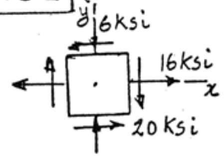
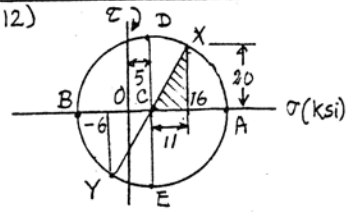
8A solutions

6.32 (Probs. 6.8 and 6.12)

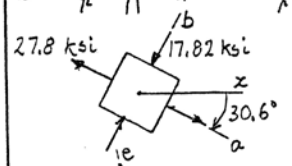




$\sigma_{ave} = \frac{1}{2}(16-6) = +5$

$\frac{\sigma_x - \sigma_y}{2} = \frac{1}{2}(16+6) = +11$ $\qquad R = \sqrt{(11)^2+(20)^2} = 22.82$

$\tan 2\theta_p = \frac{20}{11} = 1.818 \quad 2\theta_p = 61.2° \quad \theta_p = 30.6°$ ↺ and $59.4°$ ↻ ◀

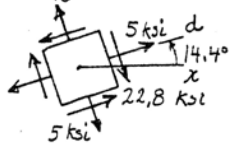


$\sigma_{max} = \sigma_{ave} + R = 5 + 22.82$

$\sigma_{max} = 27.8$ ksi ◀

$\sigma_{min} = 5 - 22.82 = -17.82$ ksi ◀

$\theta_s = \theta_p + 45° = 14.4°$ ↺ and $104.4°$ ↺ ◀



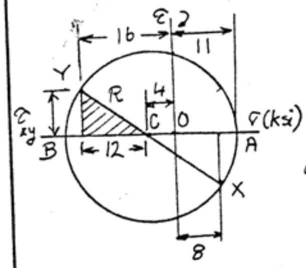
$\tau_{max} = R = 22.8$ ksi ◀

$\sigma' = \sigma_{ave} = +5.00$ ksi ◀

6.53

Given: $\sigma_x = 8$ ksi, $\sigma_y = -16$ ksi, $\tau_{max} = 11$ ksi

We compute $\tau_{ave} = \frac{1}{2}(8-16) = -4$



$\frac{1}{2}(\sigma_x - \sigma_y) = \frac{1}{2}(8+16) = 12$

$R = \sigma_{max} + |\sigma_{ave}|$

$= 11 + 14 = 15$

From shaded triangle:

$\tau_{xy} = \sqrt{R^2 - (12)^2} = \sqrt{(15)^2 - (12)^2}$

$\tau_{xy} = \pm\ 9.00$ ksi

$-9.00 \text{ ksi} \le \tau_{xy} \le 9.00 \text{ ksi}$ ◀

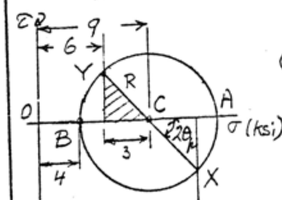
6.57

Given: $\sigma_x = 12$ ksi, $\sigma_y = 6$ ksi, $\sigma_{min} = 4$ ksi.

Also, from sense of τ_{xy}, X must be below axis

$\sigma_{ave} = \frac{1}{2}(12+6) = 9$ $\qquad \frac{1}{2}(\sigma_x - \sigma_y) = \frac{1}{2}(12-6) = +3$

$R = \sigma_{ave} - \sigma_{min} = 9 - 4 = 5$



(a) From shaded triangle:

$\cos 2\theta_p = \frac{3}{R} = \frac{3}{5}, \quad 2\theta_p = 53.13°$

$\theta_p = 26.6°$ ↺ and $63.4°$ ↺ ◀

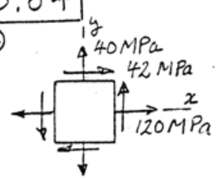
(b) $\sigma_{max} = \sigma_{ave} + R = 9 + 5 = 14$

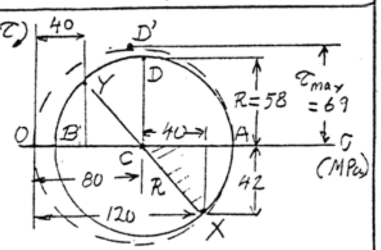
$\sigma_{max} = 14.00$ ksi ◀

(c) $\tau_{max} = R = 5.00$ ksi ◀

6.64

(a)





$\sigma_{ave} = \frac{1}{2}(120+40) = 80$

$\frac{1}{2}(\sigma_x - \sigma_y) = \frac{1}{2}(120-40) = 40 \qquad R = \sqrt{(40)^2+(42)^2} = 58$ MPa

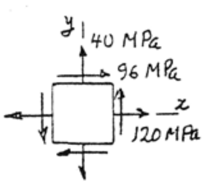
$\sigma_{max} = \sigma_{ave} + R = 80 + 58 = 138$ MPa

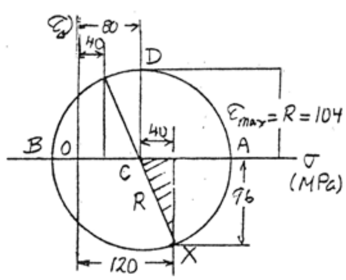
$\sigma_{min} = \sigma_{ave} - R = 80 - 58 = 22$ MPa

Since σ_{max} and σ_{min} have the same sign, τ_{max} is out of the plane of stress. Using Mohr's circle through O and A, we have

$\tau_{max} = \frac{1}{2}\sigma_{max} = \frac{1}{2}(138 \text{ MPa}) \qquad \tau_{max} = 69.0$ MPa ◀

(b)





$\sigma_{ave} = \frac{1}{2}(120+40) = 80$

$\frac{1}{2}(\sigma_x - \sigma_y) = \frac{1}{2}(120-40) = 40 \qquad R = \sqrt{(40)^2+(96)^2} = 104$ MPa

$\sigma_{max} = \sigma_{ave} + R = 80 + 104 = 184$ MPa

$\sigma_{min} = \sigma_{ave} - R = 80 - 104 = -24$ MPa

Since σ_{max} and σ_{min} have opposite signs, τ_{max} is equal to the in-plane maximum shearing stress:

$\tau_{max} = R = 104$ MPa $\qquad \tau_{max} = 104.0$ MPa ◀

8B Sol

10-2

$\epsilon_x = -130(10^{-6})$ $\epsilon_y = 280(10^{-6})$ $\gamma_{xy} = 75(10^{-6})$

$$\epsilon_{1,2} = \frac{\epsilon_x+\epsilon_y}{2} \pm \sqrt{\left(\frac{\epsilon_x-\epsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$= \left[\frac{-130+280}{2} \pm \sqrt{\left(\frac{-130-280}{2}\right)^2 + \left(\frac{75}{2}\right)^2}\right] 10^{-6}$$

(a)

$\epsilon_1 = \underline{\underline{283(10^{-6})}}$ $\epsilon_2 = \underline{\underline{-133(10^{-6})}}$

b)

$$\frac{\gamma_{\substack{max \\ in\text{-}plane}}}{2} = \sqrt{\left(\frac{\epsilon_x-\epsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$\gamma_{\substack{max \\ in\text{-}plane}} = 2\left[\sqrt{\left(\frac{-130-280}{2}\right)^2 + \left(\frac{75}{2}\right)^2}\right] 10^{-6}$$

$$= \underline{\underline{417(10^{-6})}}$$

$$\epsilon_{avg} = \frac{\epsilon_x+\epsilon_y}{2} = \left(\frac{-130+280}{2}\right)(10^{-6})$$

$$= \underline{\underline{75.0(10^{-6})}}$$

Orientation of ϵ_1 and ϵ_2

$$\tan 2\theta_p = \frac{\gamma_{xy}}{\epsilon_x-\epsilon_y} = \frac{75}{-130-280}$$

$$\theta_p = -5.18^\circ, \ 84.82^\circ$$

Use eq 10-5 to determine the direction of ϵ_1 and ϵ_2

$$\epsilon_x' = \frac{\epsilon_x+\epsilon_y}{2} + \frac{\epsilon_x-\epsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

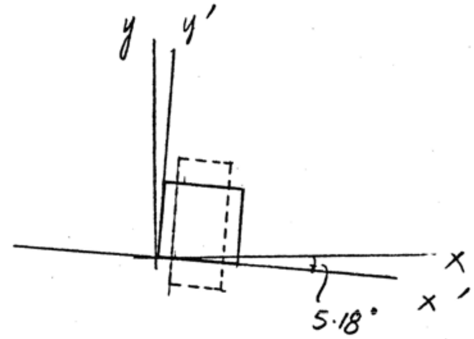
$$\theta = \theta_p = -5.18^\circ$$

cont above

$$\epsilon_x' = \left[\frac{-130+280}{2} + \frac{-130-280}{2}\cos(-10.37^\circ) + \frac{75}{2}\sin(-10.37^\circ)\right]10^{-6}$$

$$= -133(10^{-6})$$

$$\therefore \theta_{P_1} = \underline{\underline{84.8^\circ}} \quad \theta_{P_2} = \underline{\underline{-5.18^\circ}}$$



Orientation of γ_{max}

$$\tan 2\theta_s = \frac{-(\epsilon_x-\epsilon_y)}{\gamma_{xy}}$$

$$= \frac{-(-130-280)}{75}$$

$$\theta_s = \underline{\underline{39.8^\circ}}, \ \underline{\underline{130^\circ}}$$

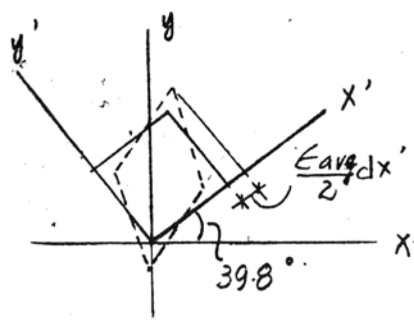
Use eq 10-11 to determine the sign of $\gamma_{\substack{max \\ in\text{-}plane}}$

$$\frac{\gamma_{x'y'}}{2} = -\frac{\epsilon_x-\epsilon_y}{2}\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta$$

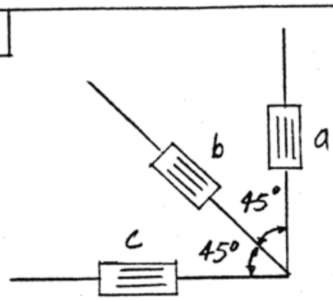
$$\theta = \theta_s = 39.8^\circ$$

$$\gamma_{x'y'} = -(-130-280)\sin 79.63^\circ + 75\cos 79.63$$

$$= 417(10^{-6})$$



10-20



$\epsilon_a = 650(10^{-6}) \quad \epsilon_b = -300(10^{-6})$

$\epsilon_c = 480(10^{-6})$

$\theta_a = 90° \quad \theta_b = 135° \quad \theta_c = 180°$

$\epsilon_a = \epsilon_x \cos^2\theta_a + \epsilon_y \sin^2\theta_a + \gamma_{xy}\sin\theta_a \cos\theta_a$

$650(10^{-6}) = \epsilon_x \cos^2 90° + \epsilon_y \sin^2 90° + \gamma_{xy} \sin 90° \cos 90°$

$\epsilon_y = 650(10^{-6})$

$\epsilon_c = \epsilon_x \cos^2\theta_c + \epsilon_y \sin^2\theta_c + \gamma_{xy}\sin\theta_c \cos\theta_c$

$480(10^{-6}) = \epsilon_x \cos^2 180° + \epsilon_y \sin^2 180° + \gamma_{xy} \sin 180° \cos 180°$

$\epsilon_x = 480(10^{-6})$

$\epsilon_b = \epsilon_x \cos^2\theta_b + \epsilon_y \sin^2\theta_b + \gamma_{xy}\sin\theta_b \cos\theta_b$

$-300(10^{-6}) = 480(10^{-6}) \cos^2 135° + 650(10^{-6}) \sin^2 135 + \gamma_{xy} \sin 135° \cos 135°$

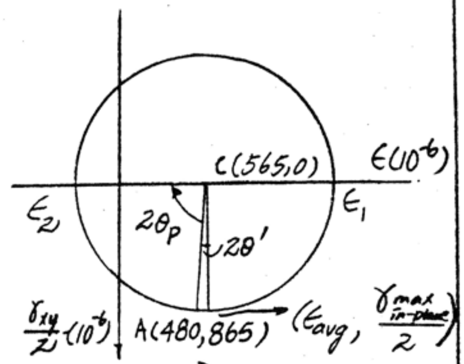
$\gamma = 1730(10^{-6})$

$\epsilon_x = 480(10^{-6}) \quad \epsilon_y = 650(10^{-6})$

$\gamma_{xy} = 1730(10^{-6}) \quad \frac{\gamma_{xy}}{2} = 865(10^{-6})$

$A(480, 865)\,10^{-6}$

$C(565, 0)\,10^{-6}$



$R = \left[\sqrt{(565-480)^2 + 865^2}\right] 10^{-6} = 869.17(10^{-6})$

a)

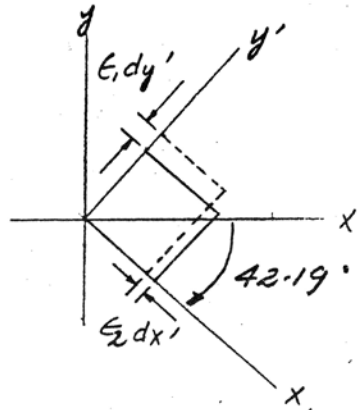
$\epsilon_1 = (565 + 869.17)10^{-6} = \underline{\underline{1434(10^{-6})}}$

$\epsilon_2 = (565 - 869.17)10^{-6} = \underline{\underline{-304(10^{-6})}}$

$\tan 2\theta_p = \frac{865}{565-480}$

$2\theta_p = 84.39°$ (Mohr's circle)

$\theta_p = 42.19°$ (element)



b) $\gamma_{\substack{max \\ in\text{-}plane}} = 2R = 2(869.17)10^{-6}$

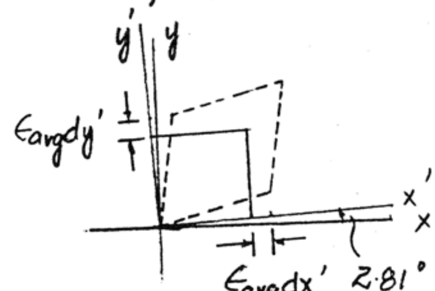
$= \underline{\underline{1738(10^{-6})}}$

$\epsilon_{ave} = 565 \times 10^{-6}$

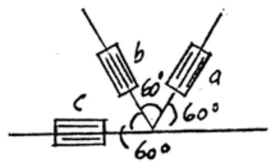
10-20 cont'd

$2\theta_s = 90 - 2\theta_p = 5.61$ (Mohr's circle)

$\theta_s = 2.81°$ (element)



10-21



$\epsilon_a = 250(10^{-6}) \quad \epsilon_b = -400(10^{-6})$

$\epsilon_c = 280(10^{-6})$

$\theta_a = 60° \quad \theta_b = 120° \quad \theta_c = 180°$

$\epsilon = \epsilon_x \cos^2\theta + \epsilon_y \sin^2\theta + \gamma_{xy} \sin\theta \cos\theta$

$280(10^{-6}) = \epsilon_x \cos^2 180° + \epsilon_y \sin^2 180$
$\quad + \gamma_{xy} \sin 180° \cos 180°$

$\epsilon_x = 280(10^{-6})$

$250(10^{-6}) = \epsilon_x \cos^2 60° + \epsilon_y \sin^2 60°$
$\quad + \gamma_{xy} \sin 60° \cos 60°$

$250(10^{-6}) = 0.25\epsilon_x + 0.75\epsilon_y + 0.433\gamma_{xy} \quad (1)$

$-400(10^{-6}) = \epsilon_x \cos^2 120° + \epsilon_y \sin^2 120°$
$\quad + \gamma_{xy} \sin 120° \cos 120°$

$-400(10^{-6}) = 0.25\epsilon_x + 0.75\epsilon_y - 0.433\gamma_{xy} \quad (2)$

cont.

(1) − (2) 10-21 cont

$650(10^{-6}) = 0.866\gamma_{xy}$

$\gamma_{xy} = 750.56(10^{-6})$

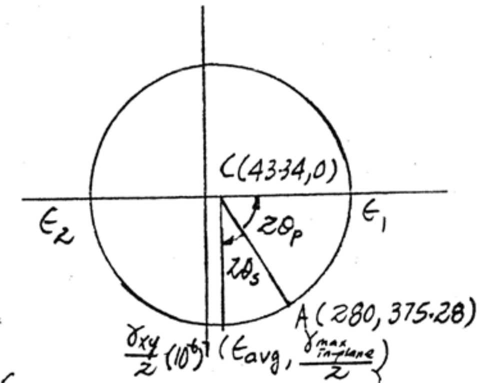
$\epsilon_y = -193.33(10^{-6})$

$\epsilon_x = 280(10^{-6}) \quad \epsilon_y = -193.33(10^{-6})$

$\gamma_{xy} = 750.56(10^{-6}) \quad \frac{\gamma_{xy}}{2} = 375.28(10^{-6})$

$A(280, 375.28)\,10^{-6}$

$C(43.34, 0)\,10^{-6}$



$R = \left[\sqrt{(280-43.34)^2 + 375.28^2}\right] 10^{-6}$

$= 443.67(10^{-6})$

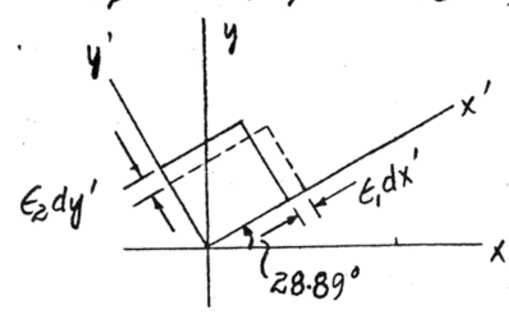
a) $\epsilon_1 = (43.34 + 443.67)10^{-6} = \underline{\underline{487(10^{-6})}}$

$\epsilon_2 = (43.34 - 443.67)10^{-6}) = \underline{\underline{-400(10^{-6})}}$

$\tan 2\theta_p = \frac{375.28}{280 - 43.34}$

$2\theta_p = 57.76°$ (Mohr's circle)

$\theta_p = 28.89°$ (element)



10-27

$\sigma_x = \frac{w_1}{t} = \frac{60}{0.002} = 0.03\text{ MPa} \quad \sigma_y = 0 \quad \sigma_z = 0$

$\epsilon_x = \frac{\sigma_x}{E} = \frac{0.03}{4} = 0.0075$

$\epsilon_y = -\frac{\nu\sigma_x}{E} = \frac{-0.42(0.03)}{4} = -0.00315$

$\epsilon_z = -\frac{\nu\sigma_x}{E} = -0.00315$

$a' = 100 + 100(-0.00315) = \underline{\underline{99.685\text{ mm}}}$

$b' = 100 + 100(0.0075) = \underline{\underline{100.750\text{ mm}}}$

$t' = 2 + 2(-0.00315) = \underline{\underline{1.9937\text{ mm}}}$.

10.34 Plane stress so $\sigma_z = 0$

$\epsilon_x = \frac{1}{E}[\sigma_x - \nu\sigma_y] \rightarrow E\epsilon_x = \sigma_x - \nu\sigma_y$

$\nu E\epsilon_x = \nu\sigma_x - \nu^2\sigma_y$ likewise

$E\epsilon_y = -\nu\sigma_x + \sigma_y$ add last 2 Eq

$\nu E\epsilon_x + E\epsilon_y = \sigma_y - \nu^2\sigma_y$

$E(\nu\epsilon_x + \epsilon_y = \sigma_y(1-\nu^2$ or

$\sigma_y = \frac{E(\nu\epsilon_x + \epsilon_y)}{1-\nu^2}$ QED

Do same for σ_x get $\sigma_x = \frac{E(\nu\epsilon_y + \epsilon_x)}{1-\nu^2}$ QED