ME 3300 TEAM QUIZ #5 S.09 (Open Book)

Names: Key

Use and energy method to determine the reaction at B due to the couple M_o applied at the left end of the cantilever beam.

EI is constant

$0<x<a$

$M=-M_o$

$a<x<2a$

$M=R_B(x-a)-M_o$

$$U=\int_0^a \frac{M_o^2}{2EI}dx+\int_a^{2a}\frac{[R_B(x-a)-M_o]^2dx}{2EI}$$

$$0=\frac{\partial U}{\partial R_A}=0+\frac{2}{2EI}\int_0^{2a}[R_B(x-a)-M_o](x-a)dx$$

$$0=\int_0^{2a}[R_B(x^2-2ax+a^2)-M_o(x-a)]dx$$

$$0=\left[R_B\left(\frac{x^3}{3}-\frac{2ax^2}{2}+a^2x\right)-M_o\left(\frac{x^2}{2}-ax\right)\right]_a^{2a}$$

R_B = $\frac{3}{2}M_o/a$

$$R_B\left[\left(\frac{8a^3}{3}-4a^3+2a^3\right)-\left(\frac{a^3}{3}-a^3+a^3\right)\right]=M_o\left[\left(\frac{4a^2}{2}-2a^2\right)-\left(\frac{a^2}{2}-a^2\right)\right]$$

$$R_B\left[\frac{8-12+6-1}{3}\right]a^3=M_oa^2\left[2-2-\frac{1}{2}+1\right]$$

$$R_Ba\left[\frac{1}{3}\right]=\frac{1}{2}M_o \qquad \boxed{R_B=\frac{3}{2}\frac{M_o}{a}}$$

alternate

$M=R_Bx-M_o$

$$U=\int_0^a\frac{(R_Bx-M_o)^2}{2EI}dx$$

$$\frac{\partial U}{\partial R_B}=0=\frac{2}{2EI}\int_0^a[R_Bx-M_o]x\,dx$$

$$0=\int_0^a R_Bx^2-M_ox\,dx=\left.\frac{R_Bx^3}{3}-\frac{M_ox^2}{2}\right]_0^a$$

$$\frac{R_Ba^3}{3}=\frac{M_oa^2}{2} \qquad \boxed{R_B=\frac{3}{2}\frac{M_o}{a}}$$