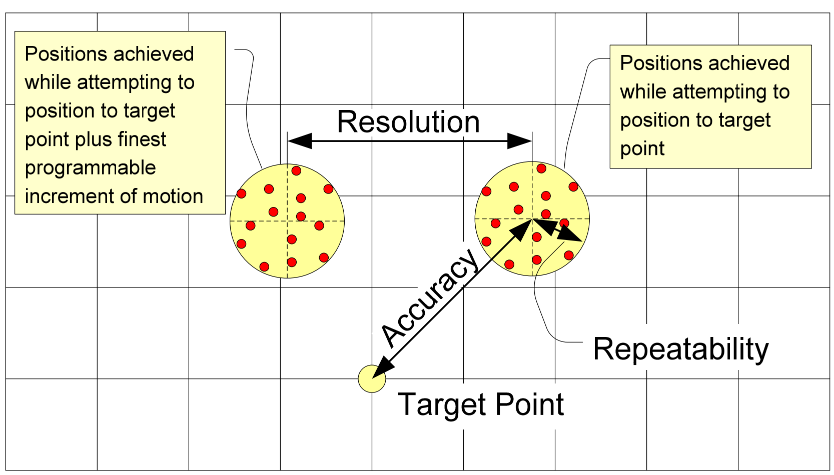
Fundamentals of Errors

ME EN 7960 – Precision Machine Design
Topic 2

Accuracy, Repeatability, and Resolution



Accuracy

- Accuracy is the ability to tell the truth or:
  - The maximum translational or rotational error between any two points in the machine's work volume
  - Linear, planar, and volumetric accuracy can all be defined for a machine
- Note: mechanical accuracy is far more costly than repeatability

Repeatability

- Repeatability (precision) is the ability to tell the same story over and over again or:
  - The error between a number of successive attempts to move the machine to the same position
  - Repeatability is often considered to be the most important parameter of a computer controlled machine (or sensor)
  - Often the intent is to map the errors and then compensate for them
- Minimize static friction, backlash, and thermal variants to get better repeatability

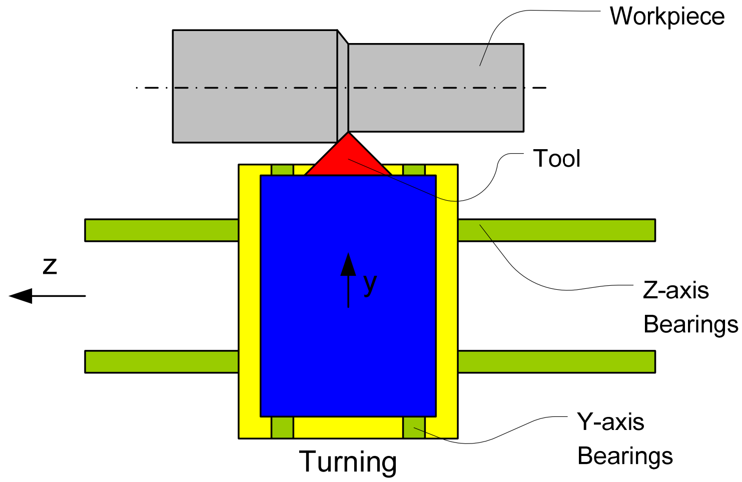
Resolution

- Resolution is how detailed your story is
  - Resolution is the larger of the smallest programmable step or:
  - The smallest mechanical step the machine can make during point to point motion
  - Resolution gives a lower bound on the repeatability
- Minimize static friction and use higher resolution feedback to get better resolution

What is the importance of errors?

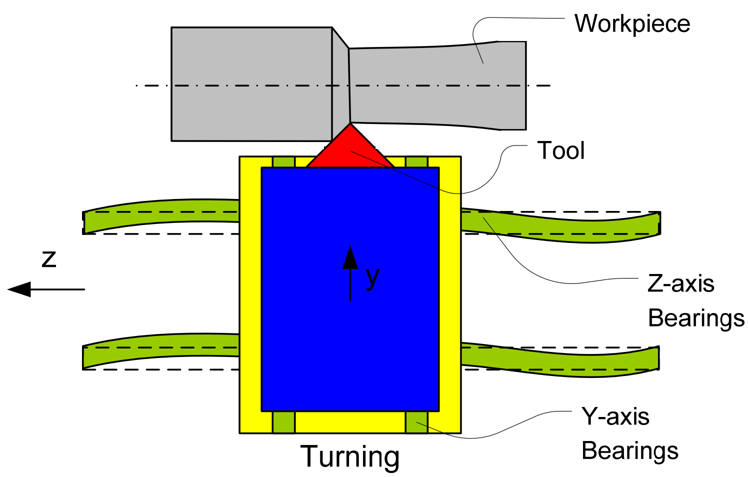
- The trajectory of the tool relative to the work piece directly determines the dimensions of all features machined
- No machine tool monitors the tool trajectory directly. Instead, the tool trajectory is monitored indirectly by monitoring the position of the moving components (axes)
- Any deviation of any moving component will imprinted onto the work piece, thereby reducing the part accuracy

Imprinting Form Errors



Case 1: Perfectly straight z-axis bearings create a perfectly cylindrical part

Imprinting Form Errors (contd.)



Case 2: The straightness errors of the z-axis bearings are transferred onto the part

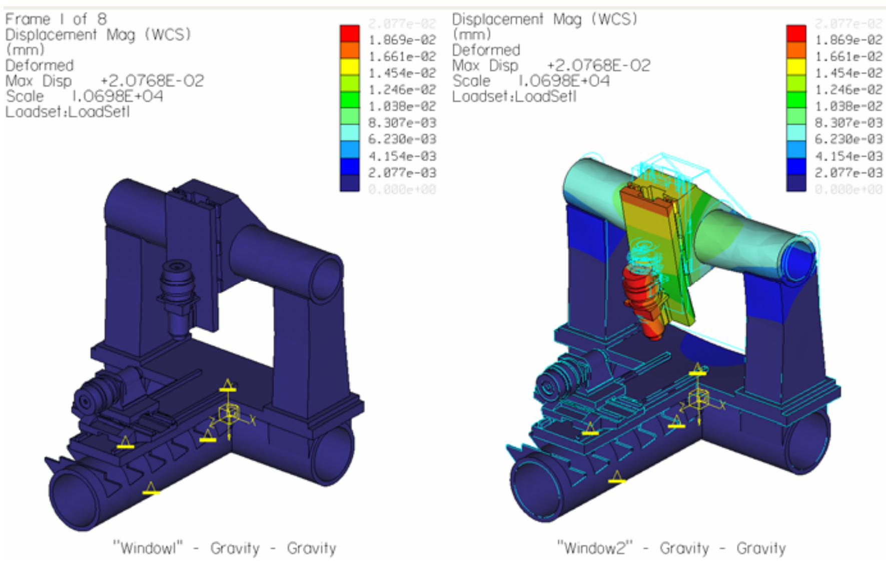
Performance Limiting Criteria

- Accuracy and Repeatability are limited by:
  – Geometric errors of all components
  – Kinematic errors
  – Load induced errors
  – Thermal errors
  – Dynamic errors
  – Calibration errors
  – Computational errors
- Resolution is limited by
  – Quality of sensors
  – Quality of control system
  – Friction (stick and slip effect)
  – Backlash

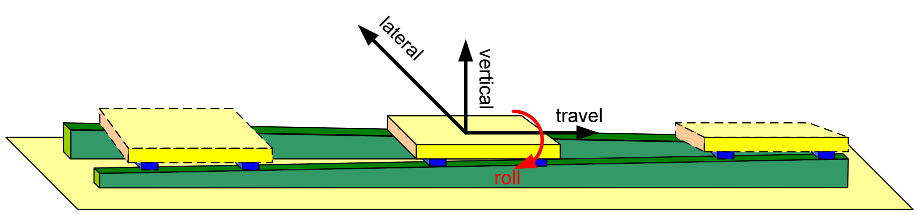
Geometric Errors

- Errors in the form of individual machine components:
  – Component straightness error due to machining errors
  – Component straightness error due to gravity loading
- Surface finish effects
- Quasi-static accuracy of surfaces moving relative to each other (e.g., linear or rotary motion axes):
  – Linear motion axis:
    - Pitch
    - Roll
    - Yaw
    - Straightness (2 components)
    - Linear displacement

Geometric Error - Gravity

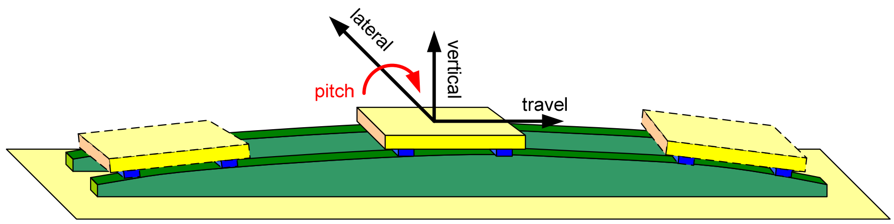


Geometric Errors - Roll



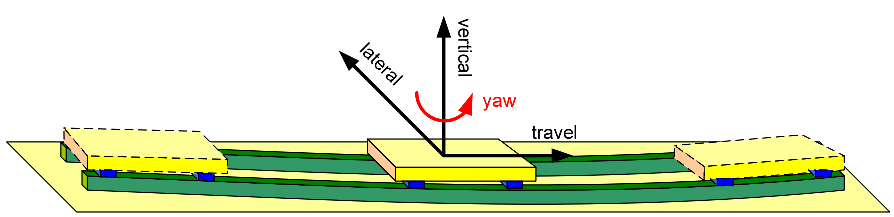
Roll as a result of different slopes of the rails

Geometric Errors - Pitch



Pitch as a result of a bow of the
rails in the vertical direction

Geometric Errors - Yaw

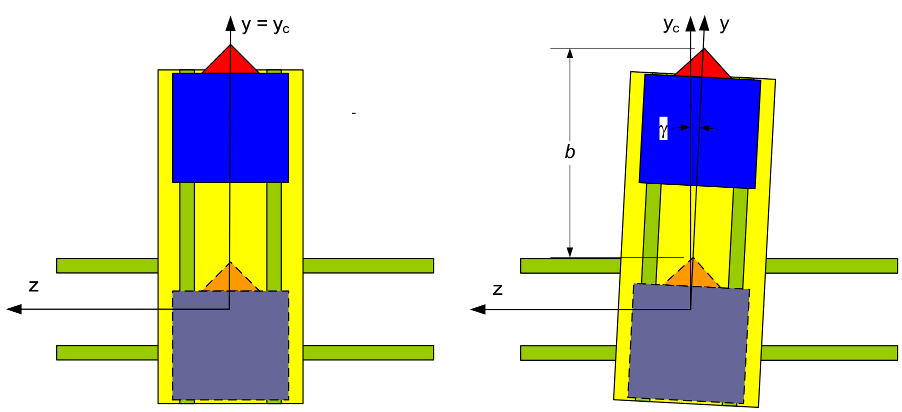


Yaw as a result of bow of the
rails in the lateral direction

Other Errors

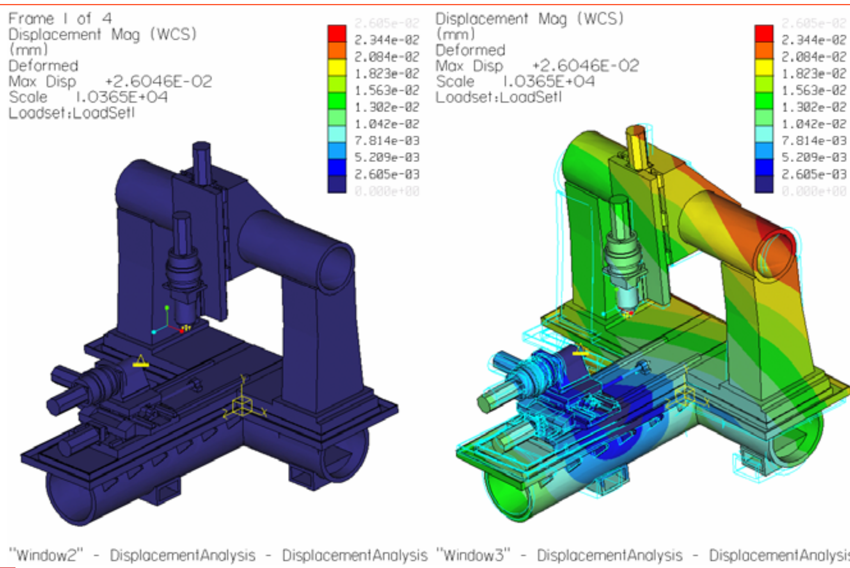
- Kinematic Errors
  - Errors in an axis's trajectory that are caused by misaligned or improperly sized components
    - Orthogonality between axes
    - Parallelism between axes
    - Error motions in a closed kinematic chain
- External load induced errors:
  - Errors due to deformation of components:
    - Gravity load induced errors
    - Cutting/probing force induced errors
    - Axis acceleration load induced errors

Kinematic Error - Orthogonality Between Axes



$$\Delta z = b \cdot \gamma$$

Geometric Error – Load Induced



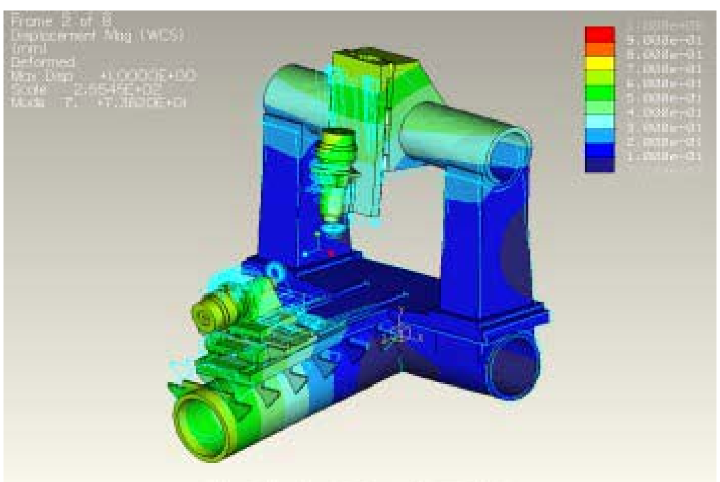
Thermal Errors

- Mean temperature other than 68 °F (20 °C)
  - Gradients in environment's temperature
  - Errors caused by thermal expansion of elements:
    - External heat sources:
      - Mean temperature of the room
      - Sun shining through the window onto the machine
      - Nearby machine's hot air vent
      - Overhead lights
      - Operator's body heat
    - Internal heat sources:
      - Motors
      - Bearings
      - Machining process
      - Pumps
      - Expansion of compressed fluids
      - Coolant

Dynamic Errors

- Errors caused by vibration or control processes:
  - Vibration:
    - External environment (usually through the ground)
    - Cutting process
    - Rotating masses
  - Control system:
    - Algorithm type (e.g., PID, adaptive, etc.)
    - Stick-slip friction
    - Varying mass
    - Varying stiffness
  - Switching amplifiers
    - Servo loop frequency excites a natural mode of the machine

Dynamic Errors



Calibration Errors

- Errors associated with sensors:
  - Intrinsic accuracy
  - Interpolation
  - Mounting errors:
    - Position
    - Mounting stress
  - Calibration (error associated with the mastering process)

Additional Errors

- Computational errors:
  - Error introduced in the analysis algorithms
  - Rounding off errors due to hardware
- Additional sources of error (often very difficult to model):
  - Humidity
  - Loose joints
  - Dirt
- Variations in supply systems:
  - Electricity
  - Fluid pressure
  - Operator inattention
  - Fluid supply cleanliness
- Operators (“52-2” factor)

Error Assessment and Budgeting

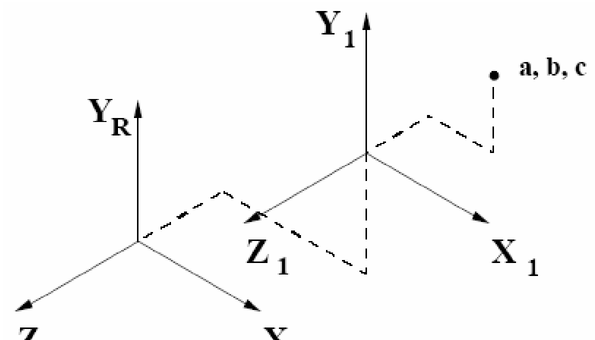
- Given all the different types of errors that can affect all different components:
  - Keeping track of all the errors is such a daunting task:
    - Most engineers don't bother and use "experience" to guide the design
    - It is left up to manufacturing and service to work the bugs out
  - This seems to be a major source of reliability and performance problems
- The solution to a successful project is a good budget:
  - A project requires a good **financial budget** to make it feasible
  - A project requires a good **time budget** to make it feasible
  - A project requires a good **error budget** to make it feasible
- In order to make a good error budget for the system, a good mathematical model is needed

Homogenous Transformation Matrices

- Allows the designer to consider one part of the machine at a time, and then link them all together like beads on a string
- Based on rigid body model of a linear series (open chain) of coordinate frames
- Takes into account linear and angular offsets between coordinate frames
- Transforms XYZ coordinates of one frame into XYZ coordinates of another frame
- Used to transform locally referenced errors into errors referenced with respect to the tool point and the work piece

Homogenous Transformation Matrices (contd.)

- Coordinate frames are placed at bearings, joints, and areas where other parameters are lumped
- Closed chains (e.g. a five point bearing mount) need to be modeled with the generation of constraint equations



Source: Alexander Slocum, *Precision Machine Design*

Structure of HTM's

Orientation of X_n with respect to adjacent coordinate frame

Orientation of Y_n with respect to adjacent coordinate frame

Orientation of Z_n with respect to adjacent coordinate frame

$$R_{T_n} = \begin{bmatrix} O_{ix} & O_{iy} & O_{iz} & P_x \\ O_{jx} & O_{jy} & O_{jz} & P_y \\ O_{kx} & O_{ky} & O_{kz} & P_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Translation of $X_nY_nZ_n$ with respect to adjacent coordinate frame

Structure of HTM's (contd.)

- The first three columns are the direction cosines (unit vectors i, j, k).
  - They represent the orientation of the X_n, Y_n, and Z_n axes with respect to an adjacent coordinate frame
- The last column is the position of the rigid body's coordinate system's origin with respect to the reference frame
- The pre-superscript represents the reference frame in which you want the result to be represented
- The post-subscript represents the reference frame from which you are transferring:

Structure of HTM's (contd.)

- The equivalent coordinates of a point in a reference frame n, in a reference frame R are:

Equivalent coordinates of a point in reference frame *n*, in reference frame *R*

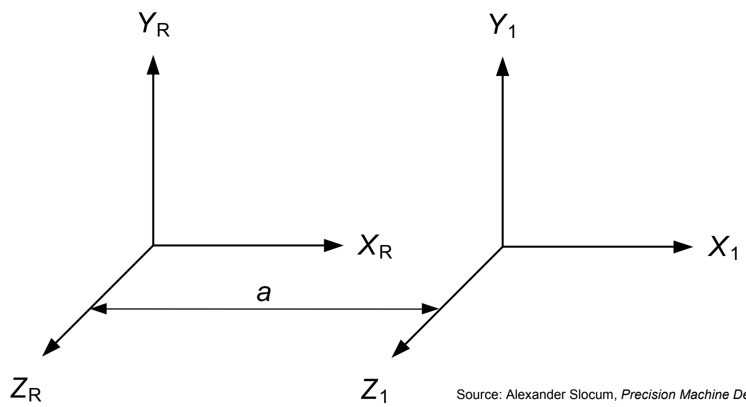
Equivalent coordinates of a point in reference frame *n*, in reference frame *R*

$$\begin{bmatrix} X_R \\ Y_R \\ Z_R \\ 1 \end{bmatrix} = {}^{R}T_{n} \begin{bmatrix} X_n \\ Y_n \\ Z_n \\ 1 \end{bmatrix}$$

HTM that describes the transformation of reference frame *n* into reference frame *R*

Example: Translation in X

- The coordinate system $X_1Y_1Z_1$ is shifted in X by an amount a:



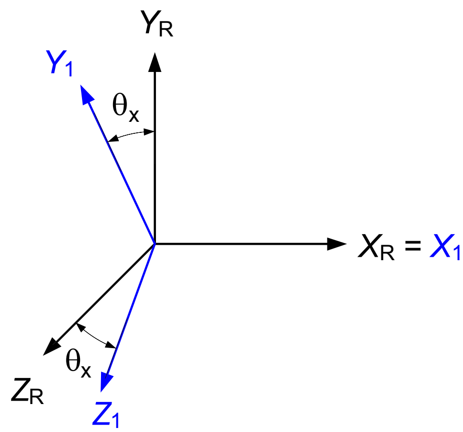
Source: Alexander Slocum, *Precision Machine Design*

HTM of Translation in x

$$
{}^{XYZ}T_{X_1Y_1Z_1} = \begin{bmatrix} 1 & 0 & 0 & a \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

Example: Rotation about X-Axis

- the $X_1Y_1Z_1$ coordinate system is rotated by an amount θ_X about the X axis:



HTM of Rotation about X

$$
{}^{XYZ}T_{X_1Y_1Z_1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta_x & -\sin\theta_x & 0 \\ 0 & \sin\theta_x & \cos\theta_x & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

Sequential Systems

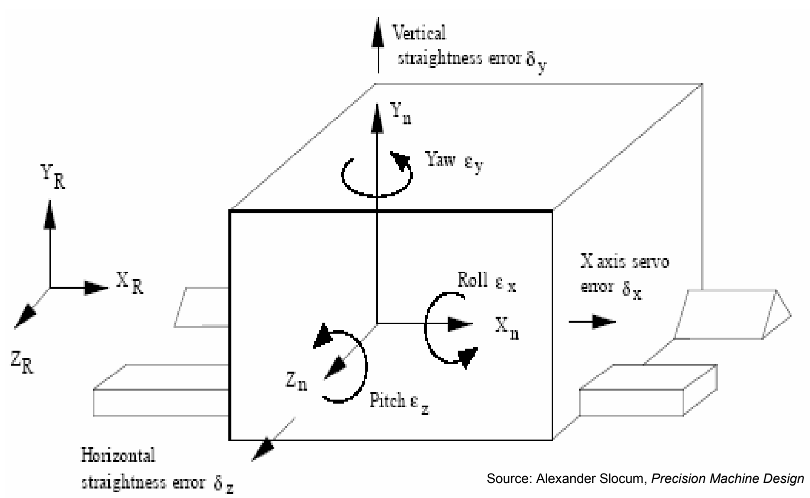
- Transformation from the N^{th} axis to the reference system will be the sequential product of all the HTMs:

$$R_{T_N} = \prod_{m=1}^{N} {}^{m-1}T_m = {}^{0}T_1 \, {}^{1}T_2 \, {}^{2}T_3 \ldots$$

- The error between the tool and the work piece is obtained from:

$$\begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \end{bmatrix} = \begin{bmatrix} P_x \\ P_y \\ P_z \end{bmatrix}_{work,R} - \begin{bmatrix} P_x \\ P_y \\ P_z \end{bmatrix}_{tool,R}$$

Rigid body models of machine components

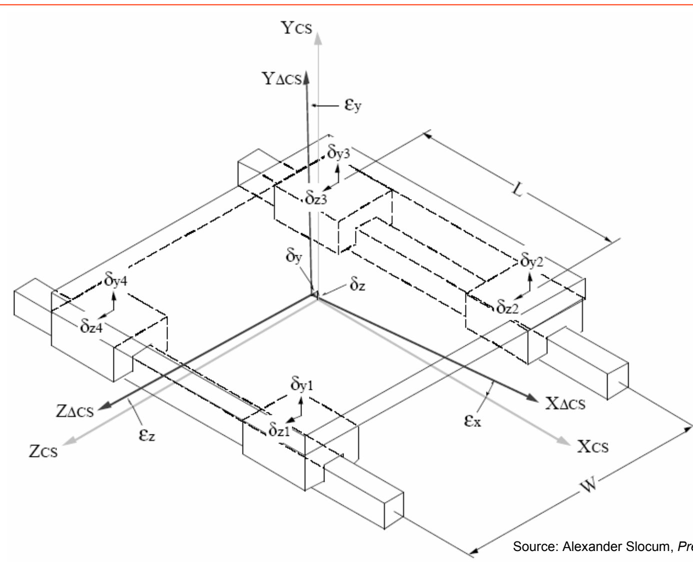


Source: Alexander Slocum, *Precision Machine Design*

HTM of Linear Motion Carriage

$$R_{T_{nerr}} = \begin{bmatrix} 1 & -\varepsilon_z & \varepsilon_y & a+\delta_x \\ \varepsilon_z & 1 & -\varepsilon_x & b+\delta_y \\ -\varepsilon_y & \varepsilon_x & 1 & c+\delta_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Estimating Position Errors from Catalogs

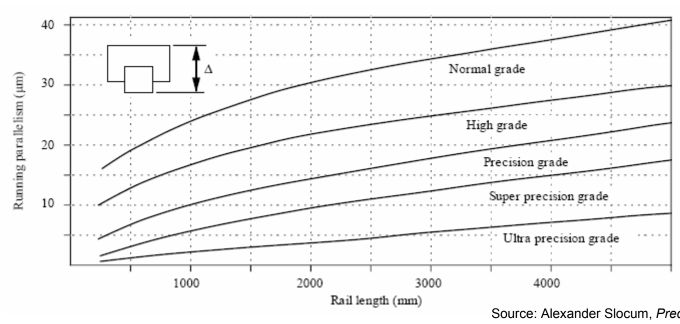


Source: Alexander Slocum, *Precision Machine Design*

HTM of Modular Bearing System

$$\begin{bmatrix} 1 & -\varepsilon_z & \varepsilon_y & a+\delta_x \\ \varepsilon_z & 1 & -\varepsilon_x & b+\delta_y \\ -\varepsilon_y & \varepsilon_x & 1 & c+\delta_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

HTM of Modular Bearing System (contd.)



Source: Alexander Slocum, *Precision Machine Design*

The translational errors are based on the average of the straightness errors experienced by the bearing block:

$$\delta_{axial} = \delta_{servo}$$

$$\delta_{lateral} = \frac{\delta_{lateral1} + \delta_{lateral2} + \delta_{lateral3} + \delta_{lateral4}}{4}$$

$$\delta_{vertical} = \frac{\delta_{vertical1} + \delta_{vertical2} + \delta_{vertical3} + \delta_{vertical4}}{4}$$

HTM of Modular Bearing System (contd.)

- The angular errors are based on the differences in the average straightness errors experienced by pairs of bearing blocks acting across the carriage:

$$\varepsilon_{axial} = \frac{\frac{\delta_{vertical2} + \delta_{vertical3}}{2} - \frac{\delta_{vertical1} + \delta_{vertical4}}{2}}{W}$$

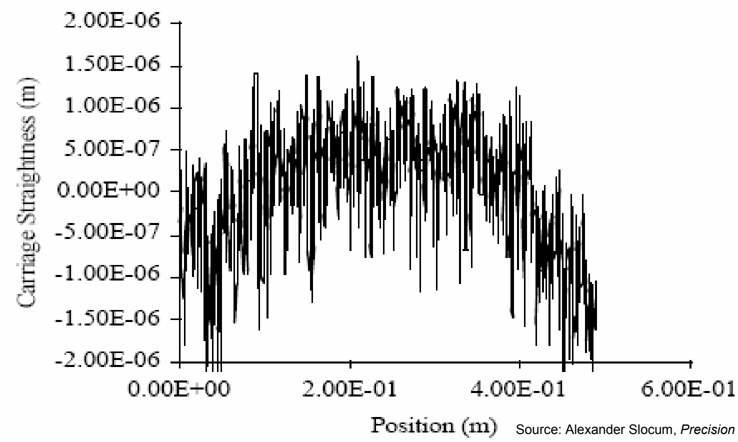
$$\varepsilon_{lateral} = \frac{\frac{\delta_{vertical1} + \delta_{vertical2}}{2} - \frac{\delta_{vertical3} + \delta_{vertical4}}{2}}{L}$$

$$\varepsilon_{vertical} = \frac{\frac{\delta_{lateral3} + \delta_{lateral4}}{2} - \frac{\delta_{lateral1} + \delta_{lateral2}}{2}}{L}$$

HTM of Modular Bearing System (contd.)

- Note we assumed for the straightness we assumed all the errors are acting in the same direction
- Here we assume one set of errors acts up, and the other acts down
- This is very conservative (makes up for other effects we might miss)

Linear Error Plot

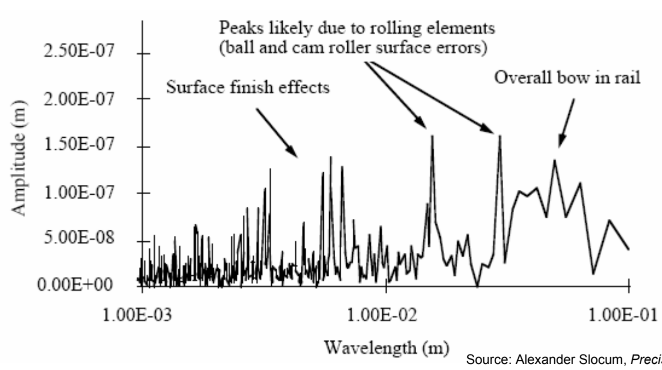


Source: Alexander Slocum, *Precision Machine Design*

Straightness error of a kinematically (statically determined) supported linear axis (5 rolling element bearings on a vee and flat)

Fourier Transformation of Linear Error Plot

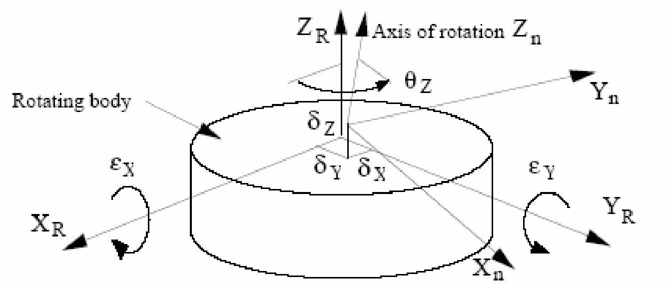
- The Fourier Transformations when plotted as the error amplitude as a function of wavelength is an invaluable diagnostic tool



Source: Alexander Slocum, *Precision Machine Design*

- The wavelength of rolling elements is $2\pi D$

HTM of Rotary Axis



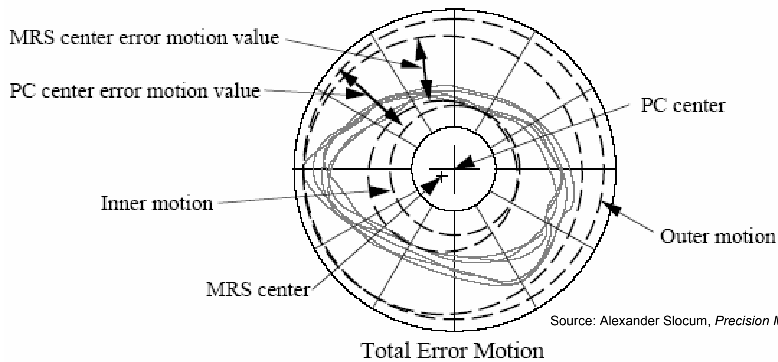
Source: Alexander Slocum, *Precision Machine Design*

$$R_{T_{nerr}} = \begin{bmatrix} \cos\varepsilon_y \cos\theta_z & -\cos\varepsilon_y \sin\theta_z & \sin\varepsilon_y & \delta_x \\ \sin\varepsilon_x \sin\varepsilon_y \cos\theta_z + \cos\varepsilon_x \sin\theta_z & \cos\varepsilon_x \cos\theta_z - \sin\varepsilon_x \sin\varepsilon_y \sin\theta_z & -\sin\varepsilon_x \cos\varepsilon_y & \delta_y \\ -\cos\varepsilon_x \sin\varepsilon_y \cos\theta_z + \sin\varepsilon_x \sin\theta_z & \sin\varepsilon_x \cos\theta_z + \cos\varepsilon_x \sin\varepsilon_y \sin\theta_z & \cos\varepsilon_x \cos\varepsilon_y & \delta_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Rotary Errors

- This general result may also be used for the case of a linear motion carriage if ε_z is substituted for θ_z
- Radial error motion is conservatively estimated to be the ABEC bore tolerance
- Tilt error motion is conservatively estimated to be the ABEC axial tolerance divided by the bearing OD

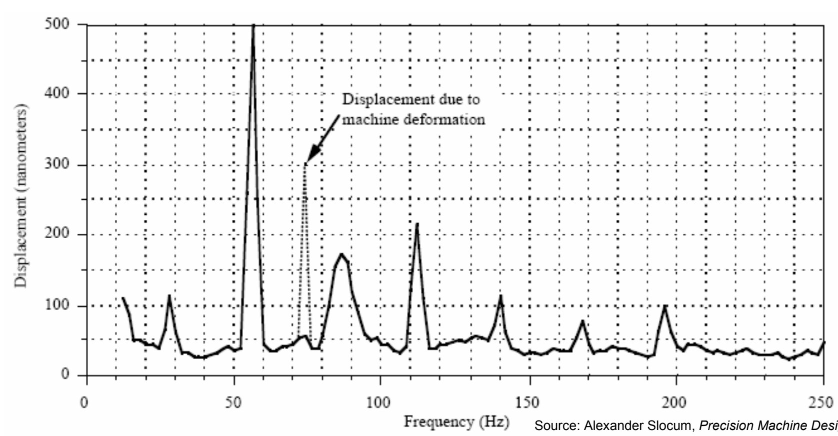
Rotary Error Plot



Source: Alexander Slocum, *Precision Machine Design*

- The average error motion is indicative of the form error that will be imparted to the part when held in a lathe spindle
- The asynchronous error motion is indicative of the surface finish that will be obtained

Fourier Transformation of Radial Error



Source: Alexander Slocum, *Precision Machine Design*

- Spindle speed: 1680 rpm (28 Hz)
- Bearing ID: 75 mm
- Bearing OD: 105 mm
- Number of balls: 20
- Ball diameter: 10 mm
- Contact angle: 15

Combinational Ruler for 3 Common Types

- **Random** - under apparently equal conditions at a given position, errors that do not always have the same value, and can only be expressed statistically
- **Systematic** - which always have the same value and sign at a given position and under given circumstances
  - Generally can be correlated with position along an axis and can be corrected
    - If the relative accompanying random error is small enough
- **Hysteresis** - a systematic error which in this instance is separated out for convenience
  - Usually repeatable, sign depends on the direction of approach, and magnitude partly dependent on the travel
  - May be compensated for if the direction of approach is known and an adequate pre-travel is made