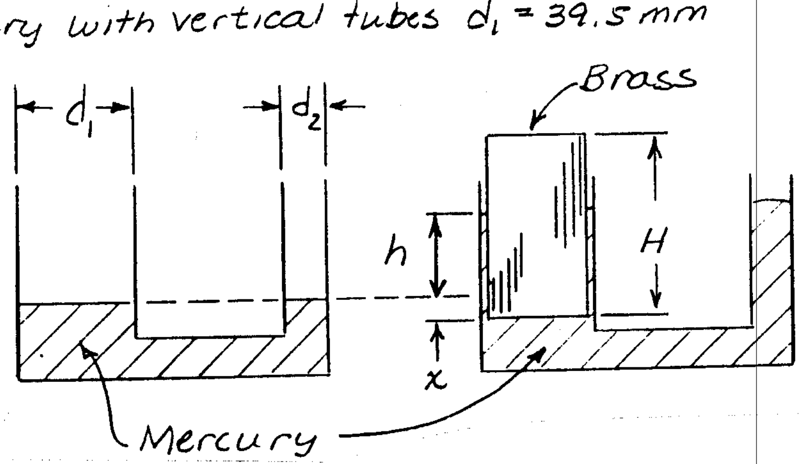
Problem 3.14

Given: Container of mercury with vertical tubes $d_1 = 39.5\ mm$ and $d_2 = 12.7\ mm$.

Brass cylinder with $D = 37.5\ mm$ and $H = 76.2\ mm$ is introduced into larger tube, where it floats.

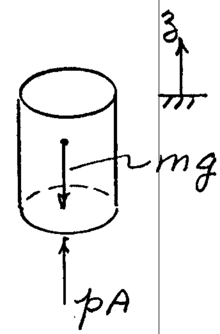


Find: (a) Pressure on bottom of cylinder.
(b) New equilibrium level, h, of mercury.

Solution: Analyze free-body diagram of cylinder, apply hydrostatics.

Computing equations: $\Sigma F_z = 0$; $\frac{dp}{dz} = -\rho g$; $\rho = SG\, \rho_{H_2O}$

Assumptions: (1) Static liquid
(2) Incompressible liquid



For the cylinder $\Sigma F_z = p \frac{\pi D^2}{4} - \rho_{brass}\, g \frac{\pi D^2}{4} H = 0$

Thus $p = \rho_{brass}\, g H = SG_{brass}\, \rho_{H_2O}\, g H$

From Table A.1, $SG_{brass} = 8.55$ at 20°C, so

$$p = 8.55 \times 1000 \frac{kg}{m^3} \times 9.81 \frac{m}{s^2} \times 0.0762\ m \times \frac{N \cdot s^2}{kg \cdot m} = 6.39\ kPa\ (gage)$$

This pressure must be produced by a column of mercury $h+x$ in height. Thus, using SG_{Hg} from Table A.1,

$$p = \rho_{Hg}\, g (h+x) = SG_{Hg}\, \rho_{H_2O}\, g\, (h+x) = SG_{brass}\, \rho_{H_2O}\, g\, H$$

Thus $h + x = \frac{SG_{brass}}{SG_{Hg}} H = \frac{8.55}{13.55} H = 0.631\ H$ (1)

But the volume of mercury must remain constant. Therefore

$$\frac{\pi D^2}{4} x = \frac{\pi (d_1^2 - D^2)}{4} h + \frac{\pi d_2^2}{4} h \quad \text{or} \quad x\left[\left(\frac{d_1}{D}\right)^2 - 1 + \left(\frac{d_2}{D}\right)^2\right] = 0.224\ h$$

Substituting into Eq. 1,

$$h + x = h + 0.224\, h = 1.224\, h = 0.631\ H \quad \text{or} \quad h = \frac{0.631}{1.224} H = 0.516\ H$$