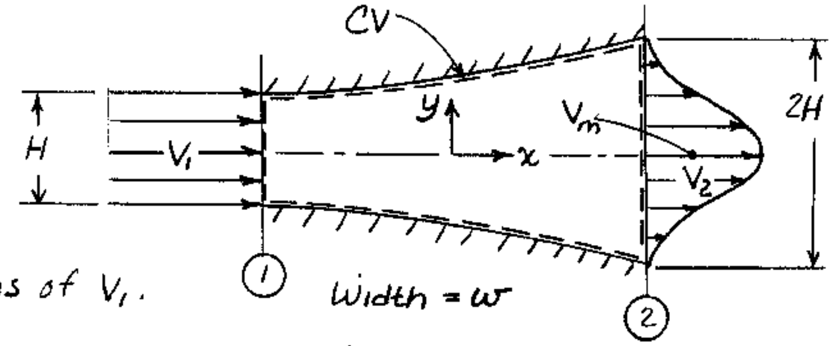
Problem 4.22

Given: Incompressible flow in a diverging channel, as shown.

$V_1 = \text{constant}$

$V_2 = V_m \cos\left(\frac{\pi y}{2H}\right)$



Find: Express V_m in terms of V_1.

Solution: Apply conservation of mass using the CV shown.

Basic equation: $$0 = \overset{=0(1)}{\cancel{\frac{\partial}{\partial t}\int_{CV}\rho\, d\forall}} + \int_{CS}\rho \vec{V}\cdot d\vec{A}$$

Assumptions: (1) Steady flow
(2) Uniform flow at section 1
(3) Incompressible flow

Then $$0 = \left\{-|\cancel{\rho} V_1 A_1|\right\} + \int_{-H}^{H}\cancel{\rho} V_2\, w\, dy$$

Since $A_1 = wH$, then $$V_1 w H = \int_{-H}^{H} V_m \cos\left(\frac{\pi}{2}\frac{y}{H}\right) w\, dy = 2\int_0^H V_m \cos\left(\frac{\pi}{2}\frac{y}{H}\right) w\, dy$$

So $$V_1 H = 2V_m\left(\frac{2H}{\pi}\right)\int_0^H \cos\left(\frac{\pi}{2}\frac{y}{H}\right) d\left(\frac{\pi}{2}\frac{y}{H}\right) = \frac{4V_m H}{\pi}\left[\sin\left(\frac{\pi}{2}\frac{y}{H}\right)\right]_0^H = \frac{4V_m H}{\pi}$$

Thus $$V_m = \frac{\pi}{4}V_1$$ V_m