ME2450 – Numerical Methods
Differential Equation Classification:

There are much more rigorous mathematical definitions than those given below however, these examples should help you understand the concept of differential equation classifications.

Differential Equations – These are problems that require the determination of a function satisfying an equation containing one or more derivatives of the unknown function.

Ordinary Differential Equations – the unknown function in the equation only depends on <u>one</u> independent variable; as a result only ordinary derivatives appear in the equation.

Partial Differential Equations – the unknown function depends on <u>more than one</u> independent variable; as a result partial derivatives appear in the equation.

Order of Differential Equations – The order of a differential equation (partial or ordinary) is the highest derivative that appears in the equation.

Linearity of Differential Equations – A differential equation is linear if the dependant variable and all of its derivatives appear in a linear fashion (i.e., they are not multiplied together or squared for example or they are not part of transcendental functions such as sins, cosines, exponentials, etc.).

ODE Examples where *y* is the dependant variable and x is the independent variable:

1. $y''+y=0$ Linear
2. $yy''+y=0$ Non-linear
3. $xy''+y=0$ Linear
4. $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}=\sin x$ Linear
5. $\frac{d^2y}{dx^2}+\sin y=0$ Non-Linear

Equation 2 is non-linear because of the *yy''* product. Equation 5 is non-linear because of the sin(*y*) term.

PDE Examples where *u* is the dependant variable and *x, y* and *t* are independent variables:

6. $\frac{\partial^2 u}{\partial x^2}+\sin y=0$ Linear
7. $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}+u^2=0$ Non-Linear
8. $u\frac{\partial^2 u}{\partial x^2}+u=0$ Non-Linear
9. $\frac{\partial^2 u}{\partial t^2}=e^{-t}\frac{\partial^2 u}{\partial x^2}+\sin t$ Linear

Equation 7 is nonlinear because of the u^2 term. Equation 8 is non-linear because of the $u\frac{\partial^2 u}{\partial x^2}$ term.

Homogeneity of Differential Equations – Given the general partial differential equation:

$$A\frac{d^2u}{dx^2}+B\frac{d^2u}{dy^2}+C\frac{du}{dx}+D\frac{du}{dy}+E\frac{du}{dy}+Fu=G(x,y)$$

where *A*,*B*,*C*,*D* and *E* are coefficients, if *G(x,y) = 0* the equation is said to be homogeneous.

ODE Examples where *y* is the dependant variable and *x* is the independent variable:

1. $y''+y=0$ homogeneous
2. $x^2y''+xy'+x^2=0$ homogeneous
3. $y''+y'+y=\sin(t)$ non-homogeneous
4. $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}=x+e^{-x}$ non-homogen

More examples:

Example 1: Equation governing the motion of a pendulum.

$$\frac{d^2\theta}{dt^2}+\frac{g}{l}\sin\theta=0 \qquad (1)$$

$$\frac{d^2\theta}{dt^2}+\frac{g}{l}\theta=0 \qquad (2)$$

Equations (1) & (2) are both 2^{nd} order, homogeneous, ODEs. Equation (1) is non-linear because of the sine function while equation (2) is linear.

$3x^2y''+2\ln(x)y'+e^xy=3x\sin(x)$: 2^{nd} order, non-homogeneous, linear ODE

$y'''+y'+e^y=3x\sin(x)$: 3^{rd} order, non-homogeneous, non-linear ODE

$\frac{\partial^4 u}{\partial t^4}=e^{-t}\frac{\partial^2 u}{\partial x^2}+\sin t$: 4^{th} order, non-homogeneous, linear PDE