25.18 Solutions:
If you solved the problem using Runge-Kutta you should get:
The 4th-order RK method with h = 0.5 gives

x	y	k_1	y_m	k_2	y_m	k_3	y_e	k_4	ϕ
0	1	-1.1	0.725	-0.75219	0.811953	-0.8424	0.578799	-0.49198	-0.79686
0.5	0.60157	-0.51133	0.473737	-0.25463	0.537912	-0.28913	0.457006	-0.0457	-0.27409
1	0.464524	-0.04645	0.452911	0.209471	0.516892	0.239062	0.584055	0.671663	0.253713
1.5	0.59138	0.680087	0.761402	1.494252	0.964943	1.893701	1.538231	4.460869	1.986144
2	1.584452	4.594911	2.73318	10.83023	4.292008	17.00708	10.08799	51.95317	18.70378

If you solved the problem 25.1 using
Euler's method with h = 0.5

x	y	dy/dx
0	1	-1.1
0.5	0.45	-0.3825
1	0.25875	-0.02588
1.5	0.245813	0.282684
2	0.387155	1.122749

27.2 Solutions: Re-express the second-order equation as a pair of ODEs:

$$\frac{dT}{dx} = z$$

$$\frac{dz}{dx} = 0.15T$$

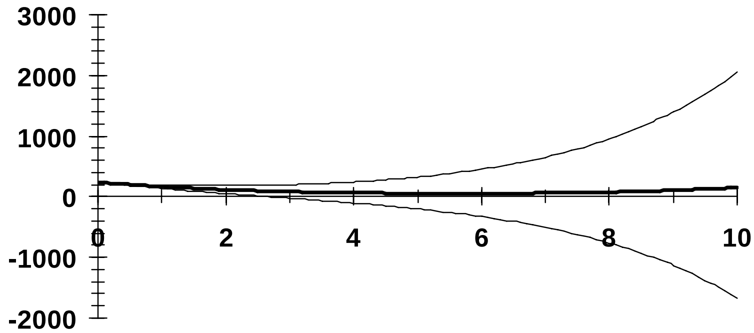
The solution was using the Heun method (without iteration) with a step-size of 0.01. An initial condition of $z = -120$ was chosen for the first shot. The first few calculation results are shown below.

x	T	z	k_{11}	k_{12}	T_{end}	z_{end}	k_{21}	k_{22}	ϕ_1	ϕ_2
0	240.000	-120.000	-120.000	36.000	228.000	-116.400	-116.400	34.200	-118.200	35.100
0.1	228.180	-116.490	-116.490	34.227	216.531	-113.067	-113.067	32.480	-114.779	33.353
0.2	216.702	-113.155	-113.155	32.505	205.387	-109.904	-109.904	30.808	-111.529	31.657
0.3	205.549	-109.989	-109.989	30.832	194.550	-106.906	-106.906	29.183	-108.447	30.007
0.4	194.704	-106.988	-106.988	29.206	184.006	-104.068	-104.068	27.601	-105.528	28.403
0.5	184.152	-104.148	-104.148	27.623	173.737	-101.386	-101.386	26.061	-102.767	26.842

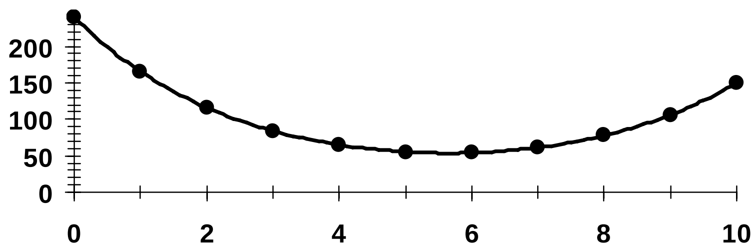
The resulting value at $x = 10$ was $T(10) = -1671.817$. A second shot using an initial condition of z(0) = –60 was attempted with the result at $x = 10$ of $T(10) = 2047.766$. These values can then be used to derive the correct initial condition,

$$z(0) = -120 + \frac{-60+120}{2047.766-(-1671.817)}(150-(-1671.817)) = -90.6126$$

The resulting fit, along with the two “shots” are displayed below:



The final shot along with the analytical solution (displayed as filled circles) shows close agreement:



27.3 Solution

A centered finite difference can be substituted for the second derivative to give,

$$\frac{T_{i+1} - 2T_i + T_{i-1}}{h^2} - 0.15T_i = 0$$

or for h = 1,

$$-T_{i-1} + 2.15T_i - T_{i+1} = 0$$

The first node would be

$$2.15T_1 - T_2 = 240$$

and the last node would be

$$-T_9 + 2.15T_{10} = 150$$

The tridiagonal system can be solved with the Thomas algorithm or Gauss-Seidel for (the analytical solution is also included)

x	*T*	Analytical
0	240	240
1	165.7573	165.3290
2	116.3782	115.7689
3	84.4558	83.7924
4	65.2018	64.5425
5	55.7281	55.0957
6	54.6136	54.0171
7	61.6911	61.1428
8	78.0223	77.5552
9	106.0569	105.7469
10	150	150

The following plot of the results (with the analytical shown as filled circles) indicates close agreement.