive used to bond the pieces of wood.

$$\bar{I} = \frac{1}{3}(3)(1)^3 + 3(1)(1)^2 = 6.75 \text{ in}^4$$

$$\tau = \frac{VQ}{I\bar{t}} = \frac{(4000)[(3 \times 1)(1)]}{(6.75)(3)} = 592.6 \text{ psi}$$

$$\sigma_c = 4.9 \text{ ksi} \quad \sigma_t = 4.9 \text{ ksi} \quad \tau_{adhesive} = 593 \text{ psi}$$

5- A part made of an aluminum (Tensile "Failure" Stress = 40,000 psi and $E = 10 \times 10^6$ psi) has principal stresses, at the critical point, of $\sigma_1 = 10,000$ psi, $\sigma_2 = -20,000$ psi and $\sigma_3 = 0$ psi. (a) Determine the part's factor of safety according to the (a) Max Normal Stress Failure Criterion, (b) Max Shear Stress (Tresca) Failure Criterion, and (c) Max Energy of Distortion (Von Mises) Failure Criterion. (d) similar to the above part with the same stresses, except here use Mohr Failure Criterion and assume that the part is made of a "brittle" material with a tensile failure stress of 20,000 psi and a compressive failure stress of 45,000 psi.

$$\text{(a) Max Normal Stress: } FS = \frac{40,000}{10,000} = 4 \quad \text{might be wrong}$$

$$\text{(b) } \frac{40,000 - (-20,000)}{2} = (FS)_T = 1.33$$

$$\text{(c) } \sigma_{VM} = \sqrt{\frac{1}{2}[(10+20)^2 + 40^2 + 20^2]} = 26,400 \text{ psi} \quad FS_{VM} = 1.51$$

$$\text{(d) } \frac{10}{20} - \frac{(-20)}{45} = 0.944 \quad FS = 1.059$$

$$\text{(a) } FS_{MNSC} = 4 \quad \text{(b) } FS_{MSSC} = 1.33 \quad \text{(c) } FS_{MISC} = 1.51 \quad \text{& (d) } FS_{MPC} = 1.06$$