# Design of Machine Structures

ME EN 7960 – Precision Machine Design
Topic 14

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# Topics

- Overall design approach for the structure
- Stiffness requirements
- Damping requirements
- Structural configurations for machine tools
- Other structural system considerations

# Design Strategies

- Strategies for Accuracy:
  - Accuracy obtained from component accuracy
    - Most machine tools are built this way
  - Accuracy obtained by error mapping
    - Most coordinate measuring machines are built this way
  - Accuracy obtained from a metrology frame
    - Special machines are built this way (usually one-of-a-kind cost-is-no-object machines)
- Kinematic design:
  - Deterministic
  - Less reliance on manufacturing
  - Stiffness and load limited, unless pot in epoxy

# Design Strategies (contd.)

- Elastically averaged design:
  - Non-deterministic
  - More reliance on manufacturing
  - Stiffness and load not limited
- Passive temperature control:
  - Minimize and isolate heat sources
  - Minimize coefficient of thermal expansion
  - Maximize thermal diffusivity
  - Insulate critical components
  - Use indirect lighting
  - Use PVC curtains to shield the machine from infrared sources

# Design Strategies (contd.)

- Active temperature control:
  - Air showers
  - Circulating temperature controlled fluid
  - Thermoelectric coolers to cool hot spots
  - Use proportional control
- Structural configurations:
  - Where are the center of mass, friction and stiffness located?
  - What does the structural loop look like?
  - Open frames (G type)
  - Closed frames (Portal type)
  - Spherical (NIST's M3)
  - Tetrahedral (Lindsey's Tetraform)
  - Hexapods (Stewart platforms)
  - Compensating curvatures
  - Counterweights

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# Design Strategies (contd.)

- Damping:
  - Passive:
    - Material and joint-µslip damping
    - Constrained layers, tuned mass dampers
  - Active:
    - Servo-controlled dampers (counter masses)
    - Active constrained layer dampers

# Summary of Strategies for Accuracy

- Accuracy obtained from component accuracy:
  - Inexpensive once the process is perfected
  - Accuracy is strongly coupled to thermal and mechanical loads on the machine
- Accuracy obtained by error mapping:
  - Inexpensive once the process is perfected
  - Accuracy is moderately coupled to thermal and mechanical loads on the machine
- Accuracy obtained from a metrology frame:
  - Expensive, but sometimes the only choice
  - Accuracy is uncoupled to thermal and mechanical loads on the machine

# Stiffness Requirements

- Engineers commonly ask "how stiff should it be?"
- A minimum specified static stiffness is a useful but not sufficient specification
- Static stiffness and damping must be specified
- Static stiffness requirements can be predicted
- Damping can be specified and designed into a machine

# Minimum Static Stiffness

- For heavily loaded machine tools, the required stiffness may be a function of cutting force
- For lightly loaded machines and quasi-statically positioning, use the following:
  - First make an estimate of the system's time constant:

$$\tau_{mech} = 2\pi\sqrt{\frac{m}{k}}$$

  - The control system loop time $\tau_{loop}$ must be at least twice as fast to avoid aliasing
- Faster servo times create an averaging effect by the factor $(\tau_{mechanical}/2\tau_{loop})^{½}$

# Minimum Static Stiffness (contd.)

- For a controller with $N$ bits of digital to analog resolution, the incremental force input is:

$$\Delta F = \frac{F_{\max}}{2^N\sqrt{\frac{\tau_{mech}}{2\tau_{servo}}}}$$

- The minimum axial stiffness is thus:

$$k \geq \left(\frac{F_{\max}\tau_{servo}^{1/2}}{2^N \pi^{1/2} m^{1/4} \delta_K}\right)^{4/3}$$

# Minimum Static Stiffness (contd.)

- While the controller is calculating the next value to send to the DAC, the power signal equals the last value in the DAC
- The motor is receiving an old signal and is therefore running open loop
- Assume that there is no damping in the system
- The error $\delta_M$ due to the mass being accelerated by the force resolution of the system for a time increment $\tau_{servo}$ is

$$\delta_M = \frac{1}{2}\frac{\Delta F}{M}\tau_{servo}^2$$

# Minimum Static Stiffness (contd.)

- The maximum allowable servo-loop time is thus

$$\tau_{servo} = \sqrt{\frac{2\delta_M M}{\Delta F}}$$

- The minimum axial stiffness is thus:

$$K \geq \frac{F_{\max}\delta_M^{1/4}}{2^{(N-0.25)}\pi^{1/2}\delta_K^{5/4}}$$

- It must also be greater than the stiffness to resist cutting loads or static loads not compensated for by the servos:

$$K \geq \frac{F_{\max}}{\delta}$$

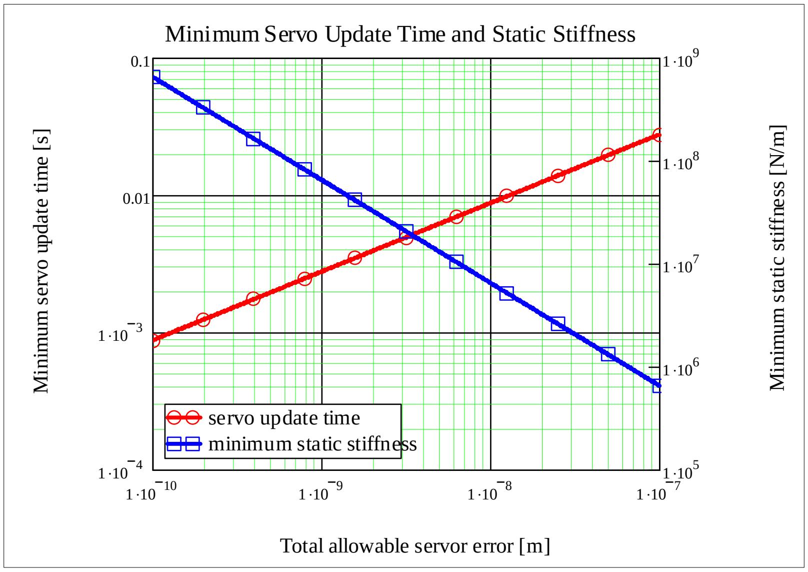
# Minimum Static Stiffness (contd.)

- The maximum servo-loop time is thus:

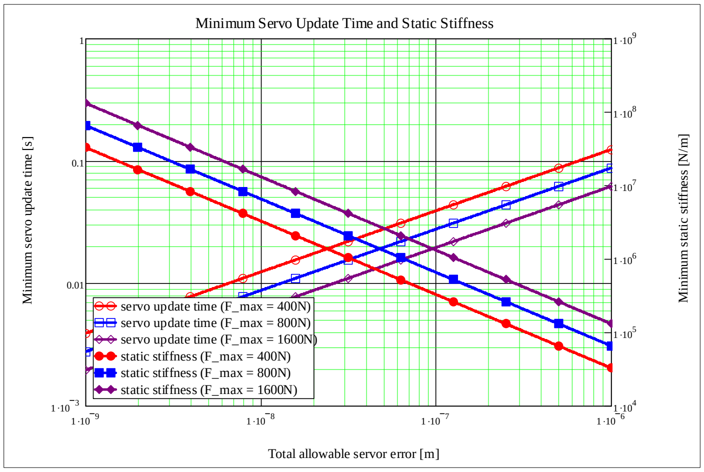
$$\tau_{servo} \le \sqrt{\frac{\pi^{1/2}\, 2^{(N+0.75)}\, \delta_M^{3/4}\, m\, \delta_K^{1/4}}{F_{\max}}}$$

- Typically, one would set $\delta_K = \delta_M = \frac{1}{2}\delta_{servo}$
- Usually, $\tau_{servo\ actual} = \tau_{servo}/L$, where $L$ is the number of past values used in a recursive digital control algorithm
- Example: Required static stiffness for a machine with 800 N max. axial force, 250 kg system mass, and 14 bit DAC:

# Example

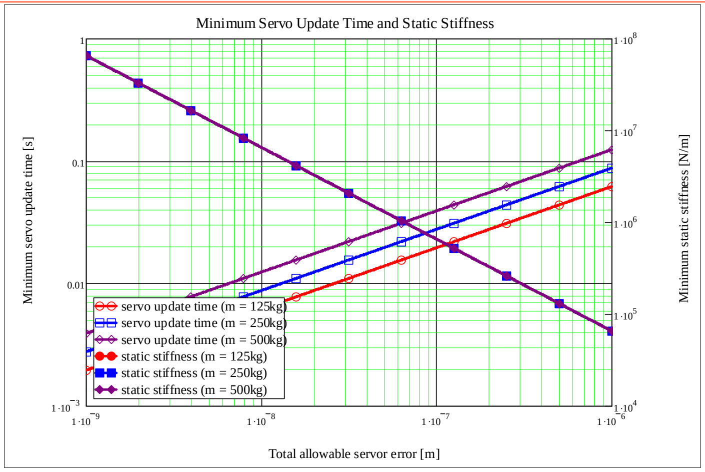


# Servo System Force Output



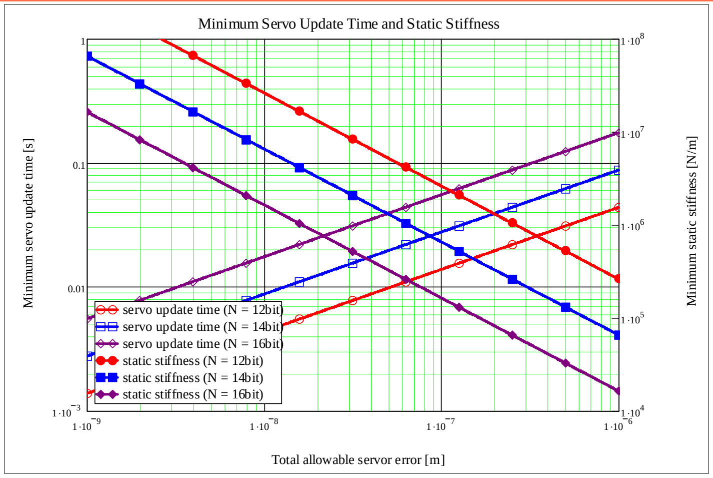
- Lower force drive system for a given servo error
  - Increases the servo update time
  - Lowers the static stiffness requirement

# System Mass



- Lower mass
  - Decreases the servo update time
  - Does not affect static stiffness requirement

# Servo DAC Resolution

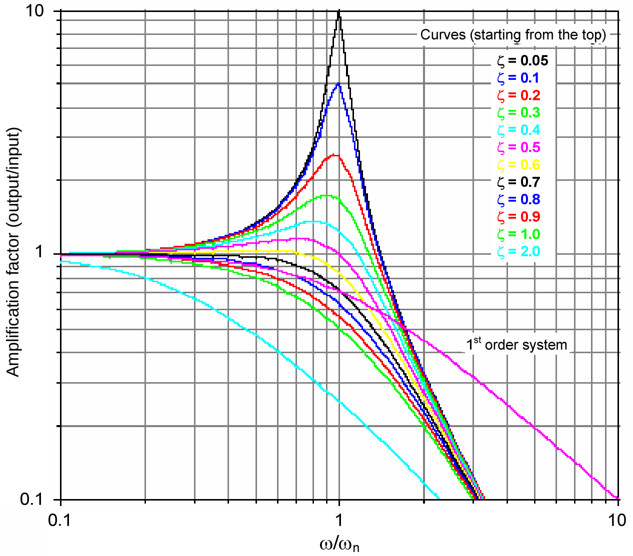


- Lower DAC resolution
  - Decreases the servo update time
  - Increases static stiffness requirement

# Dynamic Stiffness

- Dynamic stiffness is a necessary and sufficient specification
- Dynamic stiffness:
  - Stiffness of the system measured using an excitation force with a frequency equal to the damped natural frequency of the structure
- Dynamic stiffness can also be said to be equal to the static stiffness divided by the amplification (Q) at resonance

# Dynamic Stiffness (contd.)



It takes a lot of damping to reduce the amplification factor to a low level.

Source: Alexander Slocum, *Precision Machine Design*

# Dynamic Stiffness (contd.)

- Material and joint damping factors are difficult to predict and are too low anyway.
- For high speed or high accuracy machines:
  - Damping mechanisms must be designed into the structure in order to meet realistic damping levels.
- The damped natural frequency and the frequency at which maximum amplification occurs are

$$\omega_d = \omega\sqrt{1-\zeta^2}$$

$$\omega_{d\ peak} = \omega\sqrt{1-2\zeta^2}$$

# Dynamic Stiffness (contd.)

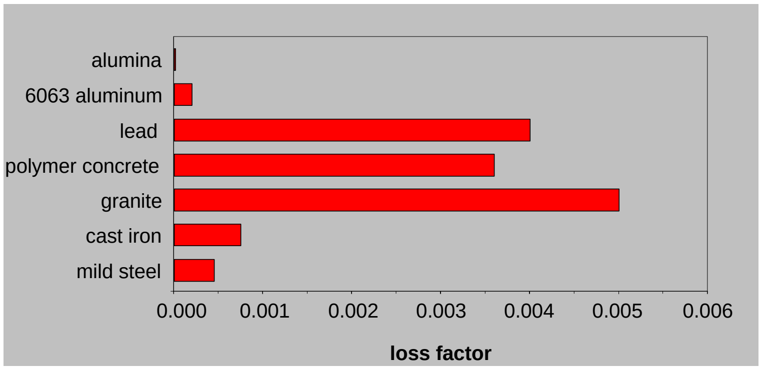
- The amplification at the damped natural frequency and the peak frequency can thus be shown to be

$$Q = \frac{Output}{Input} = \frac{1}{\sqrt{4\zeta^2 - 3\zeta^4}}$$

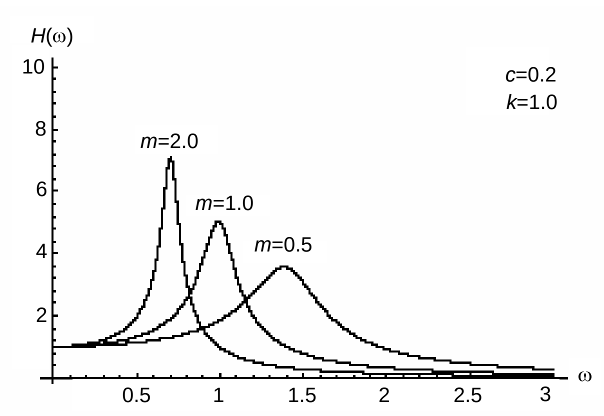
$$Q\frac{Output_{peak}}{Input} = \frac{1}{2\zeta\sqrt{1-\zeta^2}} = \frac{k_{static}}{k_{dynamic}} \approx \frac{1}{2\zeta}$$

- For unity gain or less, $\zeta$ must be greater than 0.707
- Cast iron can have a damping factor of 0.0015
- Epoxy granite can have a damping factor of 0.01-0.05
- All the components bolted to the structure (e.g., slides on bearings) help to damp the system
- To achieve more damping, a tuned mass damper or a shear damper should be used

# Material Damping



# Effects of Changing System Mass



- Adding mass:
  - Adding sand or lead shot increases mass and damping via the particles rubbing on each other
  - Higher mass slows the servo response, but helps attenuates high frequency noise

  Decreasing mass
  - Faster respond to command signals
  - Increases a higher natural frequency
    - Higher speed controller signals must be used
  - Improved damping, a result of the increased loss factor (the loss factor $\zeta = c/(2m)$
  - However, low mass systems show less noise rejection at higher frequencies
    - This suggests that the machine will be less able to attenuate noise and vibration

Source: Alexander Slocum, *Precision Machine Design*

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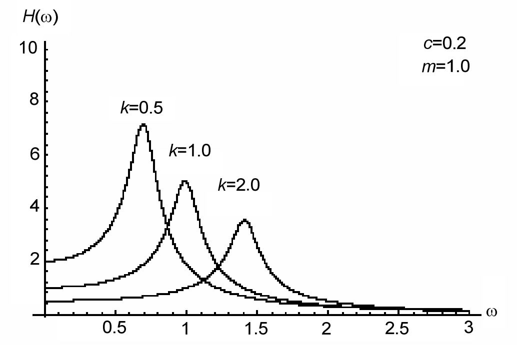
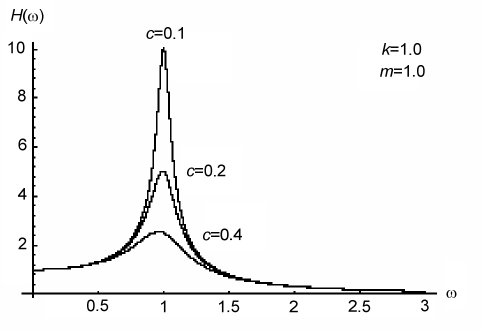
# Effects of Adding Stiffness to the Machine System



- Higher stiffness gives a flatter response at low frequencies and give smaller displacements for a given force input
- The compromise of decreased noise attenuation is not as dramatic as is the case with lowering the system mass
  - This is shown by the similar shapes in the three response curves at high frequencies
- This suggests that raising the stiffness of a system is always a desirable course of action
  - However, acoustical noise may be worsened by adding stiffness (frequency of vibration is moved to the audible region of the human ear)

Source: Alexander Slocum, *Precision Machine Design*

# Effects of Adding Damping to the Machine System



Source: Alexander Slocum, *Precision Machine Design*

- Increasing the system damping can make a dramatic improvement in the system response
- The trend is for decreasing amplification of the output at resonance with increasing damping
- The plot shows the dramatic improvement available by doubling the system damping:
  - Although a damping coefficient of 0.4 may be difficult to obtain in practice

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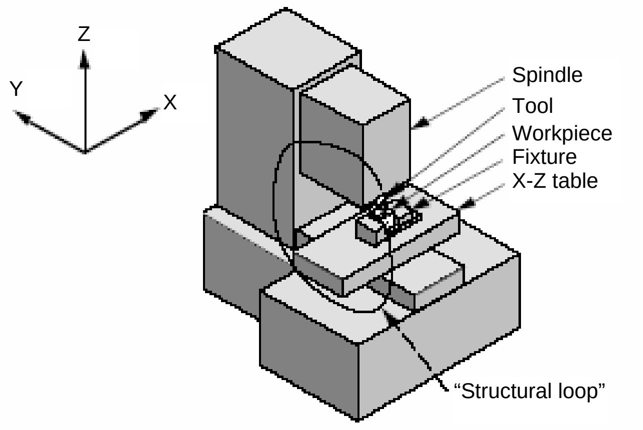
# Summary

- For a servo controlled machine:
  - The stiffness of the machine structure should be maximized to improve positioning accuracy
  - The mass should be minimized to reduce controller effort and improve the frequency response and loss factor ($\zeta$)
  - Damping, however, must be present to attenuate vibration in the machine system

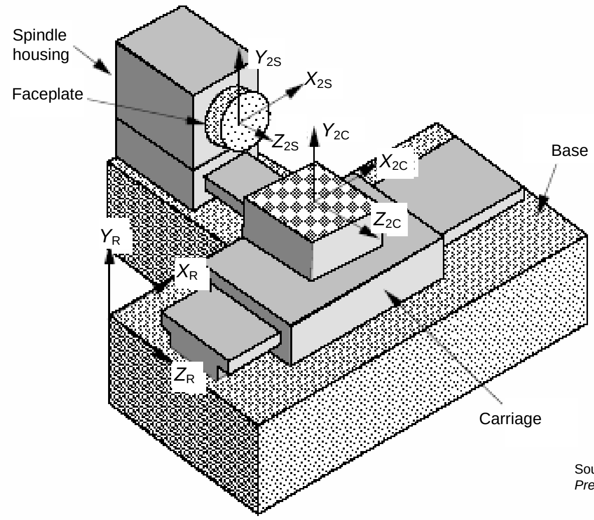
# Open Frame Structures



- Easy access to work zone
- Structural loop prone to Abbe errors (like calipers!)

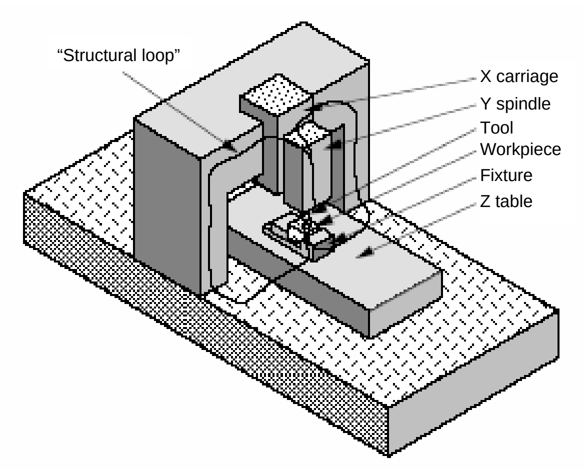
Source: Alexander Slocum, *Precision Machine Design*

# Open Frame Structures (contd.)



Source: Alexander Slocum, *Precision Machine Design*

# Closed Frame Structures



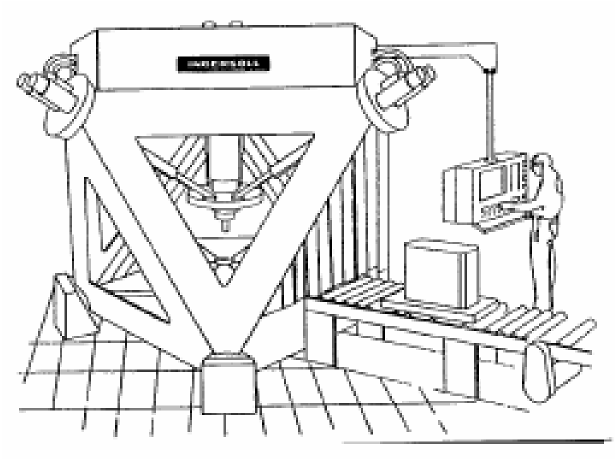
- Moderately easy access to work zone
- Moderately strong structural loop (like a micrometer!)
- Primary/follower actuator often required for the bridge
- Easier to obtain common centers of mass, stiffness, friction

Source: Alexander Slocum, *Precision Machine Design*

# Closed Frame Structures

Source: Precision Design Lab

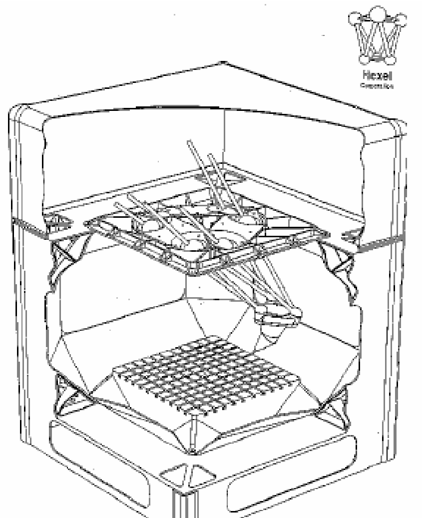
# Tetrahedral Structures



- Composed of six legs joined at spherical nodes
- Work zone in center of tetrahedron
- Bearing ways bolted to legs
- High thermal stability
- High stiffness
- Viscous shear damping mechanisms built into the legs
- Damping obtained at the leg joints by means of sliding bearing material applied to the self centering spherical joint
- Inherently stable shape

Source: Alexander Slocum, *Precision Machine Design*

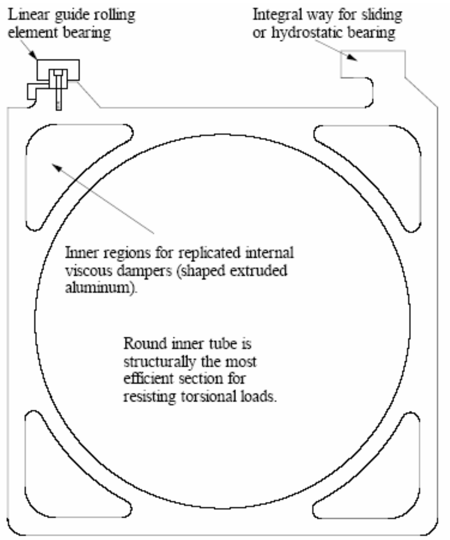
# Tetrahedral Structures



- Like the tetrahedron, the octahedron is a stable truss-type geometry (comprised of triangles)
- As the work volume increases, the structure grows less fast than a tetrahedron
- The hexapod (Stewart platform concept originally developed for flight simulators) gives six limited degrees of freedom
- The tool angle is limited to about 20 degrees from the vertical
- Advanced controller architecture and algorithms make programming possible

Source: Alexander Slocum, *Precision Machine Design*

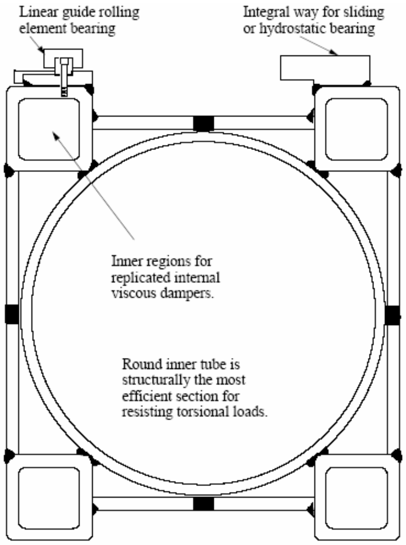
# Cast Iron Structures



- Widely used
- Stable with thermal anneal, aging, or vibration stress relieve
- Good damping and heat transfer
- Modest cost for modest sizes
- Integral ways can be cast in place
- Design rules are well established (see text)
- Economical in medium to large quantities

Source: Alexander Slocum, *Precision Machine Design*

# Welded Structures



- Often used for larger structures or small-lot sizes
- Stable with thermal anneal
- Low damping, improved with shear dampers
- Modest cost
- Integral ways can be welded in place
- Structures can be made from tubes and plates

Source: Alexander Slocum, *Precision Machine Design*

# Epoxy Granite Structures

- Can be cast with intricate passages and inserts
- $E_{\text{epoxy granite}}/E_{\text{cast iron}}$ = 5/20
- Exterior surface can be smooth and is ready to paint
- Cast iron or steel weldments can be cast in place, but beware of differential thermal expansion effects
- Epoxy granite’s lower modulus, and use of foam cores means that local plate modes require special care when designing inserts to which other structures are bolted
- Sliding contact bearing surfaces can be replicated onto the epoxy granite

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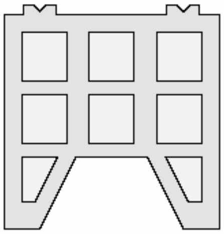
# Epoxy Granite Structures

- Foam cores reduce weight:

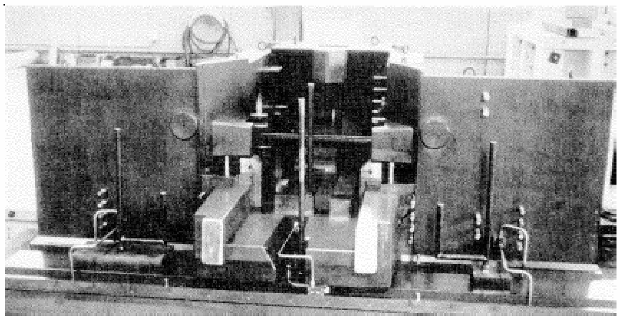


Source: Alexander Slocum, *Precision Machine Design*

- For some large one-of-a-kind machines
- A mold is made from thin welded steel plate that remains an integral part of the machine after the material is cast
- Remember to use symmetry to avoid thermal warping
- Consider the effects of differential thermal expansion when designing the steel shell
- The steel shell should be fully annealed after it is welded together

# Epoxy Granite Structures

- Instead of ribs, polymer concrete structures usually use internal foam cores to maximize the stiffness to weight ratio
- Polymer concrete castings can accommodate cast in place components (Courtesy of Fritz Studer AG.)



Source: Alexander Slocum, *Precision Machine Design*

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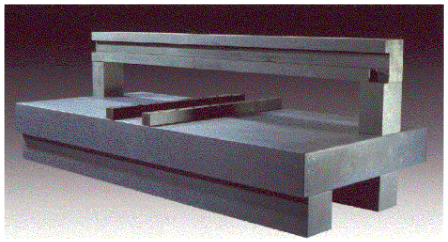
# Epoxy Granite Structures

- With appropriate section design:
  - Polymer concrete structures can have the stiffness of cast iron structures
  - They can have much greater damping
  - Highly loaded machine substructures (e.g. carriages) are still best made from cast iron
- Polymer concrete does not diffuse heat as well as cast iron
  - Attention must be paid to the isolation of heat sources to prevent the formation of hot spots
- When bolting or grouting non-epoxy granite components to an epoxy granite bed, consider the bi-material effect

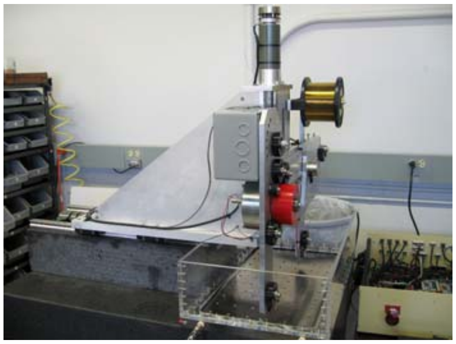
# Granite

- Dimensionally very stable
- Must be sealed to avoid absorption of water
- Can be obtained from a large number of vendors providing excellent flatness and orthogonality
- Cannot be tapped, therefore bolt holes consist of steel plugs that have been potted in place that are drilled and tapped after the epoxy has cured
- Can chip
- Provides excellent damping

# Granite (contd.)

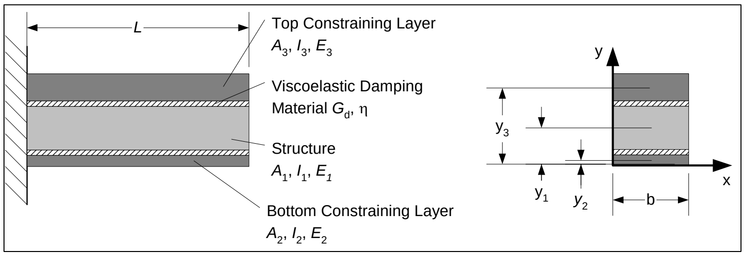


Source: Standridge Granite



Source: Precision Design Lab

# Constrained Layer Damping
How does it work?



- Visco-elastic layer damps motion between structure and constraining layer (from bending or torsion) by dissipating kinetic energy into heat

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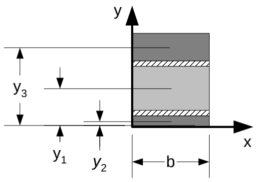
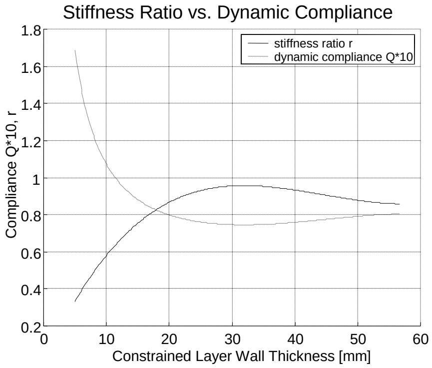
# Design parameters to tune damper

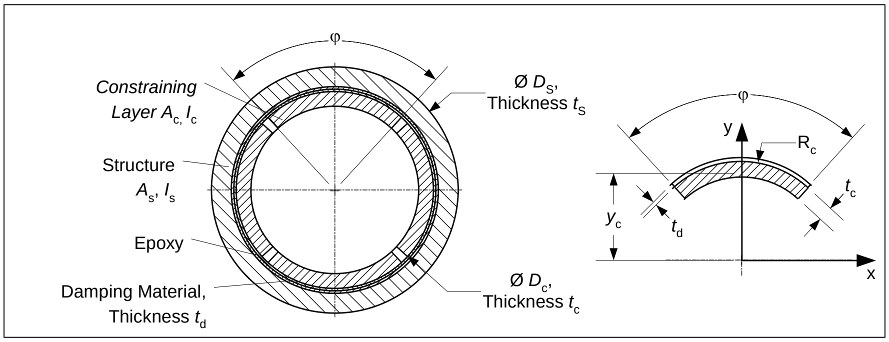




$$EI_\infty = EI_0 + \sum_i E_i A_i (y_i^2 - y_\infty^2)$$

$$r = \frac{EI_\infty}{EI_0} - 1$$

# How is it implemented?



- For round structures, inner tube serves as constraining layer (ShearDamper™)
- Constraining layer is wrapped with damping material
- Coated inner tube is inserted and gap filled with epoxy

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# ShearDamper™ - step by step

**Step 1:** split tube

**Step 2:** wrap damping sheet around

**Step 3**: fill gap with epoxy

**Step 4**: done

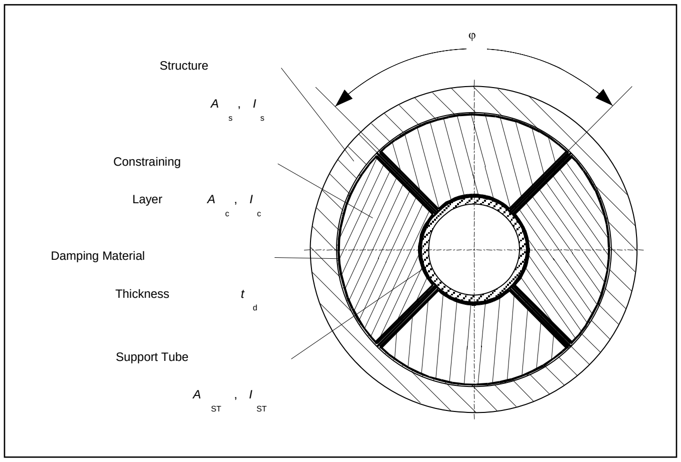
# Any tradeoffs?

- Labor intensive – inner tube needs to be split
- Inner tube and epoxy are expensive
- Added weight lowers modal frequencies
- Challenging if bottom is not accessible for sealing
- Constraining layer performance depends on available wall thickness
- Only works for round or rectangular structures where a matching split tube is available

# How can we make it better?

- Replace steel tube AND epoxy with cheaper material that has better internal damping
- Make the need for splitting the constraining layer obsolete
- Make design more flexible in terms of shape and required stiffness
- Remove design constraints

# But how?



- Four “sausage-like” damping sleeves are inserted between the outer structural and the inner support tube
- Dampers are filled with expanding concrete

E. Bamberg, A.H. Slocum, “Concrete-based constrained layer damping”, Precision Engineering, 26(4), October 2002, pp. 430-441.

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# Concrete cast – it’s simple!

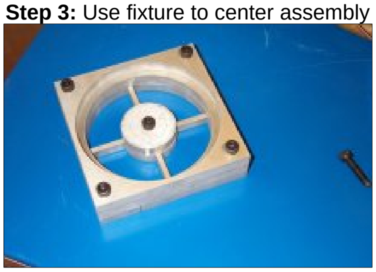
- Flexible constraining layer thickness – wide range of standard tubes can be used as support tube, wall thickness is no longer a design constraint
- Concrete provides additional damping
- Cheap
- Fast

# How is it done – part 1?

**Step 1:** Cut damping sheet

**Step 2:** Make lap joint and turn sheet into tube

**Step 3:** Use fixture to center assembly



**Step 4:** Seal bottom with cable ties

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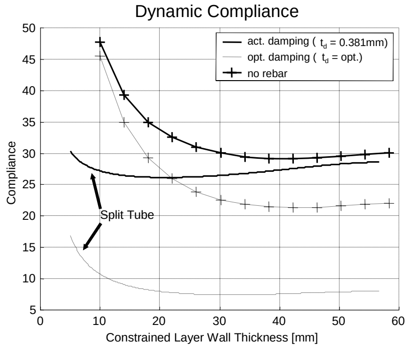
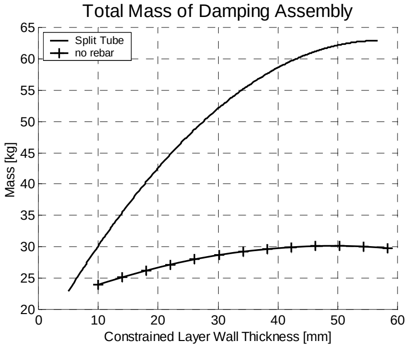
# How is it done – part 2?

**Step 5:** Pour expanding concrete

**Step 6:** After concrete has cured – cut off ends: done

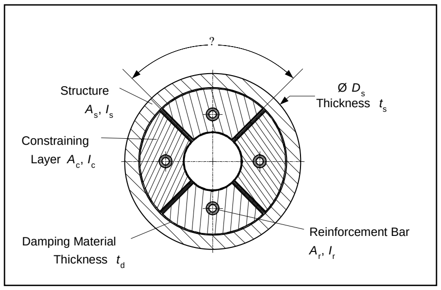
# Dynamic compliance - lower is better…

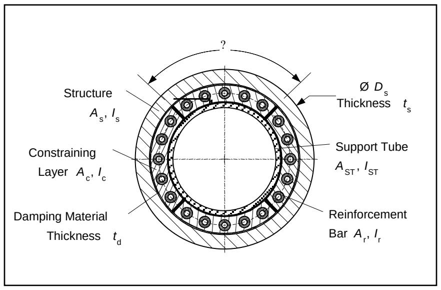




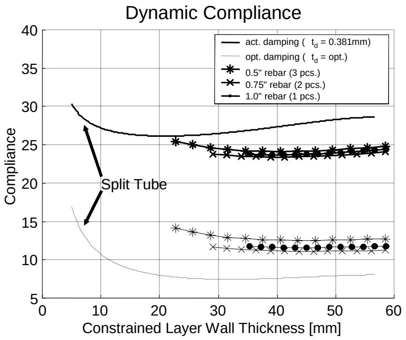
Steel constraining layer is better because it has a higher Young's modulus than concrete – an equally stiff layer is thinner and hence further away from system neutral axis → increased stiffness ratio $r$.

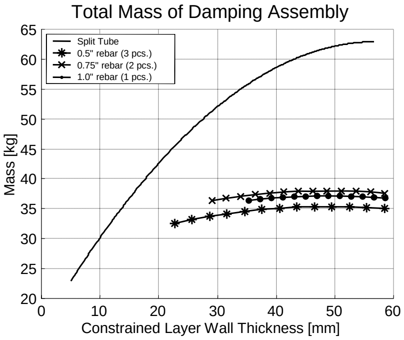
# Reinforced works even better…



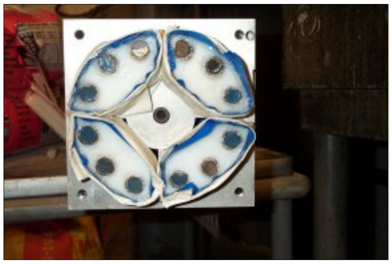


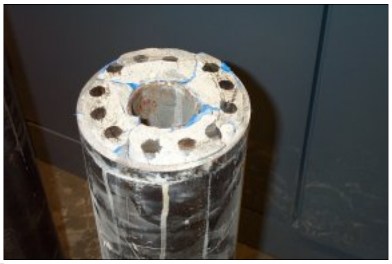
# Concrete cast rocks! (Virtually…)





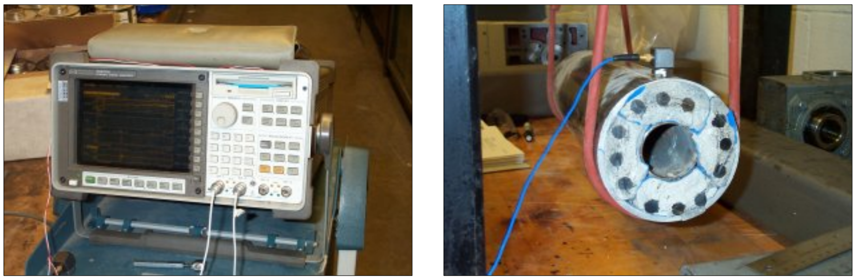
# Reinforced concrete cast





- Cable ties press damping sleeve against fixture
- Hot glue seals rebars

# Test setup



- HP 4-channel frequency analyzer
- 3-axis accelerometer
- 28 data points
- Free-free setup

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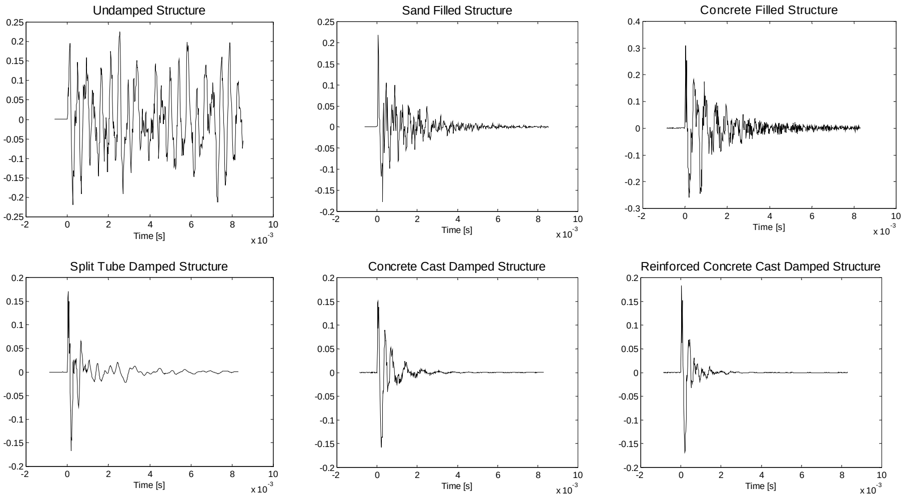
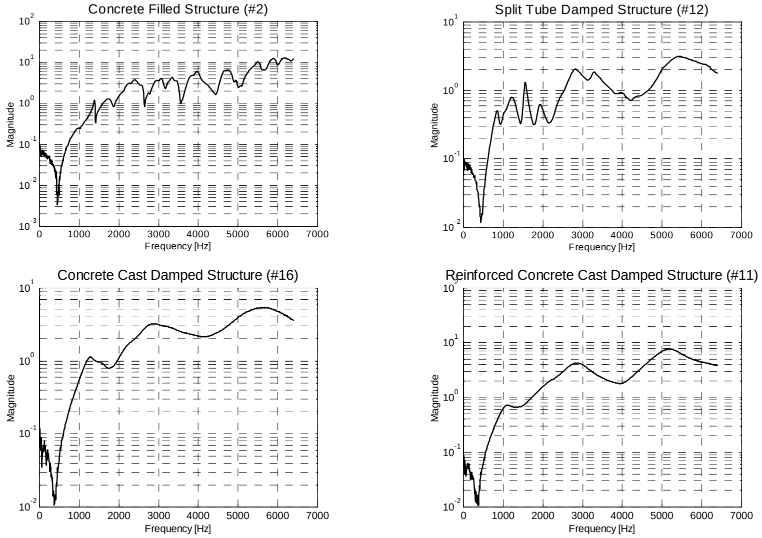
# Response to impulse



E. Bamberg, A.H. Slocum, “Concrete-based constrained layer damping”, Precision Engineering, 26(4), October 2002, pp. 430-441.

# Transfer Functions



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# Measured performance

- Split Tube
  - $f$=1530 Hz and $\eta$=0.055 (predicted: 0.032)
  - Material cost 100%
- Concrete Cast
  - $f$=1260 Hz and $\eta$=0.145 (predicted: 0.035)
  - Material cost 23.5%
- Reinforced Concrete Cast
  - $f$=1640 Hz and $\eta\approx$0.3 (predicted: 0.044)
  - Material cost 28.8%