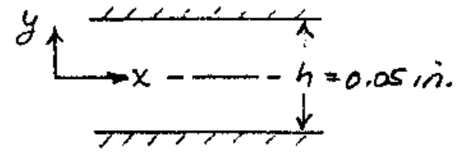
Problem 8.10

Given: Fully developed laminar flow between parallel plates.

$$\mu = 2.40 \times 10^{-5}\ \frac{\text{lbf}\cdot\text{s}}{\text{ft}^2}\ ;\ \frac{\partial p}{\partial x} = -4\ \frac{\text{lbf}}{\text{ft}^3}$$



Find: (a) Derive and plot equation for shear stress versus y.
(b) Maximum shear stress.

<u>Solution</u>: From Eq. 8.7, with $a = h$, $u = -\frac{h^2}{8\mu}\frac{\partial p}{\partial x}\left[1-\left(\frac{2y}{h}\right)^2\right]$

By symmetry, the origin for y must be located at the channel centerline. Apply Newton's law of viscosity.

$$\tau_{yx} = \mu \frac{du}{dy}$$

Assumption: Newtonian fluid

Then

$$\tau_{yx} = \mu \frac{d}{dy}\left\{-\frac{h^2}{8\mu}\frac{\partial p}{\partial x}\left[1-\left(\frac{2y}{h}\right)^2\right]\right\} = y\frac{\partial p}{\partial x}$$ τ_{yx}

For $u > 0$, $\partial p/\partial x < 0$. Thus $\tau_{yx} < 0$ for $y > 0$ and $\tau_{yx} > 0$ for $y < 0$.

On the upper plate (a minus y surface), $\tau_{yx} < 0$, so shear stress acts to the <u>right</u>.

On the lower plate (a plus y surface), $\tau_{yx} > 0$, so shear stress acts to the <u>right</u>.

The maximum stress occurs when $y = \pm h/2$. Thus

$$\tau_{max} = \tau_{yx}\left(\frac{h}{2}\right) = \frac{h}{2}\frac{\partial p}{\partial x} = \frac{1}{2} \times 0.05\,\text{in.} \times \frac{\text{ft}}{12\,\text{in.}} \times \left(-4.0\ \frac{\text{lbf}}{\text{ft}^3}\right) = -0.00835\ \frac{\text{lbf}}{\text{ft}^2}$$ τ_{ma}

or $\tau_{max} = \tau_{yx}\left(-\frac{h}{2}\right) = 0.00835\ \frac{\text{lbf}}{\text{ft}^2}$

Plot: