Control Volume Forms of the Fundamental Laws

1. Conservation of Mass
2. Conservation of Linear Momentum
3. Conservation of Angular Momentum (moment of momentum)
4. Conservation of Energy

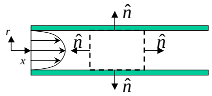
Examples of Integral Equations

- Power requirements for Hydraulic lifts
- Loads on an aircraft
- Down force on a race car
- Horsepower to pump liquids through a pipeline
- Gas pressure in an Internal Combustion

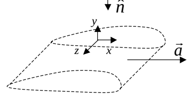
Approach to Problem Solving

1. Determine if Control Volume Analysis is appropriate (vs. differential for example)
2. Examine the behavior of Control Volume

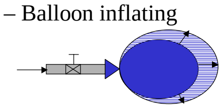
A. Fixed – Pipe flow



B. Moving – Airplane flying (Non-inertial)



C. Elastic – Balloon inflating



Derivation of the Reynolds Transport Theorem

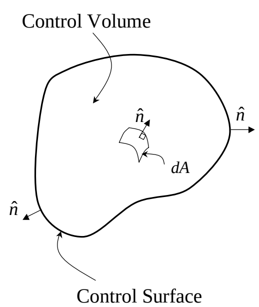
What: A formal mathematical expression which allows the time rate of change of an extensive property for a given quantity of mass, a system, to be expressed in terms of quantities related to a specific region of space, a control volume.

Why: All conservation law are written for system. We need to a way to express the time rate of change for a system in terms of a control volume. That is, since it is difficult to identify and follow the same mass of fluid, we need an Eularian description.

Definitions

- System - A fixed collection of mass particles
- Control Volume – defined region in space
- Extensive Property (N) – property of the system that depends on mass ("Stuff").
  - Momentum, internal energy, entropy
- Intensive Property (η) – property of the system that is independent of mass
  - Temperature, velocity, specific energy

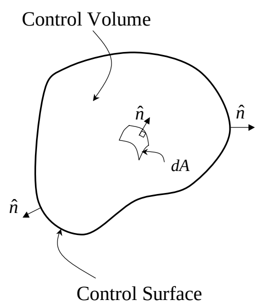
Basic Concepts



Objective:
Describe the rate at which an integral quantity associated with the system is changing as the flow passes into and out of the Control Volume.

Note: Fluid DOES NOT pass in and out of a system.

Basic Concepts



Integral Properties:

Mass flow rate:

$$\dot{m} = \int_A \rho \vec{V} \bullet \hat{n} dA$$

Mass:

$$m = \int_\forall \rho d\forall$$

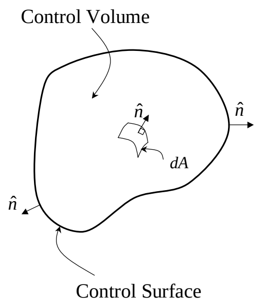
Drag Force:

$$F_d = \int_A \vec{\tau} \bullet \hat{s} dA$$

Kinetic Energy:

$$KE = \int_\forall \rho \frac{V^2}{2} d\forall$$

Extensive/Intensive Properties



$$N = \int_\forall \rho \eta d\forall$$

$$\eta = \frac{N}{m}$$

Momentum:

$$N = m\vec{V}$$

$$\eta = \frac{m\vec{V}}{m} = \vec{V}$$

Mass:

$$N = m$$

$$\eta = \frac{m}{m} = 1$$

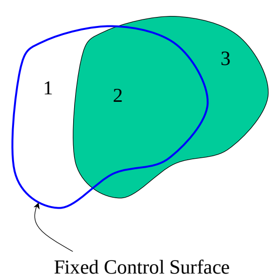
System

$$N_{sys} = \int_{\forall_{sys}} \rho \eta d\forall$$

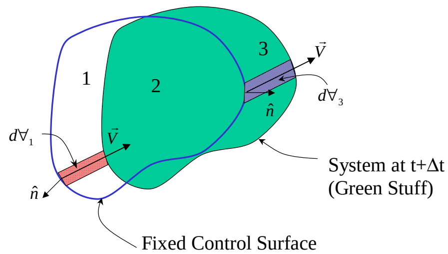
Control Volume

$$N_{C.V.} = \int_{C.V.} \rho \eta d\forall$$

Reynolds Transport Theorem for a Non-Deformable Control Volume



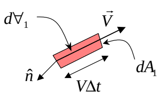
Reynolds Transport Theorem for a Non-Deformable Control Volume



$$\frac{DN_{sys}}{Dt} = \frac{D}{Dt}\int_{sys} \eta\rho\, d\forall$$

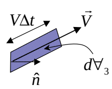
Finding the Size of the "Sweeping Volume"

Elemental volume from 1 - Incoming



$$d\forall_1 = -(V \bullet \hat{n})\Delta t\, dA_1$$

Elemental volume from 2 - Outgoing



$$d\forall_2 = (V \bullet \hat{n})\Delta t\, dA_2$$

$(V \bullet \hat{n})\Delta t$ = a length normal to the CS