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the order of the internal Rossby radius, where the energy is injected by baroclinic instability. (For the inverse cascade to occur, $\sqrt{u/\beta}$ needs to be larger than the scale at which energy is injected.)

Eventually, however, the kinetic energy has to be dissipated by molecular effects at the Kolmogorov microscale η, which is of the order of a few millimeters in the ocean and the atmosphere. A fair hypothesis is that processes such as internal waves drain energy out of the mesoscale eddies, and breaking internal waves generate three-dimensional turbulence that finally cascades energy to molecular scales.

Exercises

1. The Gulf Stream flows northward along the east coast of the United States with a surface current of average magnitude 2 m/s. If the flow is assumed to be in geostrophic balance, find the average slope of the sea surface across the current at a latitude of 45° N. [*Answer*: 2.1 cm per km]

2. A plate containing water ($\nu = 10^{-6}\,\mathrm{m^2/s}$) above it rotates at a rate of 10 revolutions per minute. Find the depth of the Ekman layer, assuming that the flow is laminar.

3. Assume that the atmospheric Ekman layer over the earth's surface at a latitude of 45° N can be approximated by an eddy viscosity of $\nu_v = 10\,\mathrm{m^2/s}$. If the geostrophic velocity above the Ekman layer is 10 m/s, what is the Ekman transport across isobars? [*Answer*: $2203\,\mathrm{m^2/s}$]

4. Find the axis ratio of a hodograph plot for a semidiurnal tide in the middle of the ocean at a latitude of 45° N. Assume that the midocean tides are rotational surface gravity waves of long wavelength and are unaffected by the proximity of coastal boundaries. If the depth of the ocean is 4 km, find the wavelength, the phase velocity, and the group velocity. Note, however, that the wavelength is comparable to the width of the ocean, so that the neglect of coastal boundaries is not very realistic.

5. An internal Kelvin wave on the thermocline of the ocean propagates along the west coast of Australia. The thermocline has a depth of 50 m and has a nearly discontinuous density change of $2\,\mathrm{kg/m^3}$ across it. The layer below the thermocline is deep. At a latitude of 30° S, find the direction and magnitude of the propagation speed and the decay scale perpendicular to the coast.

6. Using the dispersion relation $m^2 = k^2(N^2 - \omega^2)/(\omega^2 - f^2)$ for internal waves, show that the group velocity vector is given by

$$[c_{gx}, c_{gz}] = \frac{(N^2 - f^2)\,km}{(m^2 + k^2)^{3/2}(m^2 f^2 + k^2 N^2)^{1/2}}[m, -k]$$