13A 3H

2.33

$A_s = 4\left[\frac{\pi}{4}\left(\frac{3}{4}\text{in.}\right)^2\right] = 1.7671\ \text{in}^2$

$A_c = (8\ \text{in})^2 - A_s = 62.233\ \text{in}^2$

$\delta_s = \frac{P_s L}{A_s E_s} = \frac{P_s(48\ \text{in.})}{(1.7671\ \text{in}^2)(29\times10^6\ \text{psi})} = 0.93666\times10^{-6} P_s$

$\delta_c = \frac{P_c L}{A_c E_c} = \frac{P_c(48\ \text{in.})}{(62.233\ \text{in}^2)(3.6\times10^6\ \text{psi})} = 0.21425\times10^{-6} P_c$

But $\delta_s = \delta_c$: $\quad 0.93666\times10^{-6} P_s = 0.21425\times10^{-6} P_c$

$P_s = 0.22874\ P_c \qquad (1)$

Also: $P_s + P_c = P = 150\ \text{kips} \qquad (2)$

Substituting for P_s from (1) into (2):

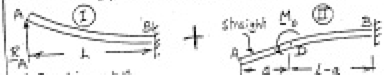
$1.22874\ P_c = 150\ \text{kips} \qquad P_c = 122.08\ \text{kips}$

From (1): $P_s = 0.22874(122.08) = 27.924\ \text{kips}$

$\sigma_s = -\frac{P_s}{A_s} = -\frac{27.924\ \text{kips}}{1.7671\ \text{in}^2} \qquad \sigma_s = -15.80\ \text{ksi}$ ◄

$\sigma_c = -\frac{P_c}{A_c} = -\frac{122.08\ \text{kips}}{62.233\ \text{in}^2} \qquad \sigma_c = -1.962\ \text{ksi}$ ◄

8.126 Beam is indeterminate to the 1st degree. We consider R_A as redundant and replace the given loading by



(a) Reaction at A

Referring to Loadings ① and ② of App. D, we write that the deflection at A must be zero:

$y_A = (y_A)_I + (y_A)_{II} = (y_A)_I + (y_D)_{II} - a(\theta_D)_{II} = 0$

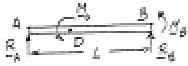
$\frac{R_A L^3}{3EI} - \frac{M_0(L-a)^2}{2EI} - a\frac{M_0(L-a)}{EI} = 0$

Multiply by $3EI$: $\frac{2}{3}R_A L^3 - M_0(L-a)(L-a+2a) = 0$

$\frac{2}{3}R_A L^3 = M_0(L^2-a^2) \qquad R_A = \frac{3M_0}{2L^3}(L^2-a^2)\uparrow$ ◄

(b) Reaction at B. Using the free-body diagram of the beam

$\Sigma F_y = 0$: $R_B = -R_A$ ◄



$+\circlearrowleft \Sigma M_B = 0$:

$M_B + M_0 - R_A L = 0$

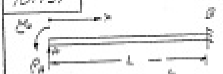
$M_B + M_0 - \frac{3M_0}{2L^2}(L^2-a^2)$

$M_B = \frac{3M_0}{2L^2}\left(L^2 - a^2 - \frac{2}{3}L^2\right) \qquad M_B = \frac{M_0}{2L^2}(L^2-3a^2)$ ◄

10.131

$M = R_A x - M_0$

$\partial M/\partial R_A = x$



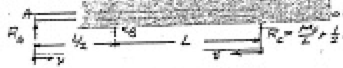
$y_A = \frac{1}{EI}\int M\frac{\partial M}{\partial R_A}dx = \frac{1}{EI}\int_0^L (R_A x - M_0)x\,dx = \frac{1}{EI}\left(R_A\frac{L^3}{3} - M_0\frac{L^2}{2}\right) = 0$

$R_A = \frac{3}{2}\frac{M_0}{L}\uparrow$ ◄

M-DIAGRAM

10.134

We consid[illegible]



$y_A = 0$

PORTION AB

$M = R_A x$

$\frac{\partial M}{\partial R_A} = x$ $\Big\}\ 0 < x < \frac{L}{2}$

PORTION BC

$M = \left(\frac{M_0}{L} + \frac{1}{2}R_A\right)v - M_0$

$\frac{\partial M}{\partial R_A} = \frac{1}{2}v$ $\Big\}\ v < L$

$y_A = \frac{1}{EI}\int M\frac{\partial M}{\partial R_A}dx = \frac{1}{EI}\int_0^{L/2}(R_A x)x\,dx + \frac{1}{EI}\int_0^L\left[\left(\frac{M_0}{L} + \frac{R_A}{2}\right)v - M_0\right]\frac{v}{2}dv$

$y_A = \frac{1}{EI}\int_0^{L/2} R_A x^2 dx + \frac{1}{EI}\int_0^L\left[\frac{M_0}{2L}v^2 + \frac{R_A}{4}v^2 - \frac{M_0}{2}v\right]dv = 0$

CANCEL EI

$R_A\frac{(L/2)^3}{3} + \frac{M_0}{2L}\frac{L^3}{3} + \frac{R_A}{4}\frac{L^3}{3} - \frac{M_0}{2}\frac{L^2}{2} = 0$

$R_A L^3\left(\frac{1}{24} + \frac{1}{12}\right) + M_0 L^2\left(\frac{1}{6} - \frac{1}{4}\right) = 0$

$\frac{1}{8}R_A - \frac{1}{12}\frac{M_0}{L} = 0 \qquad R_A = \frac{2}{3}\frac{M_0}{L}\uparrow$ ◄

$R_C = \frac{M_0}{L} + \frac{1}{2}\left(\frac{2}{3}\frac{M_0}{L}\right) = \frac{4}{3}\frac{M_0}{L} \qquad R_C = \frac{4}{3}\frac{M_0}{L}\uparrow$ ◄

FOR WHOLE BEAM

$+\uparrow\Sigma F_y = 0$: $R_A + R_B + R_C = 0$

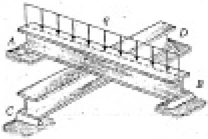
$\frac{2}{3}\frac{M_0}{L} + R_B + \frac{4}{3}\frac{M_0}{L} = 0 \qquad R_B = -2\frac{M_0}{L}$

$R_B = 2\frac{M_0}{L}\downarrow$ ◄

10B 5P

Solution to 14-15 continued

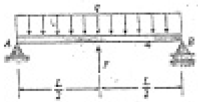
n 10.4-18 Two identical, simply supported beams AB and CD are placed they cross each other at their midpoints (see figure). Before the uniform applied, the beams just touch each other at the crossing point.
'ne the maximum bending moments $(M_{AB})_{max}$ and $(M_{CD})_{max}$ in d CD, respectively, due to the uniform load if the intensity of the $q = 6.4$ kN/m and the length of each beam is $L = 4$ m.



olution 10.4-18 Two beams that cross

= interaction force between the beams

PPER BEAM



$(\delta_B)_1$ = downward deflection due to q

$= \frac{5qL^4}{384EI}$

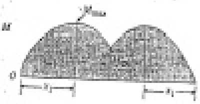
$(\delta_B)_2$ = downward deflection due to F

$= \frac{FL^3}{48EI}$

$\delta_{AB} = (\delta_B)_1 - (\delta_B)_2$

$= \frac{5qL^4}{384EI} - \frac{FL^3}{48EI}$

$x_1 = \frac{11qL}{32}$



$x_1 = \frac{11L}{32}$

$M_{max} = \frac{121qL^2}{2048}$

$M = \frac{3qL^2}{64}$ $\quad (M_{AB})_{max} = \frac{121qL^2}{2048}$ ←

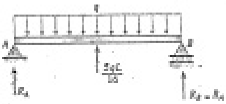
LOWER BEAM

$\delta_{CD} = \frac{FL^3}{48EI}$

COMPATIBILITY $\delta_{AB} = \delta_{CD}$

$\frac{5qL^4}{384EI} - \frac{FL^3}{48EI} = \frac{FL^3}{48EI}$ $\quad \therefore F = \frac{5qL}{16}$

UPPER BEAM



LOWER BEAM

$M_{max} = \frac{FL}{4} = \frac{5qL^2}{64}$

$(M_{CD})_{max} = \frac{5qL^2}{64}$ ←

NUMERICAL VALUES

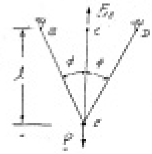
$q = 6.4$ kN/m $\quad (M_{AB})_{max} = 6.05$ kN·m

$L = 4$ m $\quad (M_{CD})_{max} = 8.0$ kN·m ←

10.136

$L_{BE} = L_{DE} = \frac{h}{\cos\phi}$

$y_C = 0$



We consider F_{CE} as redundant

Symmetry

$F_{BE} = F_{DE}$

A and E are same for each member

Joint E:

$+\uparrow \sum F_y = 0$

$(F_{BE} + F_{DE})\cos\phi + F_{CE} - P = 0$

$F_{BE} = F_{DE} = \frac{1}{2}\frac{P - F_{CE}}{\cos\phi}$ (1)

$y_C = \sum \frac{F_i L_i}{AE}\frac{\partial F_i}{\partial F_{CE}} = \frac{1}{AE}\sum F_i L_i \frac{\partial F_i}{\partial F_{CE}} = 0$

Member	F_i	L_i	$\frac{\partial F_i}{\partial F_{CE}}$	$F_i L_i \frac{\partial F_i}{\partial F_{CE}}$
BE	$\frac{P-F_{CE}}{2\cos\phi}$	$\frac{h}{\cos\phi}$	$\frac{-1}{2\cos\phi}$	$-\frac{(P-F_{CE})h}{4\cos^3\phi}$
CE	F_{CE}	h	1	$F_{CE}\,h$
DE	$\frac{P-F_{CE}}{2\cos\phi}$	$\frac{h}{\cos\phi}$	$\frac{-1}{2\cos\phi}$	$-\frac{(P-F_{CE})h}{4\cos^3\phi}$

$\sum F_i L_i \frac{\partial F_i}{\partial F_{CE}} = -\frac{2(P-F_{CE})h}{4\cos^3\phi} + F_{CE}\,h = 0$

$-P + F_{CE} + F_{CE}(2\cos^3\phi) = 0 \qquad F_{CE} = \frac{P}{1+2\cos^3\phi}$

Substitute into (1)

$F_{BE} = F_{DE} = \frac{1}{2\cos\phi}\left[P - \frac{P}{1+2\cos^3\phi}\right] = \frac{P}{2\cos\phi}\left[\frac{1+2\cos^3\phi - 1}{1+2\cos^3\phi}\right]$

$F_{BE} = F_{DE} = \frac{P}{2\cos\phi}\left[\frac{2\cos^3\phi}{1+2\cos^3\phi}\right] \qquad F_{BE} = F_{DE} = P\frac{\cos^2\phi}{1+2\cos^3\phi}$

For $\phi = 30°$, we obtain

$F_{BE} = F_{DE} = P\frac{\cos^2 30°}{1+2\cos^3 30°} = P\frac{0.75}{2.299}$ $\qquad F_{BE} = F_{DE} = 0.326\,P$ ◀

$F_{CE} = P\frac{1}{1+2\cos^3 30°} = P\frac{1}{2.299}$ $\qquad F_{CE} = 0.435\,P$ ◀