ME 2450
Numerical Methods

Exam 1 Review Notes

You are allowed 1 side of an 8 ½ x 11 sheet of paper for notes

Exam: Monday, March 6, 2006

- Exam 1 will cover Chapters 1-9 through Gauss-Jordan Method
- Students are allowed 1 side of an 8 1/2 x 11 sheet of paper for notes

CH. 1 Mathematical Modeling

1. Know concepts:

Dependent Variable = f (indep. variables, system parameters, forcing functions)

$$v = \frac{gm}{c}\left(1 - e^{-c/mt}\right)$$

2. Conservation Laws

CH. 2 Structured Programming

- Pseudocode
- Flowcharts

Sequence, selection, repetition

CH. 3 Error and Error Estimation

- Significant figures
- Accuracy and Precision

1. Truncation Error – result from an approximation of an exact mathematical procedure (I.e., truncating a Taylor Series expansion)

2. R.O. Error – results from having a limited number of significant figures
   - Examples?
   - Error definitions
     - Normalized
     - Absolute
     - Relative
     - True
3. Integer and Floating Point representation
   - How do we represent fractional quanitites?
   - Floating point operations
     - Addition, subtraction, multiplication (normalizing and chopping)
   - Problems:
     - smearing
     - Subtractive cancellation
     - Quantitized errors
     - Limited precision
     - Large numbers of computations

CH. 4 Taylor Series & Truncation Error

$$f(x_{i+1}) = f(x_i) + f'(x_i)h + \frac{f''(x_i)h^2}{2!} + \frac{f'''(x_i)h^3}{3!} + \frac{f^n(x_i)h^n}{n!} + R_n$$

$$R_n = \frac{f^{n+1}(\xi)h^{n+1}}{(n+1)!}$$

$$R_n = O\left(h^{n+1}\right)$$

1. Estimating truncation errors with Taylor series
2. Estimating derivatives with Taylor Series
   - Backward, Forward and central differencing
   - How did we derive them?
3. Error Propagation
   a. Functions of one variable f(x), estimating the error in the function.

$$\Delta f\left(\tilde{x}\right) = \left| f\left(x\right) - f\left(\tilde{x}\right)\right| = \left| f'\left(\tilde{x}\right)\right| \Delta\tilde{x}$$

Where x is the true value $\tilde{x}$ and an approximate value

   b. Functions of multiple variables

$$\Delta f\left(\tilde{x}, \tilde{y}, \tilde{z}\right) = \left|\frac{\partial f}{\partial x}\right| \Delta\tilde{x} + \left|\frac{\partial f}{\partial y}\right| \Delta\tilde{y} + \left|\frac{\partial f}{\partial z}\right| \Delta\tilde{z}$$

Ways to reduce RO and Truncation Error?

CH. 5&6 Roots of Equations

1. Bracketing Methods – Characteristics?
   a) Graphical
   b) Bisection

$$n = \frac{\log\left(\Delta x^o / E_{a,d}\right)}{\log 2}$$

   c) False Position Method

2. Open Methods - Characteristics?
   a) Fixed Point Iteration (Successive Iteration)
   - How do we formulate
   - *Linear Convergence* – the true percent relative error of each iteration is approximately proportional to the error of the previous iteration

   b) Newton-Raphson
   - *Quadratic Convergence* – the error is proportional to the square of the previous error

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

$$E_{t,i+1} = O\left(E^2{}_{t,i}\right)$$

   - Problems with NR?

   c) Secant Method – Finite Difference Approximation for Newton-Raphson method

3. Multiple Roots – functions that are tangent to the x-axis
    - Modified Newton-Raphson method

CH. 7 Roots of Polynomials

1. Mueller's Method - Involved fitting a 2nd order polynomial to our function and using the intersection with the x-axis as our root estimate
    - Allowed for Complex Roots

CH. 8 Engineering Applications

See Notes

Systems of Linear Algebraic Equations

CH. 9 Gauss Elimination $[A]\{x\}=\{b\}$

1. *Graphical Method*
2. *Cramer's Rule*
3. *Naïve Gauss Elimination*
   - During the elimination & back substitution steps division by zero can occur.
   - Round-Off Error – "Rule of Thumb" should be ok with < 100 equations
   - Ill-conditioned Systems – small changes in the coeff. Matrix result in large changes in the solution. (A wide range of answers can satisfy the equations) an ill conditioned system has D~0
   - Singular Systems – no solution or infinite solutions
     - Scaling –
       - Why: it is difficult to determine how close to zero the determinant must be to indicate ill-conditioning. (Also helps with pivoting decisions)
       - How: scale each equation so the maximum row element is unity.
     - Fast Determinant evaluation using Gauss Elim

$$D = a_{11}a'_{22}\,a''_{33}\ldots a_{nn}^{n-1}$$

       - If D=0 → Singular system

Systems of Linear Algebraic Equations

- Pivoting –
  - Why: prevent division by zero or near zero element
  - How: Switch pivot row with row containing largest coef. In the element below pivot element.

4. *Gauss – Jordan Method*: Unknown is eliminated from all equations, not just subsequent rows, all rows are normalized by dividing by the pivot element:

$$[A]\{x\} = \{b\} \Rightarrow [I]\{x\} = \{b_{\text{mod}}\}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} b'_1 \\ b'_2 \\ b'_3 \end{Bmatrix}$$