Introduction to Numerical Methods

ME 2450

Spring 2006

Lecture Objectives

- To understand what Numerical Methods are and why we (as Engineers) are interested in them
- To understand the basic concepts of mathematical modeling

Numerical Methods

- Numerical Methods are mathematically based techniques (*Tools*) that utilize computers to allow us to solve Engineering Problems that are not easily solved or even impossible to solve by analytical means.
- Usually involve large numbers of tedious arithmetic operations.

Chapter 1 – Mathematical Modeling

1. Modeling the Physics

   Mathematical Model: Equation or formulation that expresses the essential features of a physical system or process in mathematical terms (*Governing Equations*)
2. Numerical Method Selection
   a. Accuracy
   b. Implementation time
   c. Stability
   d. Ease of implementation
3. Program/debug
4. Interpret Results

Mathematical Modeling

Dependent Variable = f (indep. variables, system parameters, forcing functions)

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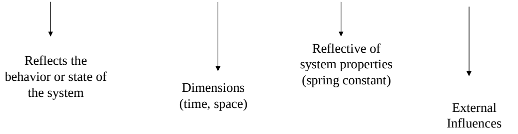

Mathematical Modeling

Dependent Variable = f (indep. variables, system parameters, forcing functions)

Reflects the behavior or state of the system

Dimensions (time, space)

Reflective of system properties (spring constant)

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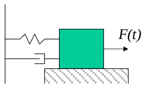
Simple Example: Mass-Spring-Damper System

$$m\ddot{x}+c\dot{x}+kx=F(t)$$

$$x=f(t,m,c,k,F)$$



Mathematical Modeling

Other Examples:

- Navier-Stokes Equation in Fluid Mechanics
- Euler’s Equations of Motion
- Energy Equation
- Conservation of Mass

- Many others

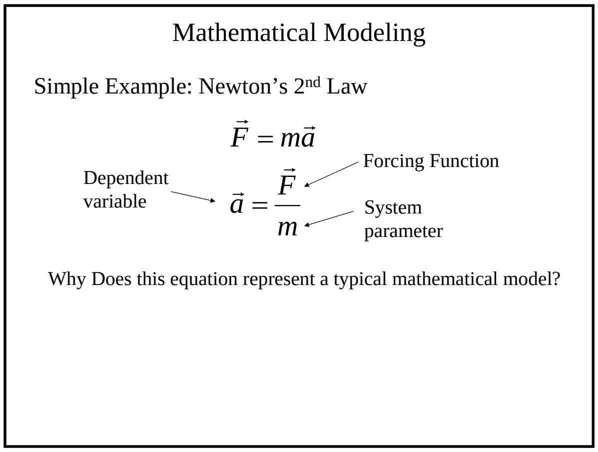
Mathematical Modeling

Simple Example: Newton’s 2nd Law

$$\vec{F}=m\vec{a}$$

Rearrange:

$$\vec{a}=\frac{\vec{F}}{m}$$



Mathematical Modeling

Simple Example: Newton's 2nd Law

$$\vec{F} = m\vec{a}$$

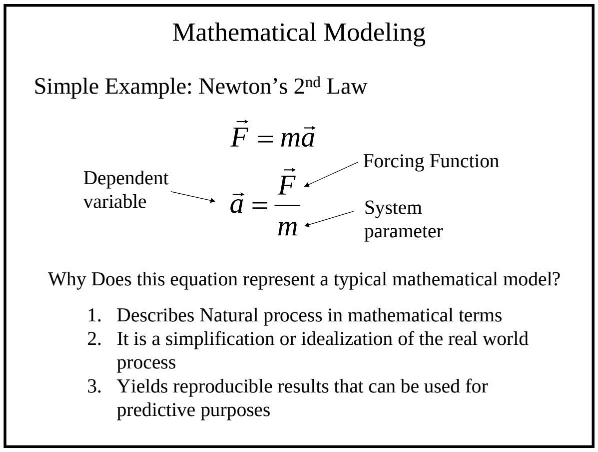
Dependent variable

$$\vec{a} = \frac{\vec{F}}{m}$$

Forcing Function

System parameter

Why Does this equation represent a typical mathematical model?



Mathematical Modeling

Simple Example: Newton's 2nd Law

$$\vec{F} = m\vec{a}$$

Dependent variable

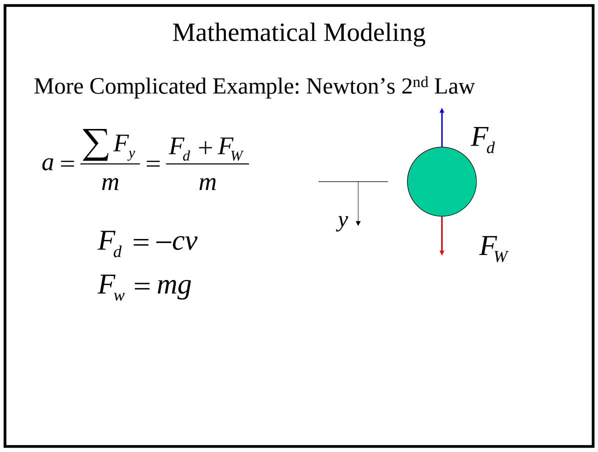
$$\vec{a} = \frac{\vec{F}}{m}$$

Forcing Function

System parameter

Why Does this equation represent a typical mathematical model?

1. Describes Natural process in mathematical terms
2. It is a simplification or idealization of the real world process
3. Yields reproducible results that can be used for predictive purposes



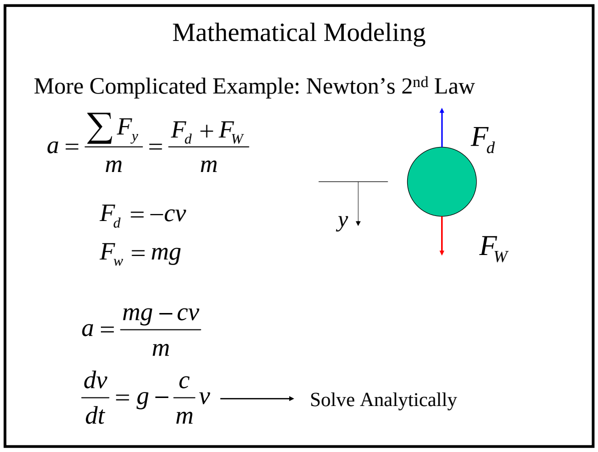
Mathematical Modeling

More Complicated Example: Newton's 2nd Law

$$a = \frac{\sum F_y}{m} = \frac{F_d + F_W}{m}$$

$$F_d = -cv$$

$$F_w = mg$$



Mathematical Modeling

More Complicated Example: Newton's 2nd Law

$$a = \frac{\sum F_y}{m} = \frac{F_d + F_W}{m}$$

$$F_d = -cv$$
$$F_w = mg$$

$$a = \frac{mg - cv}{m}$$

$$\frac{dv}{dt} = g - \frac{c}{m} v \xrightarrow{\qquad} \text{Solve Analytically}$$