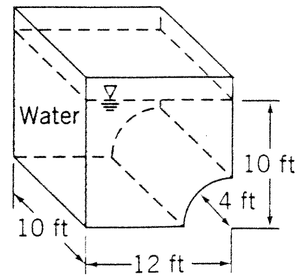
Problem 3.66



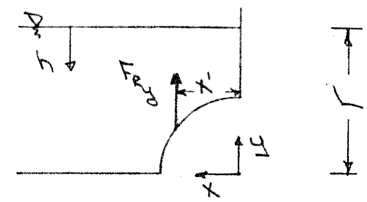
Given: Open tank as shown
width of curved surface $b = 10\,ft$

Find: (a) vertical force component, F_{R_y}, on curved surface
(b) line of action of F_{R_y}

Solution:

Basic equations: $\vec{F}_R = -\int P\,d\vec{A}$ $\quad \frac{dP}{dh} = \gamma$ $\quad \vec{r}' \times \vec{F}_R = \int \vec{r} \times d\vec{F} = -\int \vec{r} \times P\,d\vec{A}$

Assumptions: (1) static fluid
(2) gravity is only body force
(3) $\gamma = \text{constant} = 62.4\,lbf/ft^3$
(4) h is measured positive downward from free surface



$F_{R_y} = \vec{F}_R \cdot \hat{j} = -\int P\,d\vec{A} \cdot \hat{j} = -\int P\,dA_y = -\int P\,b\,dx$

We can obtain an expression for P as a function of y

$\frac{dP}{dh} = \gamma \quad dP = \gamma\,dh \quad P - P_0 = \int_{P_0}^{P} dP = \int_0^h \gamma\,dh = \gamma h$

Since atmospheric pressure acts at the free surface and on the underside of the curved surface, then the appropriate expression for P is $P = \gamma h$

Now, $h = L - y \quad \therefore P = \gamma(L - y)$

$F_{R_y} = -\int P\,b\,dx = -\int \gamma(L-y)\,b\,dx$. Along the surface $y = (R^2 - x^2)^{1/2}$ and so

$F_{R_y} = -\gamma b \int_0^R \{L - (R^2 - x^2)^{1/2}\}\,dx = -\gamma b \left[Lx - \frac{1}{2}\left(x\sqrt{R^2 - x^2} + R^2 \arcsin\frac{x}{R}\right)\right]_0^R$

$= -\gamma b \left\{LR - \frac{1}{2}(R^2 \arcsin 1) + \frac{1}{2}R^2 \arcsin 0\right\} = \gamma b R \left\{L - \frac{R}{2}\arcsin 1\right\}$

$= -\gamma b R \left\{L - R\frac{\pi}{4}\right\}$

$F_{R_y} = -62.4\,\frac{lbf}{ft^3} \times 10\,ft \times 4\,ft \times \left\{10\,ft - 4\,ft \times \frac{\pi}{4}\right\} = -17{,}100\,lbf$ (ie acts downward) $\qquad F_{R_y}$

$x'\hat{i} \times F_{R_y}\hat{j} = \int x\hat{i} \times dF_{R_y}\hat{j} = \int x\hat{i} \times (-P\,dA_y\hat{j}) = -\int x\hat{i} \times P\,b\,dx\,\hat{j}$

$x' F_{R_y}\hat{k} = -\hat{k}\int x P\,b\,dx$

$x' = -\frac{1}{F_{R_y}}\int_0^R x P\,b\,dx = -\frac{1}{F_{R_y}}\int_0^R x\,\gamma(L-y)\,b\,dx = -\frac{\gamma b}{F_{R_y}}\int_0^R x\{L - (R^2 - x^2)^{1/2}\}\,dx$

$= -\frac{\gamma b}{F_{R_y}}\left[L\frac{x^2}{2} + \frac{1}{3}\sqrt{(R^2 - x^2)^3}\right]_0^R = -\frac{\gamma b}{F_{R_y}}\left[L\frac{R^2}{2} - \frac{1}{3}R^3\right] = -\frac{\gamma b R^2}{F_{R_y}}\left[\frac{L}{2} - \frac{R}{3}\right]$

$x' = -62.4\,\frac{lbf}{ft^3} \times 10\,ft \times (4)^2\,ft^2 \times \frac{1}{(-17{,}100)\,lbf}\left[\frac{10\,ft}{2} - \frac{4\,ft}{3}\right]$

$x' = 2.14\,ft$ $\qquad x'$