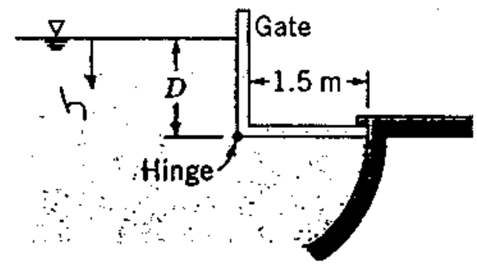
Problem 3.56

Given: Water rising on the left side of the gate causes it to open automatically. Neglect weight of gate.



Find: Depth, D, above the hinge at which the gate begins to open.

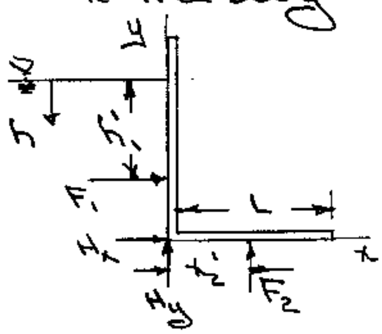
Solution:

Basic equations: $\frac{dp}{dh} = \rho g$; $\sum M_z = 0$

Computing equations: $F_R = p_c A$; $y' = y_c + \frac{I_{\hat{x}\hat{x}}}{y_c A}$; $I_{\hat{x}\hat{x}} = \frac{bD^3}{12}$

Assumptions: (1) static liquid (2) ρ = constant
(3) p_{atm} acts at free surface and on outside of gate
(4) no resisting moment in hinge.

Then on integrating $dp = \rho g\, dh$, we obtain $p = \rho g h$
The free body diagram of the gate is as shown.



F_1 is resultant of distributed force on vertical section
F_2 " " " uniform force on horizontal "

Let width of gate be b.

$F_1 = p_c A_1 = \rho g h_c bD = \rho g \frac{D}{2} bD = \frac{1}{2}\rho g b D^2$

$h_1' = h_{c_1} + \frac{bD^3}{12 h_c bD} = \frac{D}{2} + \frac{D^2}{12\frac{D}{2}} = \left(\frac{1}{2} + \frac{1}{6}\right)D = \frac{2}{3}D$

$F_2 = p_c A_2 = \rho g h_{c_2} bL = \rho g D b L$

Since the pressure is uniform over the horizontal surface, the force F_2 acts at the centroid of the surface, i.e. $x_2' = L/2$

Then summing moments about the hinge

$\sum M_{z_H} = 0 = F_2 x_2' - F_1 (D - h_1') = \rho g D b L \frac{L}{2} - \frac{1}{2}\rho g b D^2 \left(D - \frac{2}{3}D\right)$.

$\therefore \quad L^2 - \frac{D^2}{3} = 0$

$D = \sqrt{3}\,L = \sqrt{3} \times 1.5\,\text{m} = 2.60\,\text{m}$ ← D