

Some Important Thermodynamic Tools for Environmental Fluid Mechanics

Potential Temperature & Virtual Potential Temperature Calculations

1. Ideal Gas Law/Equation of State

$PV = N\mathfrak{R}T$ for a mole based system, where P is the pressure (N/m^2), V is the volume of the system (m^3), \mathfrak{R} is the universal gas constant ($8314 \text{ J}/(\text{kg}\cdot\text{mol}\cdot\text{K})$), N is the number of moles of gas in the system (kg), T is the temperature of the system (Kelvin).

$P = \rho RT$ for a mass based system: $R = \mathfrak{R} / M$ where M is the molecular weight, ρ is the density and R is the gas constant per unit mass. For air $M = 28.94 \text{ kg/mol}$ and $R = 287.3 \text{ J/kg}\cdot\text{K}$.

2. Potential Temperature (θ)

a. Static Stability

Potential temperature is the temperature that a parcel of air at pressure P and temperature T would have if it were adiabatically brought to a reference pressure P_o . The potential temperature helps determine the buoyancy of a dry displaced fluid parcel relative to its surroundings. For a static fluid, when heavier fluid lies below lighter fluid, we say that it is stably stratified (since tilting of a density surface will result in a restoring force). When lighter fluid lies below heavier fluid the equilibrium is unstable and small tilting of the density surface will grow and lead to convective motions. From the equation of state, for an adiabatic system, we know that a change in pressure results in a change in Temperature. This change must be accounted for when comparing displaced fluid parcels.

b. The 1st Law of Thermodynamics

Using the 1st Law of thermodynamics for an isentropic process (adiabatic and reversible), we can determine the potential temperature of an air parcel. The potential temperature is given by:

$$\theta = T \left(\frac{P_o}{P} \right)^\gamma$$

where P_o is a reference pressure (often taken as sea level reference - 1000mb or 100kPa), P is the pressure measured at the same height as the temperature T (Kelvin). The ratio, $\gamma = R / C_p = 0.286$, where R is the gas constant per unit mass, C_p ($\sim 1.006 \text{ kJ}/\text{kg}\cdot\text{K}$ for air at sea level and 300K) is the specific heat at constant pressure.

If the Pressure at height z , is unknown it is common to use the following approximation:

$$\theta(z) \cong T(z) + \Gamma z$$

and

$$\Gamma = \left(\frac{g}{C_p} \right) = 0.0098 K / m \text{ is the dry adiabatic lapse rate and } z \text{ is the height in meters}$$

above the level at which P_0 is measured. For example, if $P_0 \sim 1000\text{mb}$, is chosen as sea level ($z=0$), then z is just the height of the measurement above sea level.

3. Virtual Potential Temperature (θ_v)

The virtual temperature is the temperature of dry air that would have the same density and pressure as the moist air. If the atmosphere is NOT dry, we must also consider moisture content when comparing the buoyancy of fluid parcels. As a result of water vapor having a smaller molecular mass than dry air, the density of moist air is LESS than that of dry air. As a result, the more water vapor that is in the atmosphere the lower the density of the air. This is typically done using the virtual potential temperature. For unsaturated air:

e_{so} = reference saturation vapor pressure (e_s at a certain temp, usually 273.15 Kelvin)
= 6.11 hPa

T_0 = reference temperature (273.15 Kelvin)

T_d = dew point temperature (Kelvin)

T = temperature (Kelvin)

l_v = latent heat of vaporization of water (2.5×10^6 J/kg)

R_v = gas constant for water vapor (461.5 J- K / kg)

P = pressure (mb) (Note 1mb = 1hPa)

$$e = e_{so} \exp\left(\frac{l_v}{R_v} \left(\frac{1}{T_0} - \frac{1}{T_d}\right)\right) \quad \text{vapor pressure (hPa)}$$

$$e_s = e_{so} \exp\left(\frac{l_v}{R_v} \left(\frac{1}{T_0} - \frac{1}{T}\right)\right) \quad \text{saturated vapor pressure (hPa)}$$

$$RH = 100 * \left(\frac{e}{e_s}\right) \quad \text{relative humidity (\%)}$$

$$q = 0.622 \left(\frac{e}{P}\right) \quad \text{specific humidity}$$

$$\theta_v = T \left(1.0 + 0.61q\right) \left(\frac{1000.}{P}\right)^\gamma \quad \text{or} \quad \theta_v = \theta \left(1.0 + 0.61q\right)$$

Speed of sound in a gas:

$$c = \sqrt{\alpha RT}$$

since R is really a function of humidity, the virtual potential temperature is:

$$T_v = \frac{c^2}{R\alpha}$$