Given: One-dimensional, unsteady flow in a thin liquid layer is described by the equation

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = -g\frac{\partial h}{\partial x}$$

Find: Nondimensionalize the equation (using length scale, L, and velocity scale, V_o)
Obtain the dimensionless groups that characterize this flow.

Solution:

To nondimensionalize the equation, all lengths are divided by the reference length, L, velocity is divided by the reference velocity, V_o, and time is divided by the ratio, L/V_o.

Denoting the nondimensional quantities by an asterisk,

$$x^* = \frac{x}{L}, \quad h^* = \frac{h}{L}, \quad u^* = \frac{u}{V_o}, \quad t^* = \frac{t}{L/V_o}$$

Substituting into the governing equation

$$\frac{\partial(V_o u^*)}{\partial(Lt^*/V_o)} + u^* V_o \frac{\partial(V_o u^*)}{\partial(x^* L)} = -g\frac{\partial(h^* L)}{\partial(x^* L)}$$

$$\frac{V_o^2}{L}\frac{\partial u^*}{\partial t^*} + \frac{V_o^2}{L}u^*\frac{\partial u^*}{\partial x^*} = -g\frac{\partial h^*}{\partial x^*}$$

Multiplying through by L/V_o^2,

$$\frac{\partial u^*}{\partial t^*} + u^*\frac{\partial u^*}{\partial x^*} = -\frac{gL}{V_o^2}\frac{\partial h^*}{\partial x^*}$$

The dimensionless group is $\frac{gL}{V_o^2}$. This is one over the square of the Froude number.