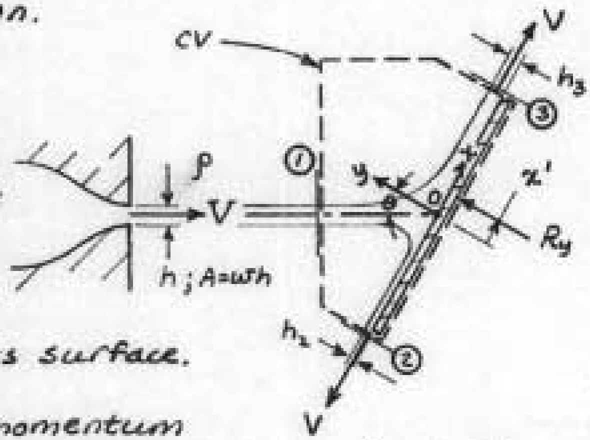
Given: Thin sheet of liquid, of width, w, and thickness, h, striking inclined flat plate, as shown.

Neglect any viscous effects.

Find: (a) Magnitude and line of action of resultant force as functions of θ.

(b) Equilibrium angle of plate if force is applied at point O, where jet centerline intersects surface.



Solution: Apply continuity, linear momentum and moment of momentum using CV and coordinates shown.

Basic equations:

$$0 = \cancelto{=0(1)}{\frac{\partial}{\partial t}\int_{CV}\rho\, d\forall} + \int_{CS}\rho\vec V\cdot d\vec A$$

$$\cancelto{=0(4)}{F_{S_x}} + \cancelto{=0(5)}{F_{B_x}} = \cancelto{=0(1)}{\frac{\partial}{\partial t}\int_{CV} u\rho\, d\forall} + \int_{CS} u\rho\vec V\cdot d\vec A$$

$$F_{S_y} + \cancelto{=0(5)}{F_{B_y}} = \cancelto{=0(1)}{\frac{\partial}{\partial t}\int_{CV} v\rho\, d\forall} + \int_{CS} v\rho\vec V\cdot d\vec A$$

$$\vec r\times\vec F_s + \cancelto{=0(5)}{\int_{CV}\vec r\times\vec g\,\rho\, d\forall} + \cancelto{=0(6)}{\vec T_{shaft}} = \cancelto{=0(1)}{\frac{\partial}{\partial t}\int_{CV}\vec r\times\vec V\rho\, d\forall} + \int_{CS}\vec r\times\vec V\rho\vec V\cdot d\vec A$$

Assumptions: (1) Steady flow
(2) Uniform flow at each section
(3) No net pressure forces; $F_{S_x} = R_x$, $F_{S_y} = R_y$
(4) No viscous effects; $R_x = 0$ and $V_1 = V_2 = V_3 = V$
(5) Neglect body forces and torques
(6) $\vec T_{shaft} = 0$
(7) Incompressible flow, ρ = constant

Then from continuity,

$$0 = \{-|\rho V w h_1|\} + \{|\rho V w h_2|\} + \{|\rho V w h_3|\} \quad \text{or} \quad h_1 = h_2 + h_3 = h \qquad (1)$$

From x momentum

$$0 = u_1\{-|\rho V w h_1|\} + u_2\{|\rho V w h_2|\} + u_3\{|\rho V w h_3|\}$$

$$u_1 = V\sin\theta \qquad u_2 = -V \qquad u_3 = V$$

$$0 = \rho V^2 w(-h_1\sin\theta - h_2 + h_3) \quad \text{or} \quad h_3 - h_2 = h_1\sin\theta = h\sin\theta \qquad (2)$$

Combining Eqs. 1 and 2,

$$h_2 = h\left(\frac{1-\sin\theta}{2}\right) \qquad (3)$$

$$h_3 = h\left(\frac{1+\sin\theta}{2}\right) \qquad (4)$$

From y momentum, $R_y = v_1\{-|\rho V w h_1|\} + v_2\{|\rho V w h_2|\} + v_3\{|\rho V w h_3|\}$

$$v_1 = -V\cos\theta \qquad v_2 = 0 \qquad v_3 = 0$$

$$R_y = \rho V^2 w h \cos\theta \qquad (5) \qquad R_y$$

From moment of momentum,

$$\vec{r}\,' \times \vec{F}_s = \vec{r}_1 \times \vec{V}_1\{-|\rho V w h_1|\} + \vec{r}_2 \times \vec{V}_2\{|\rho V w h_2|\} + \vec{r}_3 \times \vec{V}_3\{|\rho V w h_3|\}$$

$\vec{r}\,' = x'\hat{\imath}$ $\quad$ $\vec{r}_1 \times \vec{V}_1 = 0$ $\quad$ $\vec{r}_2 = \frac{h_2}{2}\hat{\jmath}$ $\quad$ $\vec{r}_3 = \frac{h_3}{2}\hat{\jmath}$

$\vec{F}_s = R_y\hat{\jmath}$ $\quad$ $\vec{V}_2 = -V\hat{\imath}$ $\quad$ $\vec{V}_3 = V\hat{\imath}$

$\vec{r}\,' \times \vec{F}_s = x' R_y \hat{k}$ $\quad$ $\vec{r}_2 \times \vec{V}_2 = \frac{h_2 V}{2}\hat{k}$ $\quad$ $\vec{r}_3 \times \vec{V}_3 = -\frac{h_3 V}{2}\hat{k}$

Combining and dropping $\hat{k}$,

$$x' R_y = \frac{1}{2}\rho V^2 w h_2^2 - \frac{1}{2}\rho V^2 w h_3^2 = \frac{1}{2}\rho V^2 w (h_2^2 - h_3^2)$$

or

$$x' = \frac{\rho V^2 w (h_2^2 - h_3^2)}{2R_y} = \frac{\rho V^2 w (h_2 + h_3)(h_2 - h_3)}{2R_y}$$

Substituting from Eqs. 3, 4 and 5,

$$x' = \frac{\rho V^2 w h^2 \left(\frac{1-\sin\theta}{2} + \frac{1+\sin\theta}{2}\right)\left(\frac{1-\sin\theta}{2} - \frac{1+\sin\theta}{2}\right)}{2\rho V^2 w h \cos\theta} = \frac{h(-\sin\theta)}{2\cos\theta}$$

or

$$x' = -\frac{h}{2}\tan\theta \qquad (6) \qquad x'$$

Note that $x' < 0$. This means that R_y must be applied <u>below</u> point 0.

If R_y is applied <u>at</u> point 0, then $x' = 0$. For equilibrium, from Eq. 6, $\theta = 0$. Thus if force is applied at point 0, plate will be in equilibrium when <u>perpendicular</u> to jet.