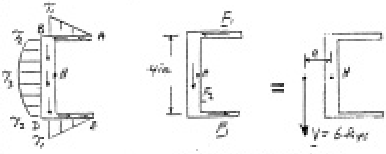
7A Solutions

5.102

$I_z = 2\left[\frac{1}{12}(1.5)(0.15)^3 + (1.5\times 0.15)(2)^2\right] + \frac{1}{12}(0.3)(4)^3$

$I_z = 3.401\ in^4$

$V = 6$ kips



$\Sigma M_H = \Sigma M_H$: $F_1(4\ in.) = Ve$ (1)

Corner B: $Q_B = (1.5\ in. \times 0.15\ in.)(2\ in.) = 0.45\ in^3$

$\tau_1 = \frac{VQ_B}{I t_{flg}} = \frac{(6\ kips)(0.45\ in^3)}{(3.401\ in^4)(0.15\ in.)} = 5.293$ ksi

$\tau_2 = \frac{VQ_B}{I t_{web}} = \frac{(6\ kips)(0.45\ in^3)}{(3.401\ in^4)(0.3\ in)} = 2.65$ ksi

Point H: $Q_H = Q_B + (2\ in. \times 0.3\ in.)(1\ in.) = 1.05\ in^3$

$\tau_m = \tau_3 = \frac{VQ}{I t_{web}} = \frac{(6\ kips)(1.05\ in^3)}{(3.401\ in^4)(0.3\ in)} = 6.17$ ksi ◀

$F_1 = \frac{1}{2}\tau_1(1.5\ in. \times 0.15\ in.) = \frac{1}{2}(5.293\ ksi)(1.5\ in. \times 0.15\ in.) = 595.5$ lb

Eq. (1): $(595.5\ lb)(4\ in.) = (6000\ lb)e$

$e = 0.397$ in. ◀

5.113

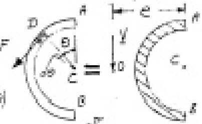
Moments of inertia

$J_0 = (2\pi a t)a^2$

$I_x = I_y$

$I_x = \frac{1}{2}J = \pi t a^3$

$I_x = \frac{1}{2}\pi t a^3$

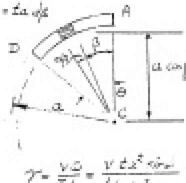


$dF = \tau dA = \tau(a t\, d\theta)$

$+\circlearrowleft\ \Sigma M_C = \Sigma M_C$: $\int_0^\pi a\, dF = Ve$

$\int_0^\pi a^2 t\, \tau\, d\theta = Ve$; $e = \frac{a^2 t}{V}\int_0^\pi \tau\, d\theta$ (1)

$dA = ta\, d\beta$



$dQ = (ta\, d\beta)(a\cos\beta)$

$Q = \int_0^\theta t a^2 \cos\beta\, d\beta = ta^2 \left|\sin\beta\right|_0^\theta$

$Q = ta^2 \sin\theta$

$\tau = \frac{VQ}{It} = \frac{V t a^2 \sin\theta}{\frac{1}{2}\pi t a^3\, t} = \frac{2V}{\pi a t}\sin\theta$ (2)

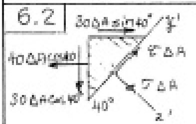
Substitute into Eq. (1):

$e = \frac{a^2 t}{V}\int_0^\pi \frac{2V}{\pi a t}\sin\theta\, d\theta = \frac{2a}{\pi}\left|-\cos\theta\right|_0^\pi$

$e = \frac{2a}{\pi}\left[-(-1)+(1)\right] = \frac{2a}{\pi}(2)$ $e = \frac{4a}{\pi}$ ◀

7B Sol.

6.2



$+\searrow \Sigma F_{x'} = 0$:

$\sigma\Delta A - 40\Delta A\cos^2 40° + 30\Delta A\cos 40°\sin 40° + 30\Delta A\sin 40°\cos 40° = 0$

$\sigma = 40\cos^2 40° - 60\cos 40°\sin 40°$

$\sigma = -6.07$ MPa ◀

$+\nearrow \Sigma F_{y'} = 0$: $\tau\Delta A + 30\Delta A\sin^2 40° - 30\Delta A\cos^2 40° - 40\Delta A\cos 40°\sin 40° = 0$

$\tau = 30(\cos^2 40° - \sin^2 40°) + 40\cos 40°\sin 40°$

$\tau = 24.9$ MPa $\angle 50°$ ◀

6.12

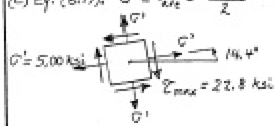
$\sigma_x = +16$ ksi, $\sigma_y = -6$ ksi, $\tau_{xy} = -20$ ksi

(a) Eq. (6.15): $\tan 2\theta_s = -\frac{\sigma_x - \sigma_y}{2\tau_{xy}} = -\frac{16+6}{2(-20)} = +0.550$

$2\theta_s = 28.81$ and $208.81°$ $\theta_s = +14.4°$ and $+104.4°$ ◀

(b) Eq. (6.16): $\tau_{max} = \sqrt{\left(\frac{16+6}{2}\right)^2 + (-20)^2} = 22.8$ ksi ◀

(c) Eq. (6.17): $\sigma' = \sigma_{ave} = \frac{16-6}{2}$ $\sigma' = +5.00$ ksi ◀



6.8

$\sigma_x = +16$ ksi, $\sigma_y = -6$ ksi, $\tau_{xy} = -20$ ksi

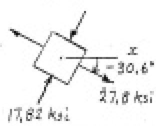
Eq. (6.12): $\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{2(-20)}{16+6} = -1.818$

$2\theta_p = -61.2°$ and $118.8°$ $\theta_p = -30.6°$ and $+59.4°$ ◀

Eq. (6.14): $\sigma_{max, min} = \frac{16-6}{2} \pm \sqrt{\left(\frac{16+6}{2}\right)^2 + (-20)^2} = 5 \pm 22.82$

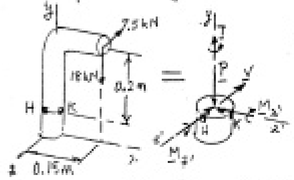
$\sigma_{max} = +27.8$ ksi ◀

$\sigma_{min} = -17.82$ ksi ◀



115 DTC (Cont'd)

6.23 Force-couple system in section through H and K equivalent to given force:



$P = 18$ kN

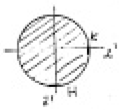
$T = 7.5 \times 0.15 = 1.125$ kN·m

$V = 7.5$ kN

$M_{x'} = 7.5 \times 0.2 = 1.5$ kN·m

$M_{z'} = 18 \times 0.15 = 2.7$ kN·m

Properties of section:



$c = 0.030\,m, \quad A = \pi c^2 = 2.8274 \times 10^{-3}\,m^2$

$I = \frac{\pi}{4} c^4 = 0.63617 \times 10^{-6}\,m^4$

$J = 2I = 1.2723 \times 10^{-6}\,m^4$

$Q = \frac{2}{3} c^3 = 18.00 \times 10^{-6}\,m^3$

$t = 2c = 0.060\,m$

(a) Stresses at H

$\sigma_x = 0, \quad \sigma_y = \frac{M_{x'} c}{I} - \frac{P}{A} = \frac{1500 \times 0.03}{0.63617 \times 10^{-6}} - \frac{18 \times 10^3}{2.8274 \times 10^{-3}}$

$= 70.736 \times 10^6 - 6.366 \times 10^6 = 64.370$ MPa

$\tau_{xy} = \frac{Tc}{J} = \frac{1125 \times 0.03}{1.2723 \times 10^{-6}} = 26.527$ MPa

Principal stresses:

Eq. (6.14): $\sigma_{max,min} = \frac{64.370}{2} \pm \sqrt{\left(\frac{64.370}{2}\right)^2 + (26.527)^2}$

$= 32.185 \pm 41.708$

$\sigma_{max} = 73.9$ MPa; $\sigma_{min} = -9.52$ MPa ◀

Maximum shearing stress:

Eq. (6.16): $\tau_{max} = \sqrt{\left(\frac{64.370}{2}\right)^2 + (26.527)^2}$

$\tau_{max} = 41.7$ MPa ◀

(b) Stresses at K

$\sigma_x = 0, \quad \sigma_y = -\frac{M_{z'} c}{I} - \frac{P}{A} = -\frac{2700 \times 0.03}{0.63617 \times 10^{-6}} - \frac{18 \times 10^3}{2.8274 \times 10^{-3}}$

$= -127.324 \times 10^6 - 6.366 \times 10^6 = -133.690$ MPa

$\tau_{xy} = \frac{Tc}{J} + \frac{VQ}{It} = 26.527 \times 10^6 + \frac{7500 \times 18 \times 10^{-6}}{(0.63617 \times 10^{-6})(0.060)}$

$= 26.527 \times 10^6 + 3.537 \times 10^6 = 30.064$ MPa

Principal stresses:

Eq. (6.14): $\sigma_{max,min} = \frac{-133.690}{2} \pm \sqrt{\left(\frac{133.690}{2}\right)^2 + (30.064)^2}$

$= -66.845 \pm 73.295$

$\sigma_{max} = 6.45$ MPa; $\sigma_{min} = -140.1$ MPa ◀

Maximum shearing stress:

Eq. (6.16): $\tau_{max} = \sqrt{\left(\frac{133.690}{2}\right)^2 + (30.064)^2}$

$\tau_{max} = 73.3$ MPa ◀