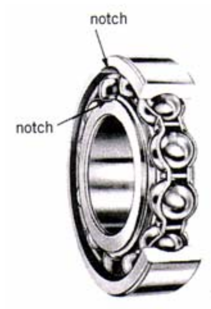
Contact Stresses and Deformations

ME EN 7960 – Precision Machine Design
Topic 7

Curved Surfaces in Contact

- The theoretical contact area of two spheres is a point (= 0-dimensional)
- The theoretical contact area of two parallel cylinders is a line (= 1-dimensional)
  - → As a result, the pressure between two curved surfaces should be infinite
  - → The infinite pressure at the contact should cause immediate yielding of both surfaces
- In reality, a small contact area is being created through elastic deformation, thereby limiting the stresses considerably
- These contact stresses are called Hertz contact stresses

Curved Surfaces in Contact – Examples



Rotary ball bearing

Rotary roller bearing

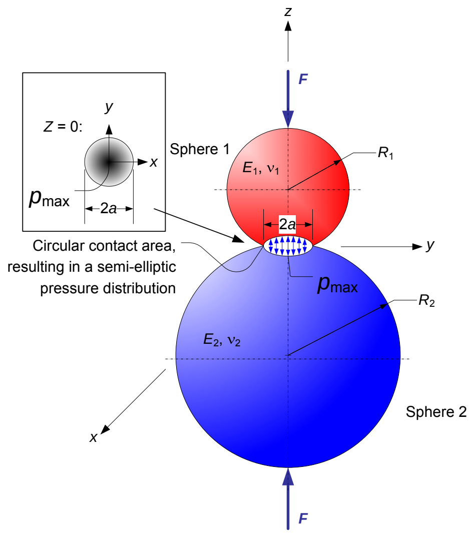
Linear bearings (ball and rollers)

Curved Surfaces in Contact – Examples (contd.)

Ball screw

Gears

Spheres in Contact



The radius of the contact area is given by:

$$a = \sqrt[3]{\frac{3F\left[\frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2}\right]}{4\left(\frac{1}{R_1} + \frac{1}{R_2}\right)}}$$

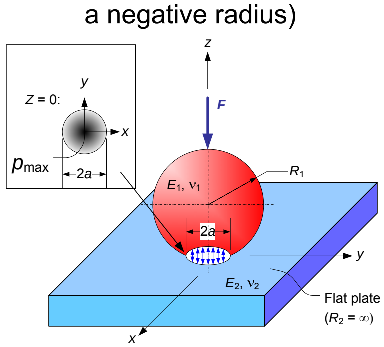
Where E_1 and E_2 are the moduli of elasticity for spheres 1 and 2 and ν_1 and ν_2 are the Poisson's ratios, respectively

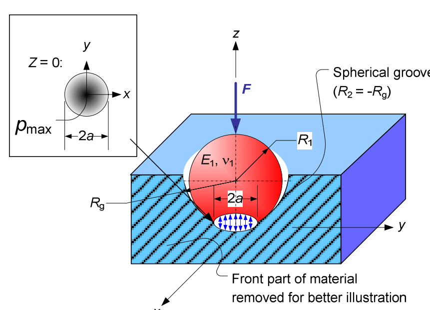
The maximum contact pressure at the center of the circular contact area is:

$$p_{max} = \frac{3F}{2\pi a^2}$$

Spheres in Contact (contd.)

- The equations for two spheres in contact are also valid for:
  - Sphere on a flat plate (a flat plate is a sphere with an infinitely large radius)
  - Sphere in a spherical groove (a spherical groove is a sphere with a negative radius)





Spheres in Contact – Principal Stresses

The principal stresses σ_1, σ_2, and σ_3 are generated on the z-axis:

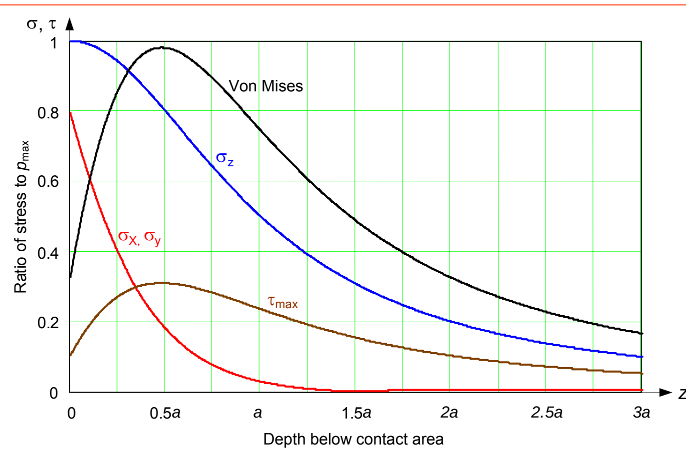
$$\sigma_1 = \sigma_2 = \sigma_x = \sigma_y = -p_{\max}\left[(1+\nu)\left(1-\left|\frac{z}{a}\right|\arctan\left|\frac{a}{z}\right|\right) - \frac{1}{2\left(\frac{z^2}{a^2}+1\right)}\right]$$

$$\sigma_3 = \sigma_z = -p_{\max}\left(\frac{z^2}{a^2}+1\right)^{-1}$$

The principal shear stresses are found as:

$$|\tau_1| = |\tau_2| = \tau_{\max} = \left|\frac{\sigma_1 - \sigma_3}{2}\right| \qquad |\tau_3| = 0$$

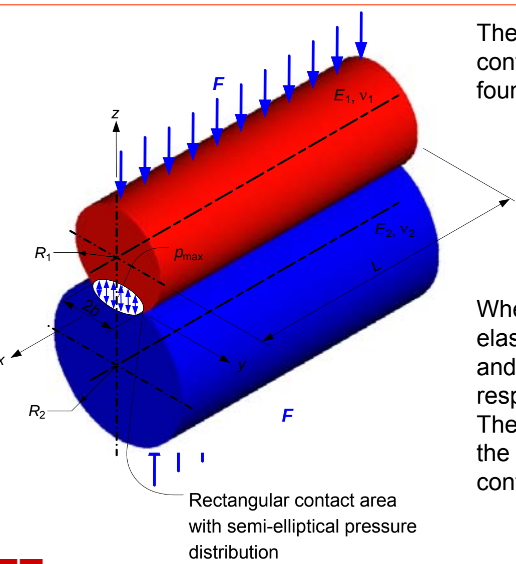
Spheres in Contact – Vertical Stress Distribution at Center of Contact Area



Plot shows material with Poisson's ratio ν = 0.3

- The maximum shear and Von Mises stress are reached below the contact area
- This causes pitting where little pieces of material break out of the surface

Cylinders in Contact



The half-width b of the rectangular contact area of two parallel cylinders is found as:

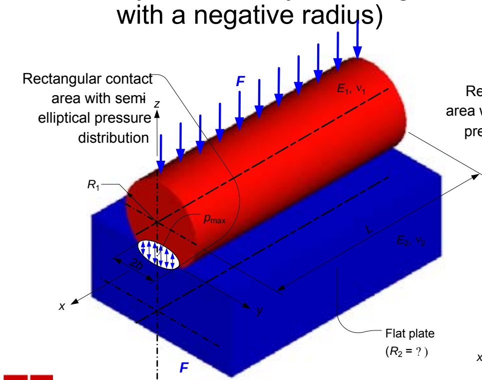
$$b = \sqrt{\frac{4F\left[\frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2}\right]}{\pi L\left(\frac{1}{R_1} + \frac{1}{R_2}\right)}}$$

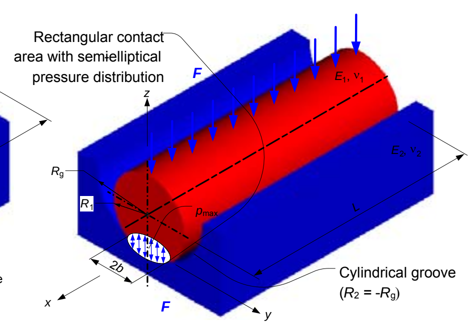
Where E_1 and E_2 are the moduli of elasticity for cylinders 1 and 2 and ν_1 and ν_2 are the Poisson's ratios, respectively. L is the length of contact. The maximum contact pressure along the center line of the rectangular contact area is:

$$p_{\max} = \frac{2F}{\pi bL}$$

Cylinders in Contact (contd.)

- The equations for two cylinders in contact are also valid for:
  - Cylinder on a flat plate (a flat plate is a cylinder with an infinitely large radius)
  - Cylinder in a cylindrical groove (a cylindrical groove is a cylinder with a negative radius)





Cylinders in Contact – Principal Stresses

The principal stresses σ_1, σ_2, and σ_3 are generated on the z-axis:

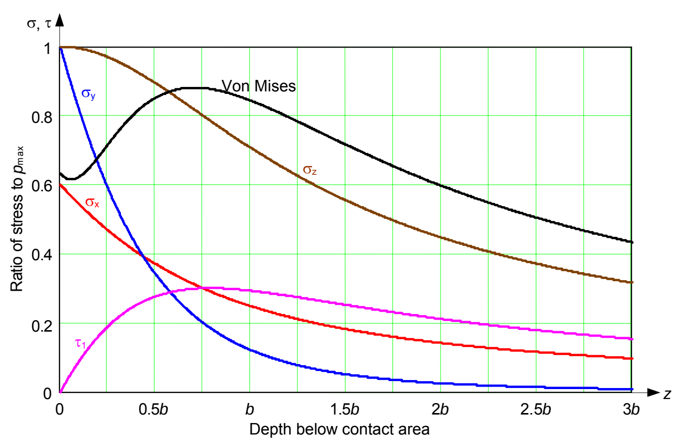
$$\sigma_1 = \sigma_x = -2\nu p_{\max}\left[\sqrt{\frac{z^2}{b^2}+1}-\left|\frac{z}{b}\right|\right]$$

$$\sigma_2 = \sigma_y = -p_{\max}\left[\left(2-\left(\frac{z^2}{b^2}+1\right)^{-1}\right)\sqrt{\frac{z^2}{b^2}+1}-2\left|\frac{z}{b}\right|\right]$$

$$\sigma_3 = \sigma_z = -p_{\max}\left[\sqrt{\frac{z^2}{b^2}+1}\right]^{-1}$$

$$\tau_1 = \left|\frac{\sigma_2-\sigma_3}{2}\right|, \quad \tau_2 = \left|\frac{\sigma_1-\sigma_3}{2}\right|, \quad \tau_3 = \left|\frac{\sigma_1-\sigma_2}{2}\right|$$

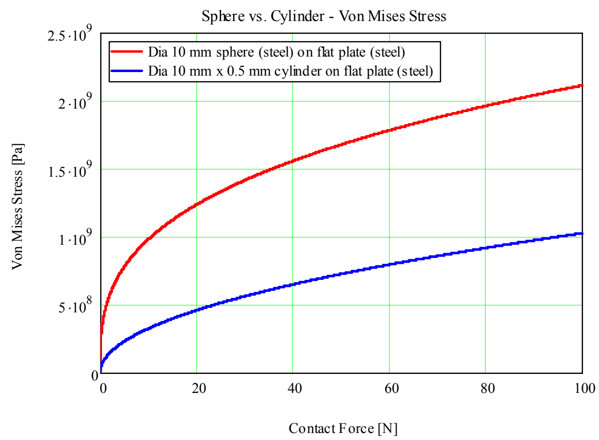
Cylinders in Contact – Vertical Stress Distribution along Centerline of Contact Area



Plot shows material with Poisson's ratio ν = 0.3

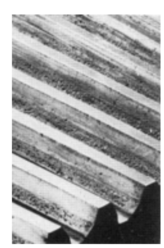
- The maximum shear and Von Mises stress are reached below the contact area
- This causes pitting where little pieces of material break out of the surface

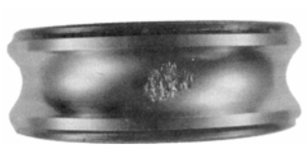
Sphere vs. Cylinder – Von Mises Stress



- The Von Mises stress does not increase linearly with the contact force
- The point contact of a sphere creates significantly larger stresses than the line contact of a cylinder

Effects of Contact Stresses - Fatigue





Elastic Deformation of Curved Surfaces

The displacement of the centers of two spheres is given by:

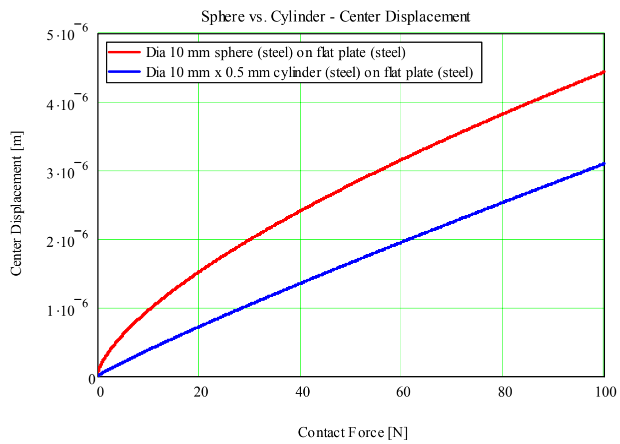
$$\delta_s = 1.04\left[F\left(\frac{1-\nu_1^2}{E_1}+\frac{1-\nu_2^2}{E_2}\right)\right]^{2/3}\left[\frac{1}{2}\left(\frac{1}{R_1}+\frac{1}{R_2}\right)\right]^{1/3}$$

The displacement of the centers of two cylinders is given by:
With $\nu_1 = \nu_2 = \nu$, and $E_1 = E_2 = E$:

$$\delta_c = \frac{2F\left(1-\nu^2\right)}{\pi LE}\left(\frac{2}{3}+\ln\frac{4R_1}{b}+\ln\frac{4R_2}{b}\right)$$

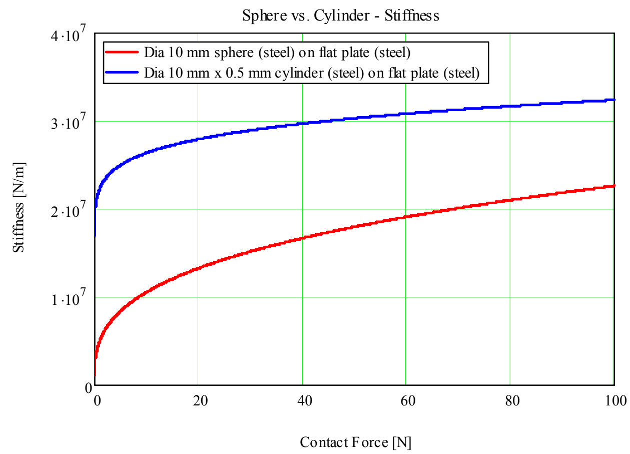
Note that the center displacements are highly nonlinear functions of the load

Sphere vs. Cylinder – Center Displacement



- The point contact of a sphere creates significantly larger center displacements than the line contact of a cylinder

Sphere vs. Cylinder – Stiffness

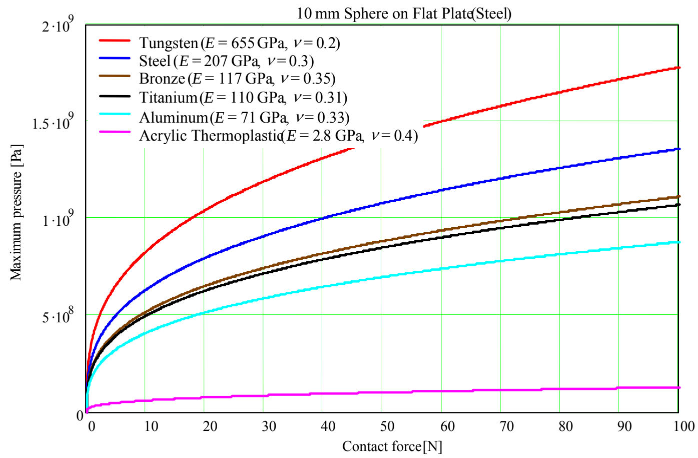


- The point contact of a sphere creates significantly lower stiffness than the line contact of a cylinder

Effects of Material Combinations

- The maximum contact pressure between two curved surfaces depends on:
  - Type of curvature (sphere vs. cylinder)
  - Radius of curvature
  - Magnitude of contact force
  - Elastic modulus and Poisson’s ratio of contact surfaces
- Through careful material pairing, contact stresses may be lowered

Contact Pressure Depending on Material Combination



- Materials with a lower modulus will experience larger deformations, resulting in a lower contact pressure

Center Displacement Depending on Material Combination