

Problem 5.4

Given: Velocity profiles listed below.

Find: Which are possible three-dimensional, incompressible cases?

Solution: Apply the continuity equation in differential form.

Basic equation: $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$

Assumption: Incompressible flow

Field	Terms	Sum	Possible
(a) $u = x + y + z^2$ $v = x - y + z$ $w = 2xy + y^2 + 4$	$\frac{\partial u}{\partial x} = 1$ $\frac{\partial v}{\partial y} = -1$ $\frac{\partial w}{\partial z} = 0$	0	Yes
(b) $u = xyz t$ $v = -xyz t^2$ $w = \frac{z^4}{2} (x t^2 - y t)$	$\frac{\partial u}{\partial x} = y z t$ $\frac{\partial v}{\partial y} = -x z t^2$ $\frac{\partial w}{\partial z} = x z t^2 - y z t$	0	Yes
(c) $u = y^2 + 2xz$ $v = -2yz + x^2 y z$ $w = \frac{x^2 z^3}{2} + x^3 y^4$	$\frac{\partial u}{\partial x} = 2z$ $\frac{\partial v}{\partial y} = -2z + x^2 z$ $\frac{\partial w}{\partial z} = x^2 z$	$\neq 0$	No