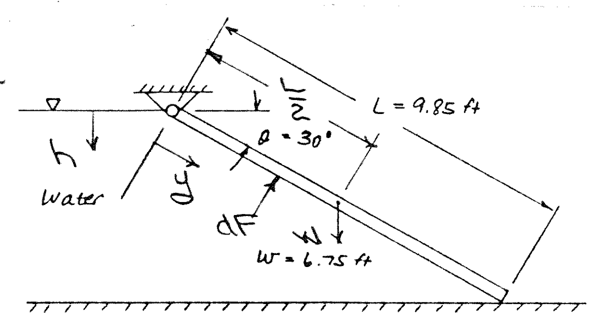
Problem 3.47

Given: Plane gate of uniform thickness and width $w = 6.75$ ft holds back a depth of water as shown.

Find: the minimum weight, W, of the gate needed to insure gate remains closed.



Solution:

Basic equations: $F = \int p\, dA$ $\qquad \frac{dp}{dh} = \rho g$

$\sum M_0 = 0 \qquad M = \int y\, dF$

Assumptions: (1) static fluid (2) ρ = constant (3) p_{atm} acts at surface of water and along top surface of the gate.

Under these assumptions, the pressure at any point in the liquid is given by $p = \rho g h = \rho g y \sin\theta$

$$\sum M_0 = 0 = \int y\, dF - W \frac{L}{2} \cos\theta \qquad \left\{ \begin{matrix} \text{moment about axis} \\ \text{through O is + cc} \end{matrix} \right\}$$

then

$$W = \frac{2}{L\cos\theta} \int y\, dF = \frac{2}{L\cos\theta} \int y\, p\, dA = \frac{2}{L\cos\theta} \int_0^L y\, \rho g y \sin\theta\, w\, dy$$

$$W = \frac{2 \rho g w \tan\theta}{L} \int_0^L y^2\, dy = \frac{2 \rho g w \tan\theta}{L} \left[\frac{y^3}{3} \right]_0^L$$

$$W = \frac{2}{3} \rho g w L^2 \tan\theta$$

$$W = \frac{2}{3} \times 1.94 \frac{\text{slug}}{\text{ft}^3} \times 32.2 \frac{\text{ft}}{\text{s}^2} \times 6.75\,\text{ft} \times (9.85)^2\,\text{ft}^2 \tan 30^\circ \times \frac{\text{lbf} \cdot \text{s}^2}{\text{ft} \cdot \text{slug}}$$

$W_{min} \approx 15{,}800$ lbf ← W_{min}