ME3700 Final Exam - Equations Sheets

$$\frac{dp}{dz} = -\rho g$$

$$\vec{F}_R = \int_A - p(\hat{n})dA$$

$$y'\vec{F}_R = \int_A yp(\hat{n})dA$$

$$\rho\frac{D\vec{V}}{Dt} = -\vec{\nabla}P + \rho\vec{g} + \mu\nabla^2\vec{V}$$

$$\vec{\nabla}\bullet\vec{V} = 0$$

$$\frac{dP}{dn} = \rho\frac{V^2}{R}$$

$$\frac{P}{\rho} + \frac{V^2}{2} + gz = Const.$$

$$\frac{DM_{sys}}{Dt} = 0 = \frac{\partial}{\partial t}\int_{CV}\rho dV + \int_{CS}\rho(\vec{V}\cdot\hat{n})dA$$

$$P = \rho RT$$

$$\vec{\omega} = \vec{\nabla}\times\vec{V}$$

$$\vec{\nabla}\bullet\vec{\omega} = 0$$

$$\Gamma = \oint_C \vec{V}\bullet d\vec{l} = \int_A \omega\bullet\hat{n}dA$$

$$\delta^* = \int_0^\delta \frac{u}{U_\infty}\left(1 - \frac{u}{U_\infty}\right)dy$$

$$\theta = \int_0^\delta \frac{u}{U_\infty}\left(1 - \frac{u}{U_\infty}\right)dy$$

$$\sum\vec{F} = \frac{D\overrightarrow{Mom}_{sys}}{Dt} = \frac{\partial}{\partial t}\int_{CV}\rho\vec{V}dV + \int_{CS}\rho\vec{V}(\vec{V}\cdot\hat{n})dA$$

$$\frac{DE_{sys}}{Dt} = \dot{Q} - \dot{W} = \frac{\partial}{\partial t}\int_{CV}\rho e dV + \int_{CS}\rho e(\vec{V}\cdot\hat{n})dA$$

$$e = \tilde{u} + \frac{V^2}{2} + gz$$

$$-\frac{W_{shaft}}{\dot{m}g} = -\left(\frac{P}{\rho} + \frac{V^2}{2} + gz\right)_1 + \left(\frac{P}{\rho} + \frac{V^2}{2} + gz\right)_2 + H_L$$

$$H_L = K\frac{V^2}{2g}$$

$$H_L = f\frac{L}{D}\frac{V^2}{2g}$$

$$\tau_w = \rho U_\infty^2\frac{d\theta}{dx}$$

$$Fr = \frac{U}{\sqrt{gL}}$$

$$St = \frac{fD}{U}$$

$$M = \frac{U}{c}$$

$$C_D = \frac{F_D}{\frac{1}{2}\rho V^2 A}$$

$$C_L = \frac{F_L}{\frac{1}{2}\rho V^2 A}$$

Compressible Flow Equations:

$$c_p = \frac{kR}{k-1}$$

$$c_v = \frac{R}{k-1}$$

$$k = \frac{c_p}{c_v}$$

$$c = \sqrt{kRT}$$

$$V_{\max} = \sqrt{2c_p T_o}$$

Stagnation Values

$$\frac{T_o}{T} = \frac{k-1}{2} M^2 + 1$$

$$\frac{p_o}{p} = \left(\frac{T_o}{T}\right)^{\frac{k}{k-1}} = \left(\frac{k-1}{2} M^2 + 1\right)^{\frac{k}{k-1}}$$

$$\frac{\rho_o}{\rho} = \left(\frac{T_o}{T}\right)^{\frac{1}{k-1}} = \left(\frac{k-1}{2} M^2 + 1\right)^{\frac{1}{k-1}}$$

Area Pressure Relationship

$$\frac{dA}{A} = \frac{dp}{\rho V^2}\left(1 - M^2\right)$$

Area Velocity Relationship

$$\frac{dA}{A} = \frac{dV}{V}\left(M^2 - 1\right)$$

Continuity Equation

$$0 = \frac{dA}{A} + \frac{d\rho}{\rho} + \frac{dV}{V}$$

$$\dot{m} = p_o \sqrt{\frac{k}{RT_o}} MA \left(1 + \frac{k-1}{2} M^2\right)^{\frac{k+1}{2(1-k)}}$$

Mach Cone Angle

$$\sin\alpha = \frac{V}{c} = \frac{1}{M}$$

Gibb's Equation

$$s_2 - s_1 = c_p \ln\frac{T_2}{T_1} - R\ln\frac{p_2}{p_1}$$

$$\frac{p_2}{p_1} = \left(\frac{T_2}{T_1}\right)^{\frac{k}{k-1}} = \left(\frac{\rho_2}{\rho_1}\right)^k$$

Isentropic Energy Equations

$$\frac{V_2^2}{2} + h_2 = \frac{V_1^2}{2} + h_1$$

$$\frac{V_2^2}{2} + c_p T_2 = \frac{V_1^2}{2} + c_p T_1$$

Critical Values

$$\frac{p^*}{p_o} = \left(\frac{2}{k+1}\right)^{\frac{k}{k-1}}$$

$$\frac{\rho^*}{\rho_o} = \left(\frac{2}{k+1}\right)^{\frac{1}{k-1}}$$

$$\frac{c^*}{c_o} = \left(\frac{2}{k+1}\right)^{\frac{1}{2}}$$

$$\frac{T^*}{T_o} = \left(\frac{2}{k+1}\right)$$

$$\frac{A}{A^*} = \frac{1}{M}\left(\frac{2 + (k-1)M^2}{k+1}\right)^{\frac{k+1}{2(k-1)}}$$