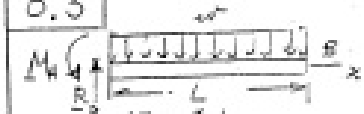
8.3

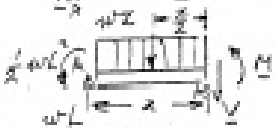


Free Body: AB:

$R_A = wL \uparrow$

$M_A = (wL)\left(\frac{L}{2}\right)$

$M_A = \frac{1}{2} wL^2$



(a) Free Body: AK

$\circlearrowleft \Sigma M_K = 0:$

$M + wx\left(\frac{x}{2}\right) - wLx + \frac{1}{2} wL^2 = 0$

$EI\frac{d^2y}{dx^2} = M = -\frac{1}{2} wx^2 + wLx - \frac{1}{2} wL^2$

$EI\frac{dy}{dx} = -\frac{1}{6} wx^3 + \frac{1}{2} wLx^2 - \frac{1}{2} wL^2x + C_1$

$\left[x=0, \frac{dy}{dx}=0\right]: \quad C_1 = 0$

$EIy = -\frac{1}{24} wx^4 + \frac{1}{6} wLx^3 - \frac{1}{4} wL^2x^2 + C_2$

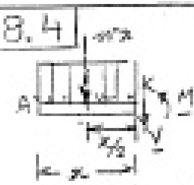
$[x=0, y=0]: \quad C_2 = 0$

$y = -\frac{w}{24EI}\left(x^4 - 4Lx^3 + 6L^2x^2\right)$ ◀

(b) $x = L: \quad y_B = -\frac{wL^4}{24EI}(1-4+6) \qquad y_B = \frac{wL^4}{8EI}\downarrow$ ◀

(c) $x = L: \left(\frac{dy}{dx}\right)_B = \frac{wL^3}{EI}\left(-\frac{1}{6}+\frac{1}{2}-\frac{1}{2}\right) \qquad \theta_B = \frac{wL^3}{6EI}$ ◀

8.4



(a) Free Body: AK:

$\circlearrowleft \Sigma M_K = 0: \quad M + wx\left(\frac{x}{2}\right) = 0$

$EI\frac{d^2y}{dx^2} = M = -\frac{1}{2} wx^2$

$EI\frac{dy}{dx} = -\frac{1}{6} wx^3 + C_1$

$\left[x=L, \frac{dy}{dx}=0\right]: \quad 0 = -\frac{1}{6} wL^3 + C_1 \qquad C_1 = \frac{1}{6} wL^3$

$EIy = -\frac{1}{24} wx^4 + \frac{1}{6} wL^3x + C_2$

$[x=L; y=0]: \quad C_2 = \left(\frac{1}{24}-\frac{1}{6}\right)wL^4 = -\frac{1}{8} wL^4$

$y = -\frac{w}{24EI}\left(x^4 - 4L^3x + 3L^4\right)$

(b) $x = 0: \quad y_A = -\frac{w}{24EI}(3L^4) \qquad y_A = \frac{wL^4}{8EI}\downarrow$

(c) $x = 0: \quad EI\left(\frac{dy}{dx}\right)_A = C_1 = \frac{1}{6} wL^3 \qquad \theta_A = \frac{wL^3}{6EI}$

8.11

Free Body: AB

$\circlearrowleft \Sigma M_B = 0: \quad R_A L - M_0 = 0$

$R_A = \frac{M_0}{L}\downarrow \qquad R_B = \frac{M_0}{L}\uparrow$

(a) Free Body: AK

$\circlearrowleft \Sigma M_K = 0: \quad M - M_0 + \frac{M_0}{L}x = 0$

$EI\frac{d^2y}{dx^2} = M = -\frac{M_0}{L}x + M_0$

$EI\frac{dy}{dx} = -\frac{1}{2}\frac{M_0}{L}x^2 + M_0x + C_1$

$EIy = -\frac{1}{6}\frac{M_0}{L}x^3 + \frac{1}{2}M_0x^2 + C_1x + C_2$

$[x=0, y=0] \qquad C_2 = 0$

$[x=L, y=0] \qquad 0 = -\frac{1}{6}\frac{M_0}{L}L^3 + \frac{1}{2}M_0L^2 + C_1L$

$C_1 = M_0L\left(\frac{1}{6}-\frac{1}{2}\right) = -\frac{1}{3}M_0L$

$y = \frac{M_0}{6EIL}\left(-x^3 + 3Lx^2 - 2L^2x\right)$ ◀

Differentiate: $\frac{dy}{dx} = \frac{M_0}{6EIL}\left(-3x^2 + 6Lx - 2L^2\right)$

(b) $x = 0: \left(\frac{dy}{dx}\right)_A = \frac{M_0}{6EIL}(-2L^2) \qquad \theta_A = \frac{M_0L}{3EI}$ ◀

(c) $x = L: \left(\frac{dy}{dx}\right)_B = \frac{M_0}{6EIL}(L^2) \qquad \theta_B = \frac{M_0L}{6EI}$ ◀

8.60

$V(x) = \frac{Pb}{L} - P\langle x-a\rangle^0$

$EI\frac{d^2y}{dx^2} = M = \frac{Pb}{L}x - P\langle x-a\rangle$

$EI\frac{dy}{dx} = \frac{1}{2}\frac{Pb}{L}x^2 - \frac{1}{2}P\langle x-a\rangle^2 + C_1 \qquad (1)$

$EI\,y = \frac{1}{6}\frac{Pb}{L}x^3 - \frac{1}{6}P\langle x-a\rangle^3 + C_1x + C_2 \qquad (2)$

$[x=0, y=0]: \quad C_2 = 0$

$[x=L, y=0]: \quad \frac{1}{6}\frac{Pb}{L}L^3 - \frac{1}{6}P(L-a)^3 + C_1L = 0$

$C_1 = \frac{P}{6L}\left(L^3 - 3L^2a + 3La^2 - a^3 - L^2b\right)$ But $b = L-a$:

$= \frac{P}{6L}\left(\cancel{L^3} - 3L^2a + 3La^2 - a^3 - \cancel{L^3} + L^2a\right)$

$= -\frac{P}{6L}\left(2L^2 - 3La + a^2\right)a = -\frac{P}{6L}(2L-a)(L-a)a$

$= -\frac{P}{6L}(L+b)b(L-b) = -\frac{Pb}{6L}(L^2-b^2)$

(CONTINUED)

8.60 CONTINUED

(a) Equation of elastic curve. Substituting for C_1 and C_2 in (2):

$y = \frac{Pb}{6EIL}\left[x^3 - (L^2-b^2)x - \frac{L}{b}\langle x-a\rangle^3\right] \qquad (3)$ ◀

(b) Slope at A. Substituting for C_1 in Eq. (1) and making $x=0$

$\theta_A = \left(\frac{dy}{dx}\right)_A = \frac{C_1}{EI} \qquad \theta_A = \frac{Pb}{6EIL}(L^2-b^2)$ ◀

(c) Deflection at C. Making $x=a$ in Eq. (3):

$y_C = \frac{Pb}{6EIL}\left[a^3 - (L^2-b^2)a\right] = \frac{Pba}{6EIL}\left[a^2 - (L-b)(L+b)\right]$

$= \frac{Pab}{6EIL}\left[a^2 - a(L+b)\right] = \frac{Pa^2b}{6EIL}\left[a - L - b\right]$

$= \frac{Pa^2b}{6EIL}[-b-b] \qquad y_C = \frac{Pa^2b^2}{3EIL}\downarrow$ ◀

8.109 Refer to App. D, Loadings ⓑ and ⓑ, using notation shown:

Loading I: (ⓑ in Table)

$$y_C = \frac{Pb}{6EIL}\left[x^3-(L^2-b^2)x\right] = \frac{P\left(\frac{L}{4}\right)}{6EIL}\left[\left(\frac{L}{2}\right)^3-\left(L^2-\frac{L^2}{16}\right)\frac{L}{2}\right] = -\frac{11PL^3}{768EI}$$

$$\theta_A = -\frac{Pb(L^2-b^2)}{6EIL} = -\frac{P\left(\frac{1}{4}L\right)\left(L^2-\frac{9}{16}L^2\right)}{6EIL} = -\frac{7PL^2}{128EI}$$

Loading II: (ⓑ in Table)

$$y_C = y_{max} = -\frac{PL^3}{48EI} \qquad \theta_A = -\frac{PL^2}{16EI}$$

(a) Deflection at C:

$$y_C = (y_C)_I + (y_C)_{II} = -\frac{11PL^3}{768EI} - \frac{PL^3}{48EI} = -\frac{27PL^3}{768EI} \qquad y_C = \frac{9PL^3}{256EI}\downarrow$$

(b) Slope at A:

$$\theta_A = (\theta_A)_I + (\theta_A)_{II} = -\frac{7PL^2}{128EI} - \frac{PL^2}{16EI} = -\frac{15PL^2}{128EI} \qquad \theta_A = \frac{15PL^2}{128EI}$$

8.112 Refer to App. D, Loading ⓐ.

Slope at C: $\theta_C = (\theta_C)_I + (\theta_C)_{II} = (\theta_C)_I + (\theta_B)_{II}$

$$\theta_C = -\frac{PL^2}{2EI} - \frac{P(L/2)^2}{2EI} = -\frac{PL^2}{8EI}(4+1) \qquad \theta_C = \frac{5PL^2}{8EI}$$

Deflection at C: $y_C = (y_C)_I + (y_C)_{II} = (y_C)_I + (y_B)_{II} + \frac{L}{2}(\theta_B)_{II}$

$$y_C = -\frac{PL^3}{3EI} - \frac{P(L/2)^3}{3EI} - \frac{P(L/2)^2}{2EI}\left(\frac{L}{2}\right) \qquad y_C = \frac{7PL^3}{16EI}\downarrow$$